Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

5-Inverse-trig-functions/5.3-Inverse-tangent/148-5.3.2-d-x-^m-a+b-arctan-c-x^n-^p

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Contents

1 Introduction **2**

2 detailed summary tables of results **20**

3 Listing of integrals **73**

4 Appendix **1065**

CHAPTER 1

Introduction

1.1 Listing of CAS systems tested . . . 3
1.2 Results . . . 4
1.3 Time and leaf size Performance . . . 7
1.4 Performance based on number of rules Rubi used . . . 9
1.5 Performance based on number of steps Rubi used . . . 10
1.6 Solved integrals histogram based on leaf size of result . . . 11
1.7 Solved integrals histogram based on CPU time used . . . 12
1.8 Leaf size vs. CPU time used . . . 13
1.9 list of integrals with no known antiderivative . . . 14
1.10 List of integrals solved by CAS but has no known antiderivative . . . 14
1.11 list of integrals solved by CAS but failed verification . . . 14
1.12 Timing . . . 15
1.13 Verification . . . 15
1.14 Important notes about some of the results . . . 16
1.15 Design of the test system . . . 19

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [166]. This is test number [148].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (166)	0.00 (0)
Mathematica	98.19 (163)	1.81 (3)
Maple	90.96 (151)	9.04 (15)
Mupad	65.06 (108)	34.94 (58)
Sympy	57.83 (96)	42.17 (70)
Maxima	56.02 (93)	43.98 (73)
Fricas	55.42 (92)	44.58 (74)
Giac	51.20 (85)	48.80 (81)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

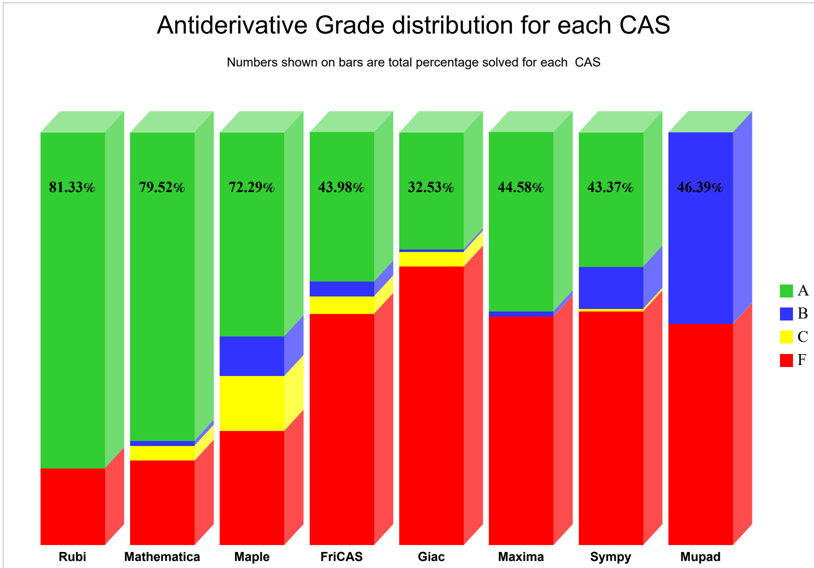
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

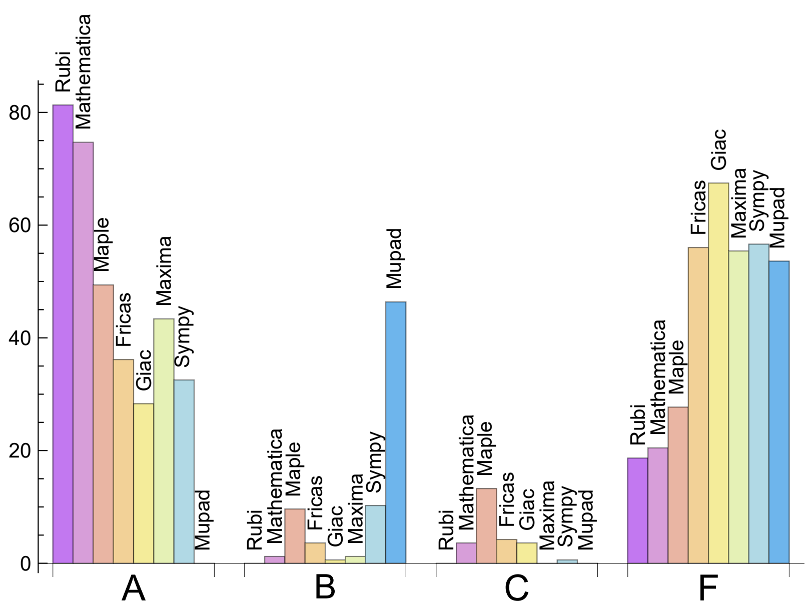
System	% A grade	% B grade	% C grade	% F grade
Rubi	80.723	0.602	0.000	18.675
Mathematica	74.699	1.205	3.614	20.482
Maple	49.398	9.639	13.253	27.711
Maxima	43.373	1.205	0.000	55.422
Fricas	36.145	3.614	4.217	56.024
Sympy	32.530	10.241	0.602	56.627
Giac	28.313	0.602	3.614	67.470
Mupad	0.000	46.386	0.000	53.614

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Maple	15	100.00	0.00	0.00
Mupad	58	0.00	100.00	0.00
Maxima	73	82.19	1.37	16.44
Fricas	74	83.78	0.00	16.22
Sympy	70	88.57	11.43	0.00
Giac	81	93.83	6.17	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.25
Maxima	0.45
Rubi	0.52
Mupad	0.56
Mathematica	0.83
Maple	3.02
Sympy	17.36
Giac	24.28

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	50.70	0.97	38.00	0.99
Giac	54.19	0.88	39.00	0.96
Fricas	73.03	1.16	45.50	1.08
Maxima	88.04	2.54	51.00	1.00
Sympy	101.60	1.48	58.00	1.09
Rubi	133.44	1.06	75.50	1.00
Mathematica	161.93	1.20	65.00	1.12
Maple	517.07	4.06	60.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

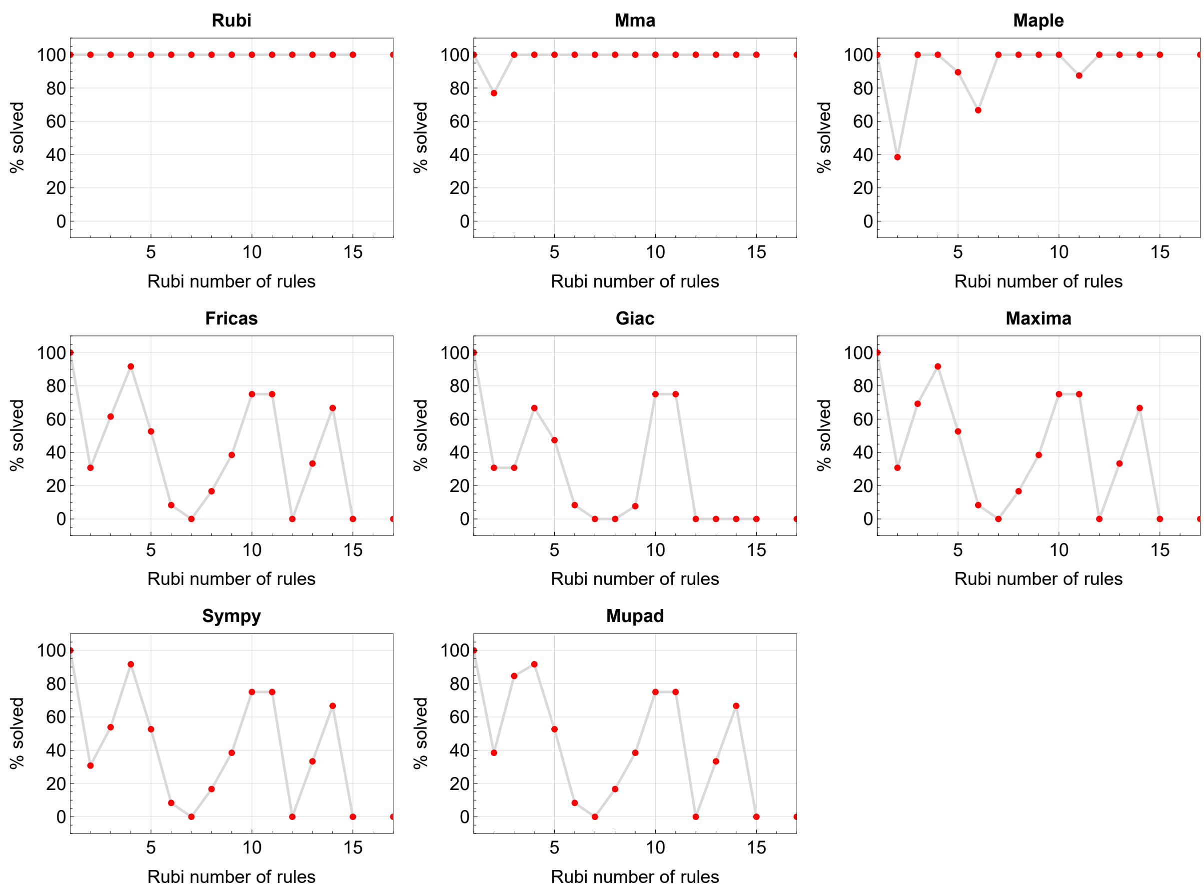


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

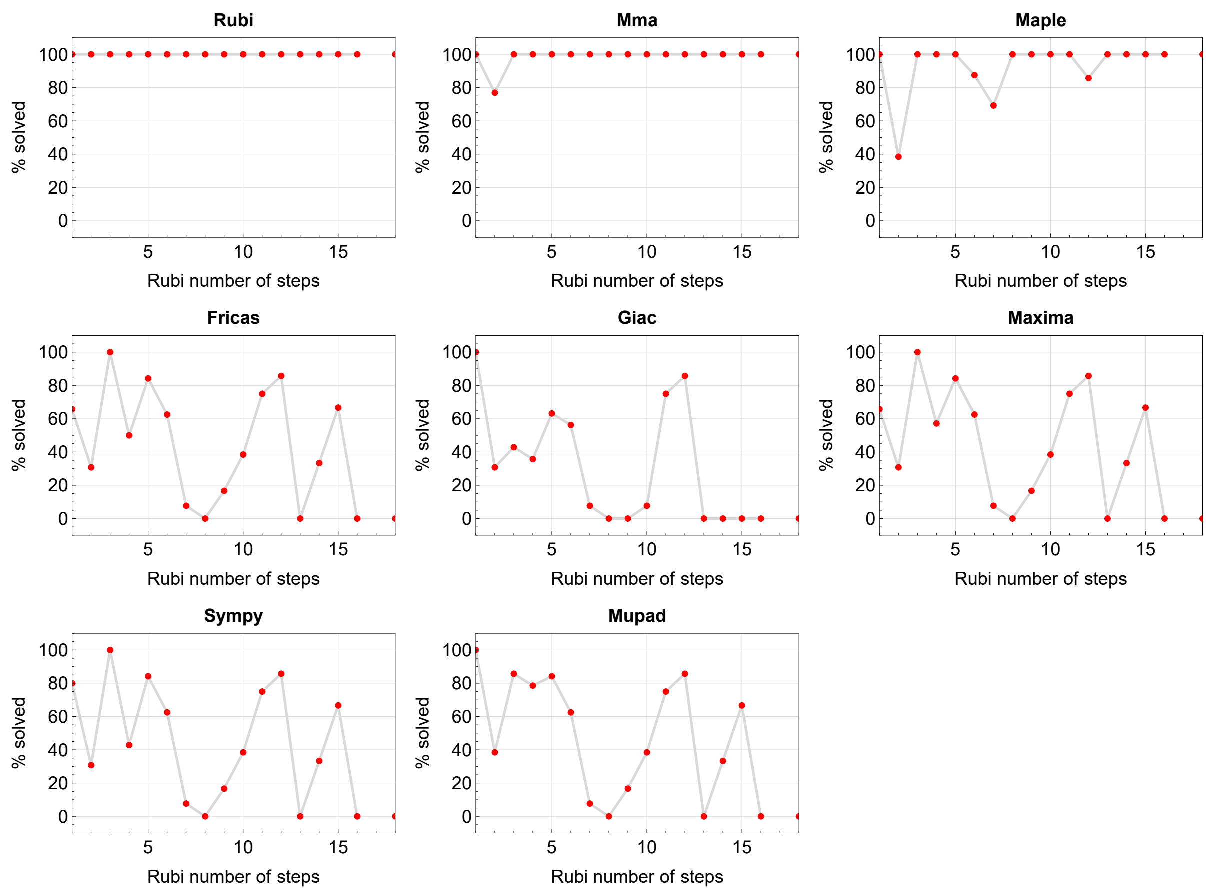


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

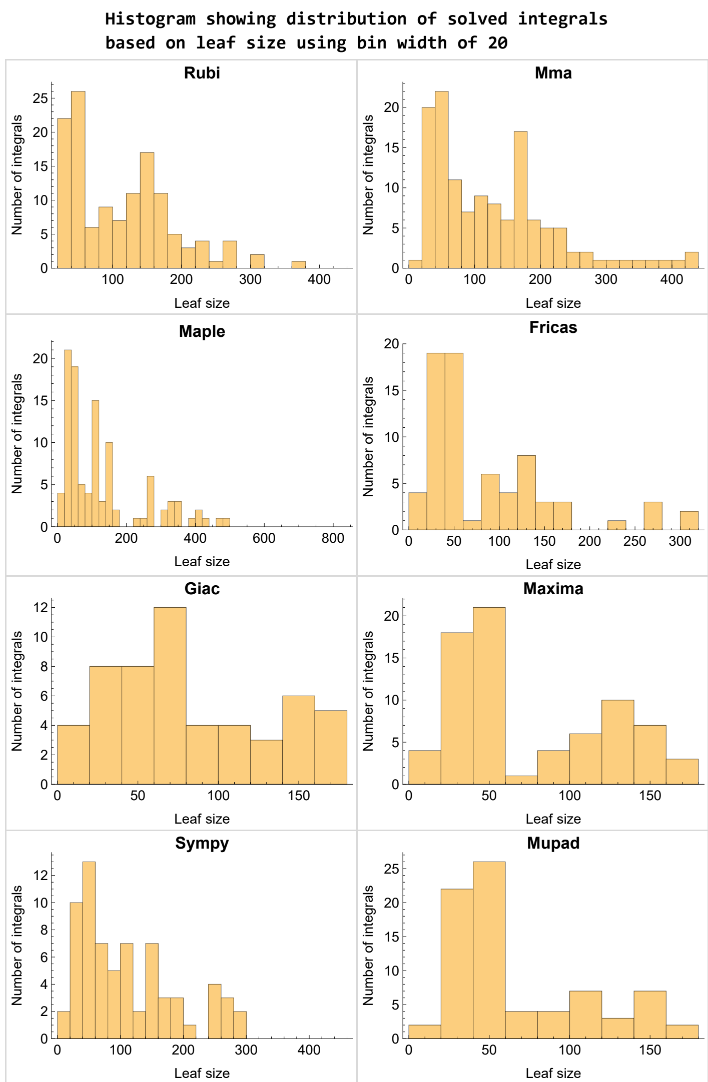


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

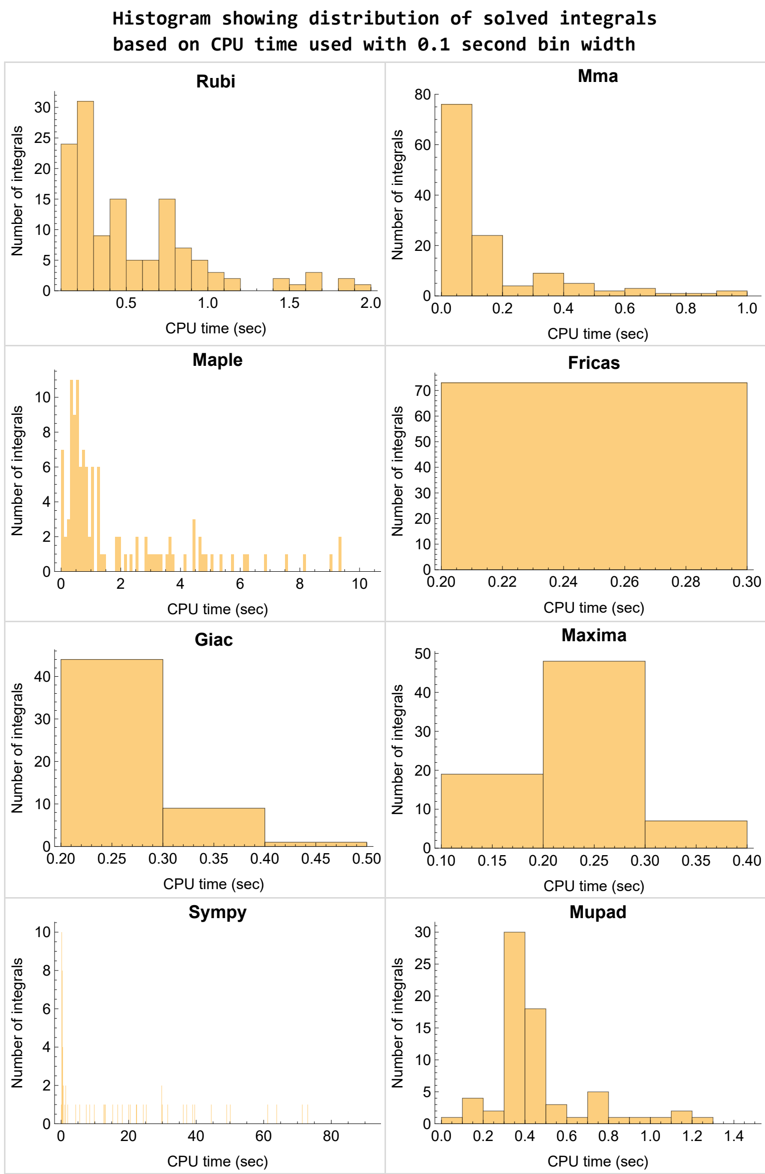


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

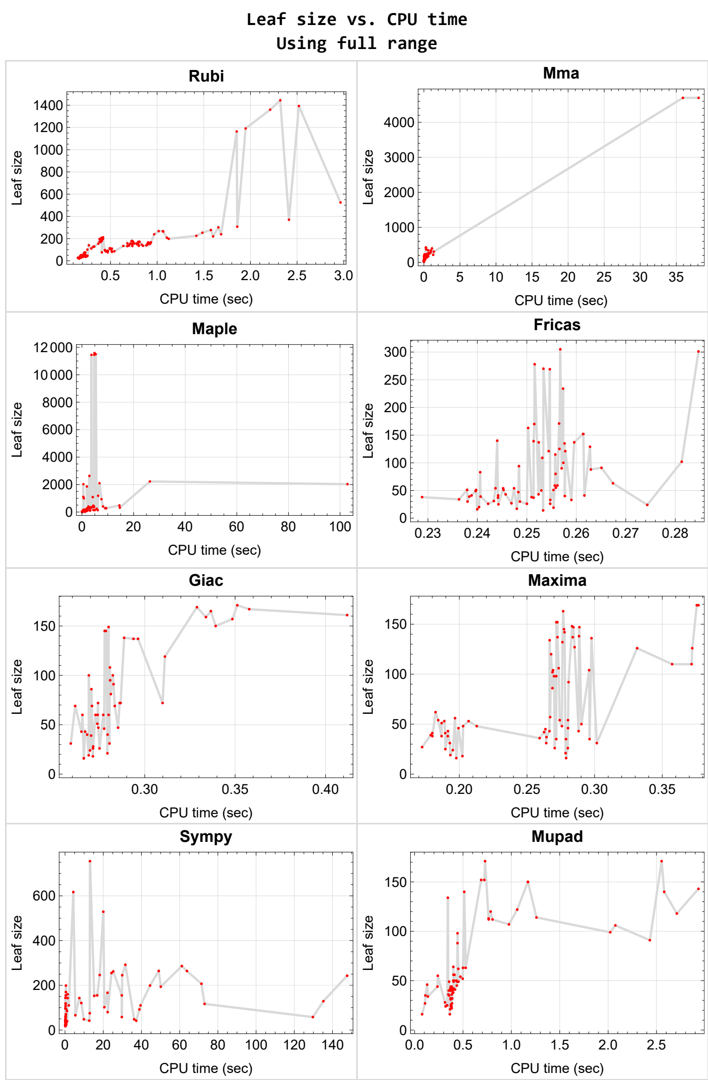


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 57, 58, 59, 91, 92, 94, 95, 127, 128, 130, 131}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {140}

Mathematica {82, 83}

Maple {19, 25, 27, 30, 31, 33, 75, 79, 86, 90, 114, 118, 120, 122, 126, 144, 148, 150, 151}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

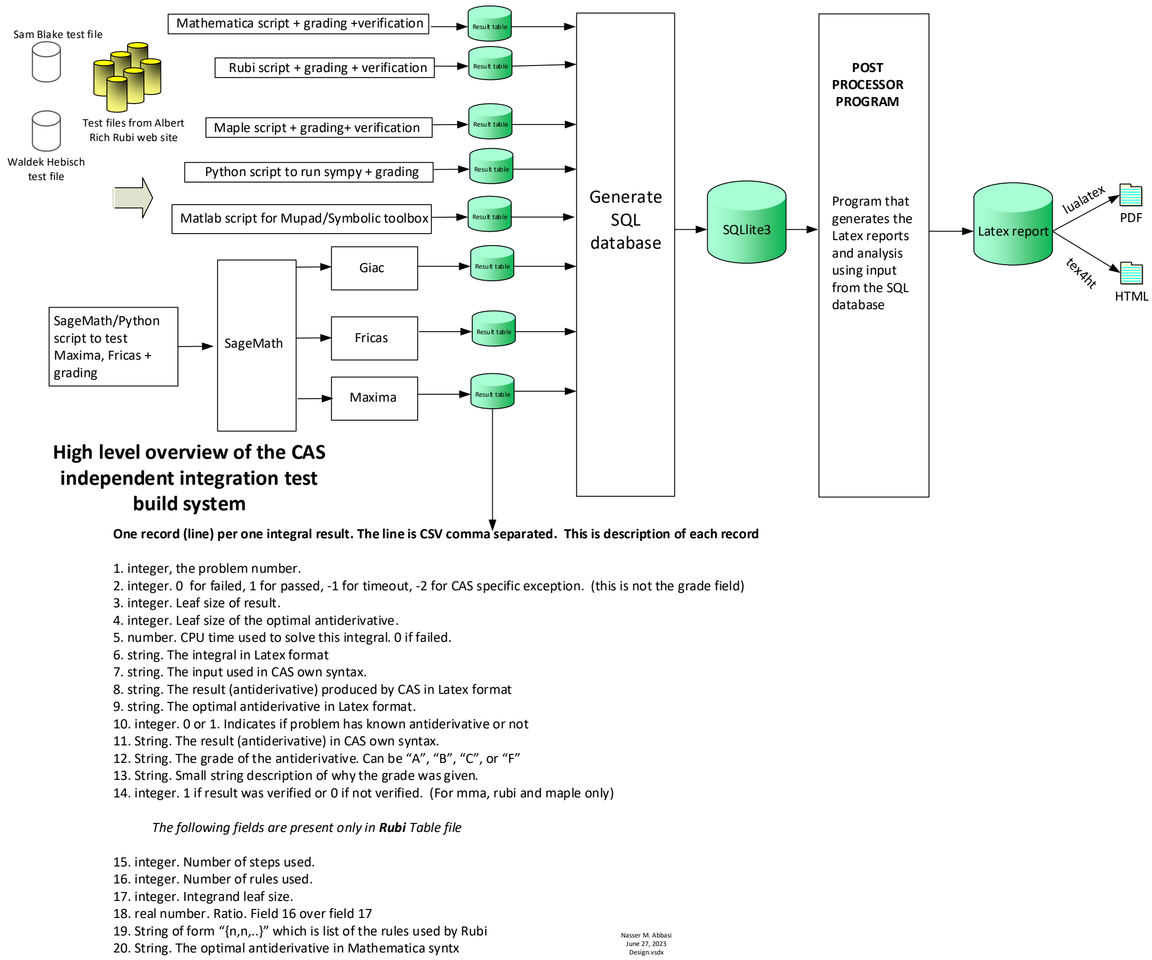
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 21
2.2 Detailed conclusion table per each integral for all CAS systems 25
2.3 Detailed conclusion table specific for Rubi results 67

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi 21
2.1.2 Mma 21
2.1.3 Maple 22
2.1.4 Fricas 22
2.1.5 Maxima 23
2.1.6 Giac 23
2.1.7 Mupad 23
2.1.8 Sympy 24

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 53, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166 }

B grade { 24 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 53, 56, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 86, 87, 88, 89, 90, 93, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166 }

B grade { 82, 83 }

C grade { 9, 11, 66, 102, 158, 159 }

F normal fail { 81, 84, 85 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 23, 24, 26, 53, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 80, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 119, 132, 133, 134, 135, 137, 138, 139, 140, 142, 145, 146, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164 }

B grade { 20, 22, 28, 29, 32, 34, 87, 123, 136, 141, 143, 147, 149, 152, 153, 166 }

C grade { 19, 25, 27, 30, 31, 33, 64, 75, 79, 86, 90, 100, 114, 118, 120, 122, 126, 144, 148, 150, 151, 165 }

F normal fail { 56, 78, 81, 82, 83, 84, 85, 88, 89, 93, 117, 121, 124, 125, 129 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 60, 61, 62, 63, 65, 66, 67, 74, 76, 80, 96, 97, 98, 99, 101, 102, 103, 105, 107, 109, 111, 113, 115, 119, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade { 104, 106, 108, 110, 112, 166 }

C grade { 53, 68, 69, 70, 71, 72, 73 }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165 }

F(-1) timedout fail { }

F(-2) exception fail { 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 119, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade { 157, 165 }

C grade { }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 166 }

F(-1) timedout fail { 34 }

F(-2) exception fail { 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52 }

2.1.6 Giac

A grade { 6, 53, 60, 61, 62, 63, 65, 67, 68, 69, 70, 71, 72, 73, 74, 76, 96, 97, 98, 99, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 133, 135, 137, 139, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade { 136 }

C grade { 66, 102, 132, 134, 138, 146 }

F normal fail { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 56, 64, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

F(-1) timedout fail { 30, 31, 32, 33, 34 }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 119, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 146, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165 }

C grade { }

F normal fail { }

F(-1) timedout fail { 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 161, 166 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 62, 65, 66, 67, 68, 69, 70, 71, 73, 74, 76, 96, 98, 102, 104, 105, 106, 108, 110, 112, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 161, 162, 163 }

B grade { 61, 63, 80, 97, 99, 101, 103, 107, 109, 111, 113, 115, 119, 158, 159, 160, 164 }

C grade { 72 }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

F(-1) timedout fail { 91, 94, 95, 127, 128, 129, 130, 131 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	57	64	52	57	54	63	0	52
N.S.	1	0.97	1.08	0.88	0.97	0.92	1.07	0.00	0.88
time (sec)	N/A	0.205	0.006	0.559	0.267	0.244	0.354	0.000	0.497

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	55	61	54	56	59	60	0	54
N.S.	1	0.98	1.09	0.96	1.00	1.05	1.07	0.00	0.96
time (sec)	N/A	0.219	0.019	0.388	0.197	0.256	0.303	0.000	0.474

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	47	53	44	48	47	53	0	44
N.S.	1	0.98	1.10	0.92	1.00	0.98	1.10	0.00	0.92
time (sec)	N/A	0.209	0.006	0.557	0.276	0.248	0.274	0.000	0.236

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	50	46	46	49	49	0	42
N.S.	1	1.00	1.11	1.02	1.02	1.09	1.09	0.00	0.93
time (sec)	N/A	0.211	0.012	0.383	0.199	0.240	0.251	0.000	0.404

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	36	37	34	42	0	34
N.S.	1	1.00	1.14	0.97	1.00	0.92	1.14	0.00	0.92
time (sec)	N/A	0.177	0.005	0.526	0.264	0.236	0.229	0.000	0.137

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	31	33	26	31	27
N.S.	1	1.00	1.00	0.97	1.07	1.14	0.90	1.07	0.93
time (sec)	N/A	0.149	0.004	0.313	0.193	0.255	0.108	0.259	0.107

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	34	0	0	0	0	28
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	0.80
time (sec)	N/A	0.199	0.005	0.480	0.000	0.000	0.000	0.000	0.314

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	37	38	39	39	37	37	0	36
N.S.	1	1.06	1.09	1.11	1.11	1.06	1.06	0.00	1.03
time (sec)	N/A	0.184	0.005	0.303	0.179	0.251	0.270	0.000	0.349

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	35	46	39	31	26	37	0	42
N.S.	1	0.95	1.24	1.05	0.84	0.70	1.00	0.00	1.14
time (sec)	N/A	0.178	0.006	0.481	0.264	0.242	0.229	0.000	0.370

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	51	54	52	51	50	61	0	46
N.S.	1	0.96	1.02	0.98	0.96	0.94	1.15	0.00	0.87
time (sec)	N/A	0.210	0.019	0.324	0.187	0.253	0.359	0.000	0.130

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	46	44	46	41	46	0	42
N.S.	1	1.00	0.96	0.92	0.96	0.85	0.96	0.00	0.88
time (sec)	N/A	0.190	0.005	0.500	0.280	0.239	0.289	0.000	0.396

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	60	69	60	62	59	71	0	56
N.S.	1	0.94	1.08	0.94	0.97	0.92	1.11	0.00	0.88
time (sec)	N/A	0.217	0.013	0.340	0.182	0.256	0.470	0.000	0.412

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	198	138	146	163	152	199	0	171
N.S.	1	1.38	0.96	1.01	1.13	1.06	1.38	0.00	1.19
time (sec)	N/A	1.112	0.137	1.002	0.277	0.261	0.478	0.000	0.728

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	208	169	266	0	0	0	0	0
N.S.	1	1.22	0.99	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.075	0.602	1.991	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	133	111	118	136	121	155	0	134
N.S.	1	1.19	0.99	1.05	1.21	1.08	1.38	0.00	1.20
time (sec)	N/A	0.711	0.100	1.024	0.298	0.254	0.383	0.000	0.343

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	151	131	238	0	0	0	0	0
N.S.	1	1.09	0.95	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.702	0.339	1.833	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	75	88	104	83	107	0	88
N.S.	1	1.00	0.99	1.16	1.37	1.09	1.41	0.00	1.16
time (sec)	N/A	0.383	0.161	0.973	0.296	0.241	0.274	0.000	0.444

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	97	90	123	0	0	0	0	0
N.S.	1	1.17	1.08	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.419	0.144	2.193	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	132	157	179	1002	0	0	0	0	0
N.S.	1	1.19	1.36	7.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.654	0.189	0.743	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	93	102	270	0	0	0	0	0
N.S.	1	1.13	1.24	3.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.197	2.348	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	76	90	98	98	94	119	0	140
N.S.	1	0.96	1.14	1.24	1.24	1.19	1.51	0.00	1.77
time (sec)	N/A	0.453	0.081	0.725	0.272	0.248	0.326	0.000	2.580

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	137	153	310	0	0	0	0	0
N.S.	1	0.98	1.09	2.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.672	0.473	2.908	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	135	128	132	152	135	170	0	171
N.S.	1	1.16	1.10	1.14	1.31	1.16	1.47	0.00	1.47
time (sec)	N/A	0.772	0.114	0.974	0.272	0.258	0.437	0.000	2.553

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	525	291	402	0	0	0	0	0
N.S.	1	2.06	1.14	1.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.900	0.953	2.524	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	271	370	396	1185	0	0	0	0	0
N.S.	1	1.37	1.46	4.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.384	1.124	6.276	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	307	225	340	0	0	0	0	0
N.S.	1	1.58	1.16	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.765	0.626	2.563	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	206	225	269	1088	0	0	0	0	0
N.S.	1	1.09	1.31	5.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.394	0.680	4.188	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	146	152	276	0	0	0	0	0
N.S.	1	1.11	1.16	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.809	0.427	2.888	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	134	192	240	0	0	0	0	0
N.S.	1	1.13	1.61	2.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.619	0.157	5.707	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	206	239	368	2026	0	0	0	0	0
N.S.	1	1.16	1.79	9.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.956	0.321	0.575	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	138	214	1862	0	0	0	0	0
N.S.	1	1.19	1.84	16.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.684	1.296	1.912	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	135	176	354	0	0	0	0	0
N.S.	1	1.02	1.32	2.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.829	0.376	4.621	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	213	220	305	2097	0	0	0	0	0
N.S.	1	1.03	1.43	9.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.603	1.383	6.803	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	277	265	423	0	0	0	0	0
N.S.	1	1.40	1.34	2.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.541	0.864	4.497	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	8	10	10	7	3	10
N.S.	1	1.00	1.25	1.00	1.25	1.25	0.88	0.38	1.25
time (sec)	N/A	0.154	0.742	10.581	0.255	0.231	0.269	23.919	0.293

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6	6	8	6	8	8	7	3	8
N.S.	1	1.00	1.33	1.00	1.33	1.33	1.17	0.50	1.33
time (sec)	N/A	0.148	0.020	4.118	0.231	0.250	0.257	21.736	0.291

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	3	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	0.30	1.20
time (sec)	N/A	0.162	0.514	4.589	0.238	0.233	0.368	24.442	0.274

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	8	45	10	8	3	10
N.S.	1	1.00	1.25	1.00	5.62	1.25	1.00	0.38	1.25
time (sec)	N/A	0.156	0.715	10.509	0.258	0.234	0.310	49.107	0.294

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6	6	8	6	39	8	8	3	8
N.S.	1	1.00	1.33	1.00	6.50	1.33	1.33	0.50	1.33
time (sec)	N/A	0.147	1.095	5.540	0.234	0.251	0.298	47.802	0.300

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	51	12	10	3	12
N.S.	1	1.00	1.20	1.00	5.10	1.20	1.00	0.30	1.20
time (sec)	N/A	0.164	1.238	4.234	0.265	0.245	0.375	49.411	0.299

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	0	0	10	3	10
N.S.	1	1.00	1.20	0.80	0.00	0.00	1.00	0.30	1.00
time (sec)	N/A	0.152	1.483	2.086	0.000	0.000	0.400	53.543	0.307

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	6	0	0	8	3	8
N.S.	1	1.00	1.25	0.75	0.00	0.00	1.00	0.38	1.00
time (sec)	N/A	0.149	1.812	1.641	0.000	0.000	0.315	53.952	0.326

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	10	3	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	0.83	0.25	1.00
time (sec)	N/A	0.161	1.102	1.947	0.000	0.000	0.361	148.199	0.293

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	0	0	10	3	10
N.S.	1	1.00	1.20	0.80	0.00	0.00	1.00	0.30	1.00
time (sec)	N/A	0.154	0.839	1.792	0.000	0.000	1.406	81.002	0.301

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	6	0	0	8	3	8
N.S.	1	1.00	1.25	0.75	0.00	0.00	1.00	0.38	1.00
time (sec)	N/A	0.147	1.711	1.527	0.000	0.000	0.852	81.253	0.323

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	10	3	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	0.83	0.25	1.00
time (sec)	N/A	0.172	0.957	2.726	0.000	0.000	0.858	142.203	0.316

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	0	0	10	3	10
N.S.	1	1.00	1.20	0.80	0.00	0.00	1.00	0.30	1.00
time (sec)	N/A	0.155	0.935	1.751	0.000	0.000	0.364	70.975	0.292

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	6	0	0	10	3	8
N.S.	1	1.00	1.25	0.75	0.00	0.00	1.25	0.38	1.00
time (sec)	N/A	0.148	0.006	1.588	0.000	0.000	0.324	64.900	0.313

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	3	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	0.25	1.00
time (sec)	N/A	0.159	1.304	2.352	0.000	0.000	0.476	68.146	0.296

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	0	0	10	3	10
N.S.	1	1.00	1.20	0.80	0.00	0.00	1.00	0.30	1.00
time (sec)	N/A	0.158	1.035	1.943	0.000	0.000	0.670	249.366	0.329

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	6	0	0	10	3	8
N.S.	1	1.00	1.25	0.75	0.00	0.00	1.25	0.38	1.00
time (sec)	N/A	0.147	1.735	1.520	0.000	0.000	0.638	239.105	0.351

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	3	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	0.25	1.00
time (sec)	N/A	0.158	2.338	2.635	0.000	0.000	0.909	184.339	0.354

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	128	108	69	86	80	110	86	49
N.S.	1	1.09	0.92	0.59	0.74	0.68	0.94	0.74	0.42
time (sec)	N/A	0.316	0.035	0.253	0.269	0.256	1.953	0.271	0.357

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	387	44	15	3	18
N.S.	1	1.00	1.12	1.00	24.19	2.75	0.94	0.19	1.12
time (sec)	N/A	0.182	4.832	2.559	3.735	0.261	7.445	72.748	0.650

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	295	30	15	3	18
N.S.	1	1.00	1.12	1.00	18.44	1.88	0.94	0.19	1.12
time (sec)	N/A	0.182	3.147	3.820	2.202	0.248	4.351	72.162	0.622

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	3	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	0.19	1.12
time (sec)	N/A	0.179	0.346	3.526	0.268	0.245	1.260	87.381	0.273

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	3	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	0.30	1.20
time (sec)	N/A	0.156	0.499	1.693	0.371	0.239	1.814	80.207	0.406

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	3	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	0.19	1.12
time (sec)	N/A	0.179	0.418	4.359	0.538	0.250	155.449	79.272	0.406

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	53	59	50	54	51	58	60	49
N.S.	1	0.98	1.09	0.93	1.00	0.94	1.07	1.11	0.91
time (sec)	N/A	0.227	0.012	0.583	0.280	0.245	29.725	0.274	0.403

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	45	48	51	80	47	44
N.S.	1	1.00	1.11	0.96	1.02	1.09	1.70	1.00	0.94
time (sec)	N/A	0.218	0.017	0.661	0.213	0.255	22.305	0.274	0.374

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	43	38	48	43	40
N.S.	1	1.00	1.12	0.95	1.00	0.88	1.12	1.00	0.93
time (sec)	N/A	0.198	0.008	0.734	0.288	0.229	9.849	0.267	0.353

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	41	36	38	39	66	40	35
N.S.	1	1.00	1.14	1.00	1.06	1.08	1.83	1.11	0.97
time (sec)	N/A	0.179	0.024	0.536	0.191	0.238	5.410	0.279	0.357

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	45	39	63	0	0	0	0	32
N.S.	1	1.15	1.00	1.62	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.243	0.008	0.747	0.000	0.000	0.000	0.000	0.389

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	41	44	39	41	43	75	60	38
N.S.	1	1.05	1.13	1.00	1.05	1.10	1.92	1.54	0.97
time (sec)	N/A	0.190	0.010	0.266	0.190	0.252	12.925	0.273	0.395

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	39	48	39	35	30	42	72	41
N.S.	1	0.95	1.17	0.95	0.85	0.73	1.02	1.76	1.00
time (sec)	N/A	0.193	0.009	0.375	0.272	0.238	12.686	0.286	0.406

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	53	60	53	53	54	92	69	50
N.S.	1	0.96	1.09	0.96	0.96	0.98	1.67	1.25	0.91
time (sec)	N/A	0.223	0.016	0.303	0.189	0.247	38.959	0.262	0.408

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	180	179	121	147	171	153	169	64
N.S.	1	1.12	1.11	0.75	0.91	1.06	0.95	1.05	0.40
time (sec)	N/A	0.389	0.068	0.869	0.289	0.257	15.270	0.329	0.400

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	169	177	118	145	152	143	165	62
N.S.	1	1.06	1.11	0.74	0.91	0.96	0.90	1.04	0.39
time (sec)	N/A	0.382	0.056	0.535	0.277	0.261	7.490	0.337	0.452

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	107	103	127	140	617	149	49
N.S.	1	1.00	0.76	0.74	0.91	1.00	4.41	1.06	0.35
time (sec)	N/A	0.264	0.061	0.358	0.285	0.244	4.336	0.280	0.453

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	154	158	107	132	139	121	138	55
N.S.	1	1.08	1.10	0.75	0.92	0.97	0.85	0.97	0.38
time (sec)	N/A	0.360	0.074	0.349	0.276	0.251	8.534	0.289	0.240

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	175	177	115	142	170	529	159	63
N.S.	1	1.10	1.11	0.72	0.89	1.07	3.33	1.00	0.40
time (sec)	N/A	0.378	0.085	0.417	0.278	0.252	20.038	0.334	0.500

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	171	177	118	138	163	155	150	63
N.S.	1	1.08	1.11	0.74	0.87	1.03	0.97	0.94	0.40
time (sec)	N/A	0.375	0.086	0.468	0.288	0.250	29.701	0.339	0.528

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	149	121	151	169	137	199	145	150
N.S.	1	1.20	0.98	1.22	1.36	1.10	1.60	1.17	1.21
time (sec)	N/A	0.773	0.120	0.855	0.377	0.252	44.486	0.279	1.170

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	171	141	333	0	0	0	0	0
N.S.	1	1.11	0.92	2.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.792	0.364	3.561	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	85	110	126	100	155	100	112
N.S.	1	1.00	0.94	1.22	1.40	1.11	1.72	1.11	1.24
time (sec)	N/A	0.448	0.083	1.290	0.331	0.257	16.832	0.269	0.805

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	113	107	140	0	0	0	0	0
N.S.	1	1.12	1.06	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.475	0.152	4.438	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	177	201	0	0	0	0	0	0
N.S.	1	1.17	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.717	0.172	0.000	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	97	107	127	339	0	0	0	0	0
N.S.	1	1.10	1.31	3.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.490	0.194	3.328	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	86	98	118	110	115	167	0	152
N.S.	1	0.99	1.13	1.36	1.26	1.32	1.92	0.00	1.75
time (sec)	N/A	0.521	0.107	0.651	0.357	0.256	22.225	0.000	0.688

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1393	1393	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.556	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1191	1191	4697	0	0	0	0	0	0
N.S.	1	1.00	3.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.917	38.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1164	1164	4697	0	0	0	0	0	0
N.S.	1	1.00	4.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.856	35.915	0.000	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1360	1360	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.265	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1444	1444	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.337	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	149	166	170	399	0	0	0	0	0
N.S.	1	1.11	1.14	2.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.897	0.512	4.665	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	154	224	275	0	0	0	0	0
N.S.	1	1.07	1.56	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.695	0.134	9.388	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	267	417	0	0	0	0	0	0
N.S.	1	1.17	1.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.040	0.301	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	156	239	0	0	0	0	0	0
N.S.	1	1.13	1.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.762	0.469	0.000	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	149	153	196	465	0	0	0	0	0
N.S.	1	1.03	1.32	3.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.882	0.369	4.421	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	406	50	0	20	20
N.S.	1	1.00	1.11	1.00	22.56	2.78	0.00	1.11	1.11
time (sec)	N/A	0.179	2.263	0.171	3.974	0.244	0.000	0.423	0.336

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	303	34	17	20	20
N.S.	1	1.00	1.11	1.00	16.83	1.89	0.94	1.11	1.11
time (sec)	N/A	0.183	1.467	0.176	2.349	0.247	111.972	0.521	0.320

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	0	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.11
time (sec)	N/A	0.178	0.457	0.188	0.240	0.247	0.000	0.278	0.283

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	124	36	0	20	20
N.S.	1	1.00	1.11	1.00	6.89	2.00	0.00	1.11	1.11
time (sec)	N/A	0.181	0.478	0.164	0.394	0.238	0.000	0.300	0.351

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	53	59	50	54	51	58	60	49
N.S.	1	0.98	1.09	0.93	1.00	0.94	1.07	1.11	0.91
time (sec)	N/A	0.218	0.012	0.750	0.274	0.238	129.726	0.266	0.439

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	45	48	51	117	47	44
N.S.	1	1.00	1.11	0.96	1.02	1.09	2.49	1.00	0.94
time (sec)	N/A	0.226	0.019	0.688	0.203	0.240	73.035	0.285	0.384

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	43	38	48	43	40
N.S.	1	1.00	1.12	0.95	1.00	0.88	1.12	1.00	0.93
time (sec)	N/A	0.204	0.010	0.816	0.267	0.251	36.158	0.265	0.384

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	41	36	38	39	102	40	35
N.S.	1	1.00	1.14	1.00	1.06	1.08	2.83	1.11	0.97
time (sec)	N/A	0.181	0.023	0.589	0.180	0.241	20.583	0.268	0.111

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	45	39	63	0	0	0	0	32
N.S.	1	1.15	1.00	1.62	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.243	0.009	0.517	0.000	0.000	0.000	0.000	0.375

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	41	44	39	41	43	110	60	38
N.S.	1	1.05	1.13	1.00	1.05	1.10	2.82	1.54	0.97
time (sec)	N/A	0.185	0.010	0.260	0.180	0.246	39.528	0.280	0.399

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	39	48	39	35	30	42	72	41
N.S.	1	0.95	1.17	0.95	0.85	0.73	1.02	1.76	1.00
time (sec)	N/A	0.186	0.010	0.437	0.296	0.249	37.249	0.287	0.418

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	53	60	53	53	54	129	69	50
N.S.	1	0.96	1.09	0.96	0.96	0.98	2.35	1.25	0.91
time (sec)	N/A	0.213	0.019	0.410	0.207	0.245	135.233	0.283	0.433

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	197	179	155	148	269	255	167	114
N.S.	1	1.13	1.03	0.89	0.85	1.55	1.47	0.96	0.66
time (sec)	N/A	0.414	0.080	0.804	0.283	0.255	24.304	0.358	1.259

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	131	98	92	234	755	95	91
N.S.	1	1.00	1.30	0.97	0.91	2.32	7.48	0.94	0.90
time (sec)	N/A	0.255	0.047	0.418	0.281	0.257	13.066	0.281	2.432

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	186	170	144	137	270	245	137	107
N.S.	1	1.13	1.03	0.87	0.83	1.64	1.48	0.83	0.65
time (sec)	N/A	0.394	0.087	0.485	0.284	0.253	29.934	0.296	0.975

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	129	183	105	102	121	286	108	118
N.S.	1	1.12	1.59	0.91	0.89	1.05	2.49	0.94	1.03
time (sec)	N/A	0.315	0.067	0.765	0.269	0.258	61.135	0.281	2.710

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	212	181	167	152	305	264	171	122
N.S.	1	1.20	1.03	0.95	0.86	1.73	1.50	0.97	0.69
time (sec)	N/A	0.415	0.115	1.270	0.273	0.257	63.808	0.351	1.060

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	126	185	113	106	137	292	119	106
N.S.	1	1.08	1.58	0.97	0.91	1.17	2.50	1.02	0.91
time (sec)	N/A	0.306	0.050	0.844	0.273	0.260	31.576	0.311	2.075

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	196	170	153	137	278	246	157	113
N.S.	1	1.19	1.03	0.93	0.83	1.68	1.49	0.95	0.68
time (sec)	N/A	0.382	0.056	0.661	0.273	0.252	18.140	0.349	0.764

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	111	170	104	98	90	262	91	99
N.S.	1	1.07	1.63	1.00	0.94	0.87	2.52	0.88	0.95
time (sec)	N/A	0.290	0.074	0.494	0.270	0.257	25.230	0.283	2.022

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	207	179	164	147	301	264	161	120
N.S.	1	1.19	1.03	0.94	0.84	1.73	1.52	0.93	0.69
time (sec)	N/A	0.403	0.075	0.638	0.284	0.285	49.046	0.412	0.786

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	149	121	151	169	129	243	145	150
N.S.	1	1.20	0.98	1.22	1.36	1.04	1.96	1.17	1.21
time (sec)	N/A	0.779	0.111	1.043	0.376	0.263	147.668	0.278	1.172

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	171	141	11449	0	0	0	0	0
N.S.	1	1.11	0.92	74.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.774	0.117	3.690	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	85	110	126	91	194	100	112
N.S.	1	1.00	0.94	1.22	1.40	1.01	2.16	1.11	1.24
time (sec)	N/A	0.451	0.073	1.474	0.372	0.265	50.123	0.282	0.766

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	113	107	140	0	0	0	0	0
N.S.	1	1.09	1.03	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.481	0.092	5.020	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	177	201	0	0	0	0	0	0
N.S.	1	1.15	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.696	0.182	0.000	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	100	107	125	11455	0	0	0	0	0
N.S.	1	1.07	1.25	114.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.488	0.194	4.777	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	86	98	118	110	102	207	0	152
N.S.	1	0.99	1.13	1.36	1.26	1.17	2.38	0.00	1.75
time (sec)	N/A	0.504	0.101	0.785	0.372	0.281	71.423	0.000	0.721

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	155	167	11496	0	0	0	0	0
N.S.	1	1.01	1.08	74.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.740	0.458	5.340	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	253	346	0	0	0	0	0	0
N.S.	1	1.05	1.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.435	0.591	0.000	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	147	166	170	935	0	0	0	0	0
N.S.	1	1.13	1.16	6.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.883	0.145	7.574	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	154	224	275	0	0	0	0	0
N.S.	1	1.11	1.61	1.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.704	0.077	9.008	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	232	267	421	0	0	0	0	0	0
N.S.	1	1.15	1.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.007	0.288	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	156	240	0	0	0	0	0	0
N.S.	1	1.17	1.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.741	0.447	0.000	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	146	153	196	11581	0	0	0	0	0
N.S.	1	1.05	1.34	79.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.881	0.372	4.884	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	407	50	0	20	20
N.S.	1	1.00	1.11	1.00	22.61	2.78	0.00	1.11	1.11
time (sec)	N/A	0.178	2.233	0.194	4.058	0.261	0.000	0.435	0.354

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	304	34	0	20	20
N.S.	1	1.00	1.11	1.00	16.89	1.89	0.00	1.11	1.11
time (sec)	N/A	0.179	1.528	0.186	2.368	0.271	0.000	0.532	0.338

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	0.093	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	0	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.11
time (sec)	N/A	0.176	0.483	0.178	0.248	0.259	0.000	0.288	0.303

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	132	36	0	20	20
N.S.	1	1.00	1.11	1.00	7.33	2.00	0.00	1.11	1.11
time (sec)	N/A	0.178	0.495	0.187	0.385	0.240	0.000	0.282	0.375

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	48	55	46	45	41	46	81	45
N.S.	1	0.96	1.10	0.92	0.90	0.82	0.92	1.62	0.90
time (sec)	N/A	0.210	0.028	1.278	0.264	0.262	0.142	0.281	0.436

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	48	47	43	40	41	69	40
N.S.	1	0.95	1.12	1.09	1.00	0.93	0.95	1.60	0.93
time (sec)	N/A	0.207	0.022	1.010	0.192	0.258	0.121	0.271	0.368

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	35	44	37	36	31	36	72	36
N.S.	1	0.90	1.13	0.95	0.92	0.79	0.92	1.85	0.92
time (sec)	N/A	0.181	0.022	1.250	0.259	0.243	0.108	0.274	0.384

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	27	27	25	22	46	25
N.S.	1	1.00	1.00	1.00	1.00	0.93	0.81	1.70	0.93
time (sec)	N/A	0.144	0.004	1.223	0.172	0.244	0.081	0.278	0.337

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	42	39	87	0	0	0	72	32
N.S.	1	1.08	1.00	2.23	0.00	0.00	0.00	1.85	0.82
time (sec)	N/A	0.242	0.010	1.332	0.000	0.000	0.000	0.310	0.372

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	37	36	38	41	36	39	43
N.S.	1	1.00	1.09	1.06	1.12	1.21	1.06	1.15	1.26
time (sec)	N/A	0.179	0.025	0.694	0.187	0.244	0.278	0.270	0.375

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	44	48	41	42	37	44	60	50
N.S.	1	1.02	1.12	0.95	0.98	0.86	1.02	1.40	1.16
time (sec)	N/A	0.194	0.025	1.233	0.263	0.244	0.298	0.277	0.400

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	54	60	45	54	55	60	51	56
N.S.	1	0.98	1.09	0.82	0.98	1.00	1.09	0.93	1.02
time (sec)	N/A	0.222	0.026	1.057	0.184	0.256	0.355	0.274	0.401

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	139	111	141	134	125	144	0	140
N.S.	1	1.14	0.91	1.16	1.10	1.02	1.18	0.00	1.15
time (sec)	N/A	0.846	0.098	3.218	0.267	0.257	0.202	0.000	0.513

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	149	152	358	0	0	0	0	0
N.S.	1	0.98	1.00	2.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.764	0.399	3.776	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	81	73	105	104	88	97	0	98
N.S.	1	0.99	0.89	1.28	1.27	1.07	1.18	0.00	1.20
time (sec)	N/A	0.511	0.059	3.170	0.269	0.263	0.156	0.000	0.444

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	89	105	308	0	0	0	0	0
N.S.	1	1.07	1.27	3.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.457	0.128	3.619	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	173	203	1106	0	0	0	0	0
N.S.	1	1.17	1.37	7.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.731	0.166	0.549	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	110	107	142	0	0	0	0	0
N.S.	1	1.15	1.11	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.497	0.115	6.113	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	85	99	103	120	109	117	137	143
N.S.	1	1.01	1.18	1.23	1.43	1.30	1.39	1.63	1.70
time (sec)	N/A	0.434	0.091	3.085	0.268	0.253	0.337	0.294	2.934

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	301	253	480	0	0	0	0	0
N.S.	1	1.41	1.18	2.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.600	0.726	14.518	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	239	336	2634	0	0	0	0	0
N.S.	1	1.04	1.47	11.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.640	0.961	2.888	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	147	174	405	0	0	0	0	0
N.S.	1	1.01	1.20	2.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.874	0.293	8.159	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	119	132	215	2028	0	0	0	0	0
N.S.	1	1.11	1.81	17.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.660	0.277	102.875	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	230	263	422	2225	0	0	0	0	0
N.S.	1	1.14	1.83	9.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.033	0.274	26.327	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	151	222	275	0	0	0	0	0
N.S.	1	1.11	1.63	2.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.713	0.126	9.309	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	162	178	321	0	0	0	0	0
N.S.	1	1.10	1.21	2.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.914	0.319	14.681	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	34	30	31	27	39	31	31
N.S.	1	1.02	0.67	0.59	0.61	0.53	0.76	0.61	0.61
time (sec)	N/A	0.180	0.017	0.162	0.302	0.247	1.131	0.281	0.373

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	43	28	25	26	20	32	26	26
N.S.	1	1.02	0.67	0.60	0.62	0.48	0.76	0.62	0.62
time (sec)	N/A	0.169	0.014	0.039	0.280	0.240	0.632	0.275	0.377

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	29	18	17	16	14	19	16	16
N.S.	1	1.32	0.82	0.77	0.73	0.64	0.86	0.73	0.73
time (sec)	N/A	0.155	0.021	0.031	0.279	0.253	0.396	0.266	0.078

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	37	31	24	35	0	0	0	24
N.S.	1	1.19	1.00	0.77	1.13	0.00	0.00	0.00	0.77
time (sec)	N/A	0.222	0.007	1.046	0.279	0.000	0.000	0.000	0.321

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	32	30	19	21	17	94	21	21
N.S.	1	1.19	1.11	0.70	0.78	0.63	3.48	0.78	0.78
time (sec)	N/A	0.168	0.012	0.037	0.279	0.248	0.557	0.279	0.366

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	43	34	27	26	26	160	26	24
N.S.	1	1.02	0.81	0.64	0.62	0.62	3.81	0.62	0.57
time (sec)	N/A	0.172	0.014	0.045	0.270	0.250	1.346	0.271	0.375

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	35	30	25	24	24	85	24	24
N.S.	1	0.97	0.83	0.69	0.67	0.67	2.36	0.67	0.67
time (sec)	N/A	0.178	0.019	0.036	0.195	0.274	1.010	0.270	0.381

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	28	25	20	19	19	24	19	0
N.S.	1	0.97	0.86	0.69	0.66	0.66	0.83	0.66	0.00
time (sec)	N/A	0.177	0.013	0.036	0.193	0.255	0.606	0.269	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	17	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.85	0.80	0.80
time (sec)	N/A	0.158	0.009	0.055	0.198	0.240	0.136	0.266	0.364

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	20	18	22
N.S.	1	1.00	1.00	0.86	0.82	1.18	0.91	0.82	1.00
time (sec)	N/A	0.159	0.015	0.196	0.202	0.255	0.404	0.271	0.383

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	34	31	26	25	33	143	28	27
N.S.	1	0.92	0.84	0.70	0.68	0.89	3.86	0.76	0.73
time (sec)	N/A	0.178	0.023	0.310	0.190	0.259	1.355	0.271	0.387

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	37	33	57	50	0	0	0	25
N.S.	1	1.12	1.00	1.73	1.52	0.00	0.00	0.00	0.76
time (sec)	N/A	0.226	0.010	0.882	0.290	0.000	0.000	0.000	0.334

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	37	32	75	0	63	0	0	0
N.S.	1	0.95	0.82	1.92	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.235	0.016	1.899	0.000	0.267	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [53] had the largest ratio of [1.37500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	0.97	12	0.250
2	A	5	4	0.98	12	0.333
3	A	3	3	0.98	12	0.250
4	A	5	4	1.00	12	0.333
5	A	3	3	1.00	10	0.300
6	A	1	1	1.00	8	0.125
7	A	2	2	1.00	12	0.167
8	A	6	5	1.06	12	0.417
9	A	3	3	0.95	12	0.250
10	A	5	4	0.96	12	0.333
11	A	4	4	1.00	12	0.333
12	A	5	4	0.94	12	0.333
13	A	15	14	1.38	14	1.000
14	A	14	13	1.22	14	0.929
15	A	10	9	1.19	14	0.643
16	A	10	9	1.09	14	0.643
17	A	4	4	1.00	12	0.333
18	A	6	5	1.17	10	0.500
19	A	4	4	1.19	14	0.286
20	A	4	4	1.13	14	0.286
21	A	9	8	0.96	14	0.571
22	A	8	8	0.98	14	0.571

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	14	13	1.16	14	0.929
24	B	18	17	2.06	14	1.214
25	A	16	15	1.37	14	1.071
26	A	14	13	1.58	14	0.929
27	A	10	10	1.09	14	0.714
28	A	9	8	1.11	12	0.667
29	A	5	5	1.13	10	0.500
30	A	5	5	1.16	14	0.357
31	A	5	5	1.19	14	0.357
32	A	7	7	1.02	14	0.500
33	A	15	14	1.03	14	1.000
34	A	11	11	1.40	14	0.786
35	N/A	1	0	1.00	8	0.000
36	N/A	1	0	1.00	6	0.000
37	N/A	1	0	1.00	10	0.000
38	N/A	1	0	1.00	8	0.000
39	N/A	1	0	1.00	6	0.000
40	N/A	1	0	1.00	10	0.000
41	N/A	1	0	1.00	10	0.000
42	N/A	1	0	1.00	8	0.000
43	N/A	1	0	1.00	12	0.000
44	N/A	1	0	1.00	10	0.000
45	N/A	1	0	1.00	8	0.000
46	N/A	1	0	1.00	12	0.000
47	N/A	1	0	1.00	10	0.000
48	N/A	1	0	1.00	8	0.000
49	N/A	1	0	1.00	12	0.000
50	N/A	1	0	1.00	10	0.000
51	N/A	1	0	1.00	8	0.000
52	N/A	1	0	1.00	12	0.000
53	A	12	11	1.09	8	1.375

Continued on next page

2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	N/A	1	0	1.00	16	0.000
55	N/A	1	0	1.00	16	0.000
56	A	2	2	1.00	14	0.143
57	N/A	1	0	1.00	16	0.000
58	N/A	1	0	1.00	10	0.000
59	N/A	1	0	1.00	16	0.000
60	A	5	4	0.98	14	0.286
61	A	5	4	1.00	14	0.286
62	A	5	4	1.00	14	0.286
63	A	2	2	1.00	12	0.167
64	A	4	3	1.15	14	0.214
65	A	6	5	1.05	14	0.357
66	A	5	4	0.95	14	0.286
67	A	5	4	0.96	14	0.286
68	A	11	10	1.12	14	0.714
69	A	11	10	1.06	14	0.714
70	A	1	1	1.00	10	0.100
71	A	10	9	1.08	14	0.643
72	A	11	10	1.10	14	0.714
73	A	11	10	1.08	14	0.714
74	A	11	10	1.20	16	0.625
75	A	11	10	1.11	16	0.625
76	A	6	5	1.00	16	0.312
77	A	7	6	1.12	14	0.429
78	A	6	5	1.17	16	0.312
79	A	6	5	1.10	16	0.312
80	A	10	9	0.99	16	0.562
81	A	2	2	1.00	16	0.125
82	A	2	2	1.00	12	0.167
83	A	2	2	1.00	16	0.125
84	A	2	2	1.00	16	0.125
85	A	2	2	1.00	16	0.125
86	A	10	9	1.11	16	0.562

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
87	A	7	6	1.07	14	0.429
88	A	7	6	1.17	16	0.375
89	A	7	6	1.13	16	0.375
90	A	9	8	1.03	16	0.500
91	N/A	1	0	1.00	18	0.000
92	N/A	1	0	1.00	18	0.000
93	A	2	2	1.00	16	0.125
94	N/A	1	0	1.00	18	0.000
95	N/A	1	0	1.00	18	0.000
96	A	5	4	0.98	14	0.286
97	A	5	4	1.00	14	0.286
98	A	5	4	1.00	14	0.286
99	A	2	2	1.00	14	0.143
100	A	4	3	1.15	14	0.214
101	A	6	5	1.05	14	0.357
102	A	5	4	0.95	14	0.286
103	A	5	4	0.96	14	0.286
104	A	12	11	1.13	14	0.786
105	A	1	1	1.00	10	0.100
106	A	11	10	1.13	14	0.714
107	A	12	11	1.12	14	0.786
108	A	12	11	1.20	14	0.786
109	A	12	11	1.08	14	0.786
110	A	11	10	1.19	12	0.833
111	A	11	10	1.07	14	0.714
112	A	12	11	1.19	14	0.786
113	A	11	10	1.20	16	0.625
114	A	11	10	1.11	16	0.625
115	A	6	5	1.00	16	0.312
116	A	7	6	1.09	16	0.375
117	A	6	5	1.15	16	0.312
118	A	6	5	1.07	16	0.312
119	A	10	9	0.99	16	0.562

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	10	9	1.01	16	0.562
121	A	12	11	1.05	16	0.688
122	A	10	9	1.13	16	0.562
123	A	7	6	1.11	16	0.375
124	A	7	6	1.15	16	0.375
125	A	7	6	1.17	16	0.375
126	A	9	8	1.05	16	0.500
127	N/A	1	0	1.00	18	0.000
128	N/A	1	0	1.00	18	0.000
129	A	2	2	1.00	16	0.125
130	N/A	1	0	1.00	18	0.000
131	N/A	1	0	1.00	18	0.000
132	A	4	4	0.96	14	0.286
133	A	6	5	0.95	14	0.357
134	A	4	4	0.90	12	0.333
135	A	1	1	1.00	10	0.100
136	A	4	3	1.08	14	0.214
137	A	2	2	1.00	14	0.143
138	A	4	4	1.02	14	0.286
139	A	6	5	0.98	14	0.357
140	A	15	14	1.14	16	0.875
141	A	10	9	0.98	16	0.562
142	A	10	9	0.99	14	0.643
143	A	10	9	1.07	12	0.750
144	A	6	5	1.17	16	0.312
145	A	7	6	1.15	16	0.375
146	A	6	5	1.01	16	0.312
147	A	13	12	1.41	16	0.750
148	A	16	15	1.04	16	0.938
149	A	9	8	1.01	14	0.571
150	A	9	9	1.11	12	0.750
151	A	7	6	1.14	16	0.375
152	A	7	6	1.11	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	A	10	9	1.10	16	0.562
154	A	7	6	1.02	10	0.600
155	A	6	5	1.02	8	0.625
156	A	5	4	1.32	6	0.667
157	A	4	3	1.19	10	0.300
158	A	5	4	1.19	10	0.400
159	A	6	5	1.02	10	0.500
160	A	3	3	0.97	12	0.250
161	A	3	3	0.97	12	0.250
162	A	2	2	1.00	12	0.167
163	A	4	4	1.00	12	0.333
164	A	3	3	0.92	12	0.250
165	A	4	3	1.12	10	0.300
166	A	4	3	0.95	10	0.300

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int x^5(a+b\arctan(cx))\,dx$ . . . 78
3.2 $\int x^4(a+b\arctan(cx))\,dx$ . . . 83
3.3 $\int x^3(a+b\arctan(cx))\,dx$ . . . 88
3.4 $\int x^2(a+b\arctan(cx))\,dx$ . . . 93
3.5 $\int x(a+b\arctan(cx))\,dx$ . . . 98
3.6 $\int (a+b\arctan(cx))\,dx$ . . . 103
3.7 $\int \frac{a+b\arctan(cx)}{x}\,dx$ . . . 107
3.8 $\int \frac{a+b\arctan(cx)}{x^2}\,dx$ . . . 111
3.9 $\int \frac{a+b\arctan(cx)}{x^3}\,dx$ . . . 116
3.10 $\int \frac{a+b\arctan(cx)}{x^4}\,dx$ . . . 121
3.11 $\int \frac{a+b\arctan(cx)}{x^5}\,dx$ . . . 126
3.12 $\int \frac{a+b\arctan(cx)}{x^6}\,dx$ . . . 131
3.13 $\int x^5(a+b\arctan(cx))^2\,dx$ . . . 136
3.14 $\int x^4(a+b\arctan(cx))^2\,dx$ . . . 144
3.15 $\int x^3(a+b\arctan(cx))^2\,dx$ . . . 152
3.16 $\int x^2(a+b\arctan(cx))^2\,dx$ . . . 159
3.17 $\int x(a+b\arctan(cx))^2\,dx$ . . . 166
3.18 $\int (a+b\arctan(cx))^2\,dx$ . . . 172
3.19 $\int \frac{(a+b\arctan(cx))^2}{x}\,dx$ . . . 177
3.20 $\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx$ . . . 183
3.21 $\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx$ . . . 189
3.22 $\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx$ . . . 196
3.23 $\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$ . . . 203
3.24 $\int x^5(a+b\arctan(cx))^3\,dx$ . . . 211
3.25 $\int x^4(a+b\arctan(cx))^3\,dx$ . . . 222
3.26 $\int x^3(a+b\arctan(cx))^3\,dx$ . . . 232
3.27 $\int x^2(a+b\arctan(cx))^3\,dx$ . . . 242
3.28 $\int x(a+b\arctan(cx))^3\,dx$ . . . 250

3.29 $\int (a+b\arctan(cx))^3\,dx$ . . . 257
3.30 $\int \frac{(a+b\arctan(cx))^3}{x}\,dx$ . . . 263
3.31 $\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx$ . . . 270
3.32 $\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx$ . . . 277
3.33 $\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx$ . . . 283
3.34 $\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx$ . . . 292
3.35 $\int \frac{x}{\arctan(ax)}\,dx$ . . . 300
3.36 $\int \frac{1}{\arctan(ax)}\,dx$ . . . 304
3.37 $\int \frac{1}{x\arctan(ax)}\,dx$ . . . 308
3.38 $\int \frac{x}{\arctan(ax)^2}\,dx$ . . . 312
3.39 $\int \frac{1}{\arctan(ax)^2}\,dx$ . . . 316
3.40 $\int \frac{1}{x\arctan(ax)^2}\,dx$ . . . 320
3.41 $\int x\sqrt{\arctan(ax)}\,dx$ . . . 324
3.42 $\int \sqrt{\arctan(ax)}\,dx$ . . . 328
3.43 $\int \frac{\sqrt{\arctan(ax)}}{x}\,dx$ . . . 332
3.44 $\int x\arctan(ax)^{3/2}\,dx$ . . . 336
3.45 $\int \arctan(ax)^{3/2}\,dx$ . . . 340
3.46 $\int \frac{\arctan(ax)^{3/2}}{x}\,dx$ . . . 344
3.47 $\int \frac{x}{\sqrt{\arctan(ax)}}\,dx$ . . . 348
3.48 $\int \frac{1}{\sqrt{\arctan(ax)}}\,dx$ . . . 352
3.49 $\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$ . . . 356
3.50 $\int \frac{x}{\arctan(ax)^{3/2}}\,dx$ . . . 360
3.51 $\int \frac{1}{\arctan(ax)^{3/2}}\,dx$ . . . 364
3.52 $\int \frac{1}{x\arctan(ax)^{3/2}}\,dx$ . . . 368
3.53 $\int \sqrt{x}\arctan(x)\,dx$ . . . 372
3.54 $\int (dx)^m(a+b\arctan(cx))^3\,dx$ . . . 380
3.55 $\int (dx)^m(a+b\arctan(cx))^2\,dx$ . . . 384
3.56 $\int (dx)^m(a+b\arctan(cx))\,dx$ . . . 388
3.57 $\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$ . . . 392
3.58 $\int (a+b\arctan(cx))^p\,dx$ . . . 396
3.59 $\int (dx)^m(a+b\arctan(cx))^p\,dx$ . . . 400
3.60 $\int x^7(a+b\arctan(cx^2))\,dx$ . . . 404
3.61 $\int x^5(a+b\arctan(cx^2))\,dx$ . . . 409
3.62 $\int x^3(a+b\arctan(cx^2))\,dx$ . . . 414
3.63 $\int x(a+b\arctan(cx^2))\,dx$ . . . 419
3.64 $\int \frac{a+b\arctan(cx^2)}{x}\,dx$ . . . 424
3.65 $\int \frac{a+b\arctan(cx^2)}{x^3}\,dx$ . . . 429
3.66 $\int \frac{a+b\arctan(cx^2)}{x^5}\,dx$ . . . 434

3.67 $\int \frac{a+b\arctan(cx^2)}{x^7}\,dx$ . . . 439
3.68 $\int x^4(a+b\arctan(cx^2))\,dx$ . . . 444
3.69 $\int x^2(a+b\arctan(cx^2))\,dx$ . . . 453
3.70 $\int (a+b\arctan(cx^2))\,dx$ . . . 462
3.71 $\int \frac{a+b\arctan(cx^2)}{x^2}\,dx$ . . . 468
3.72 $\int \frac{a+b\arctan(cx^2)}{x^4}\,dx$ . . . 476
3.73 $\int \frac{a+b\arctan(cx^2)}{x^6}\,dx$ . . . 485
3.74 $\int x^7(a+b\arctan(cx^2))^2\,dx$ . . . 494
3.75 $\int x^5(a+b\arctan(cx^2))^2\,dx$ . . . 501
3.76 $\int x^3(a+b\arctan(cx^2))^2\,dx$ . . . 508
3.77 $\int x(a+b\arctan(cx^2))^2\,dx$ . . . 514
3.78 $\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx$ . . . 520
3.79 $\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx$ . . . 526
3.80 $\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx$ . . . 532
3.81 $\int x^2(a+b\arctan(cx^2))^2\,dx$ . . . 539
3.82 $\int (a+b\arctan(cx^2))^2\,dx$ . . . 545
3.83 $\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx$ . . . 552
3.84 $\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx$ . . . 559
3.85 $\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx$ . . . 565
3.86 $\int x^3(a+b\arctan(cx^2))^3\,dx$ . . . 571
3.87 $\int x(a+b\arctan(cx^2))^3\,dx$ . . . 579
3.88 $\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx$ . . . 585
3.89 $\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx$ . . . 592
3.90 $\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx$ . . . 598
3.91 $\int (dx)^m(a+b\arctan(cx^2))^3\,dx$ . . . 605
3.92 $\int (dx)^m(a+b\arctan(cx^2))^2\,dx$ . . . 609
3.93 $\int (dx)^m(a+b\arctan(cx^2))\,dx$ . . . 613
3.94 $\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx$ . . . 617
3.95 $\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx$ . . . 621
3.96 $\int x^{11}(a+b\arctan(cx^3))\,dx$ . . . 625
3.97 $\int x^8(a+b\arctan(cx^3))\,dx$ . . . 630
3.98 $\int x^5(a+b\arctan(cx^3))\,dx$ . . . 635
3.99 $\int x^2(a+b\arctan(cx^3))\,dx$ . . . 640
3.100 $\int \frac{a+b\arctan(cx^3)}{x}\,dx$ . . . 645
3.101 $\int \frac{a+b\arctan(cx^3)}{x^4}\,dx$ . . . 650
3.102 $\int \frac{a+b\arctan(cx^3)}{x^7}\,dx$ . . . 655
3.103 $\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx$ . . . 660

3.104 $\int x^3(a+b\arctan(cx^3))\,dx$ . . . 665
3.105 $\int (a+b\arctan(cx^3))\,dx$ . . . 675
3.106 $\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$ . . . 681
3.107 $\int \frac{a+b\arctan(cx^3)}{x^6}\,dx$ . . . 690
3.108 $\int x^7(a+b\arctan(cx^3))\,dx$ . . . 699
3.109 $\int x^4(a+b\arctan(cx^3))\,dx$ . . . 710
3.110 $\int x(a+b\arctan(cx^3))\,dx$ . . . 719
3.111 $\int \frac{a+b\arctan(cx^3)}{x^2}\,dx$ . . . 729
3.112 $\int \frac{a+b\arctan(cx^3)}{x^5}\,dx$ . . . 737
3.113 $\int x^{11}(a+b\arctan(cx^3))^2\,dx$ . . . 747
3.114 $\int x^8(a+b\arctan(cx^3))^2\,dx$ . . . 754
3.115 $\int x^5(a+b\arctan(cx^3))^2\,dx$ . . . 761
3.116 $\int x^2(a+b\arctan(cx^3))^2\,dx$ . . . 767
3.117 $\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx$ . . . 773
3.118 $\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx$ . . . 779
3.119 $\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx$ . . . 784
3.120 $\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx$ . . . 791
3.121 $\int x^8(a+b\arctan(cx^3))^3\,dx$ . . . 798
3.122 $\int x^5(a+b\arctan(cx^3))^3\,dx$ . . . 806
3.123 $\int x^2(a+b\arctan(cx^3))^3\,dx$ . . . 814
3.124 $\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx$ . . . 820
3.125 $\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx$ . . . 827
3.126 $\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx$ . . . 833
3.127 $\int (dx)^m(a+b\arctan(cx^3))^3\,dx$ . . . 839
3.128 $\int (dx)^m(a+b\arctan(cx^3))^2\,dx$ . . . 843
3.129 $\int (dx)^m(a+b\arctan(cx^3))\,dx$ . . . 847
3.130 $\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx$ . . . 851
3.131 $\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx$ . . . 855
3.132 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$ . . . 859
3.133 $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$ . . . 864
3.134 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$ . . . 869
3.135 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$ . . . 874
3.136 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx$ . . . 878
3.137 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx$ . . . 883
3.138 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx$ . . . 888
3.139 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx$ . . . 893
3.140 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$ . . . 898

3.141 $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$ 906
3.142 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$ 913
3.143 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$ 920
3.144 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx$ 927
3.145 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$ 933
3.146 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$ 939
3.147 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$ 945
3.148 $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$ 954
3.149 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$ 963
3.150 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$ 970
3.151 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx$ 978
3.152 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$ 986
3.153 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$ 993
3.154 $\int x^2\arctan\left(\sqrt{x}\right)\,dx$ 1001
3.155 $\int x\arctan\left(\sqrt{x}\right)\,dx$ 1006
3.156 $\int \arctan\left(\sqrt{x}\right)\,dx$ 1011
3.157 $\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx$ 1016
3.158 $\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx$ 1021
3.159 $\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx$ 1026
3.160 $\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx$ 1031
3.161 $\int \sqrt{x}\arctan\left(\sqrt{x}\right)\,dx$ 1036
3.162 $\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$ 1041
3.163 $\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx$ 1045
3.164 $\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx$ 1050
3.165 $\int \frac{\arctan\left(ax^5\right)}{x}\,dx$ 1055
3.166 $\int \frac{\arctan\left(ax^n\right)}{x}\,dx$ 1060

3.1 $\int x^5(a+b\arctan(cx))\,dx$

3.1.1 Optimal result 78
3.1.2 Mathematica [A] (verified) 78
3.1.3 Rubi [A] (verified) 79
3.1.4 Maple [A] (verified) 80
3.1.5 Fricas [A] (verification not implemented) 80
3.1.6 Sympy [A] (verification not implemented) 81
3.1.7 Maxima [A] (verification not implemented) 81
3.1.8 Giac [F] 81
3.1.9 Mupad [B] (verification not implemented) 82

3.1.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^5(a+b\arctan(cx))\,dx=-\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{b\arctan(cx)}{6c^6}+\frac{1}{6}x^6(a+b\arctan(cx))$$

output
```
-1/6*b*x/c^5+1/18*b*x^3/c^3-1/30*b*x^5/c+1/6*b*arctan(c*x)/c^6+1/6*x^6*(a+
b*arctan(c*x))
```

3.1.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int x^5(a+b\arctan(cx))\,dx=-\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{ax^6}{6}+\frac{b\arctan(cx)}{6c^6}+\frac{1}{6}bx^6\arctan(cx)$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x]),x]
```

output
```
-1/6*(b*x)/c^5 + (b*x^3)/(18*c^3) - (b*x^5)/(30*c) + (a*x^6)/6 + (b*ArcTan
[c*x])/(6*c^6) + (b*x^6*ArcTan[c*x])/6
```

3.1.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5361, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))-\frac{1}{6}bc\int\frac{x^6}{c^2x^2+1}dx$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))-\frac{1}{6}bc\int\left(\frac{x^4}{c^2}-\frac{x^2}{c^4}-\frac{1}{c^6(c^2x^2+1)}+\frac{1}{c^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))-\frac{1}{6}bc\left(-\frac{\arctan(cx)}{c^7}+\frac{x}{c^6}-\frac{x^3}{3c^4}+\frac{x^5}{5c^2}\right)$$

input
```
Int[x^5*(a + b*ArcTan[c*x]),x]
```

output
```
(x^6*(a + b*ArcTan[c*x]))/6 - (b*c*(x/c^6 - x^3/(3*c^4) + x^5/(5*c^2) - Ar
cTan[c*x]/c^7))/6
```

3.1.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.1.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	size
parts	$\frac{a\,x^{6}}{6}+\frac{b\left(\frac{c^{6}x^{6}\arctan(cx)}{6}-\frac{c^{5}x^{5}}{30}+\frac{c^{3}x^{3}}{18}-\frac{cx}{6}+\frac{\arctan(cx)}{6}\right)}{c^{6}}$	52
derivativedivides	$\frac{\frac{a\,c^{6}x^{6}}{6}+b\left(\frac{c^{6}x^{6}\arctan(cx)}{6}-\frac{c^{5}x^{5}}{30}+\frac{c^{3}x^{3}}{18}-\frac{cx}{6}+\frac{\arctan(cx)}{6}\right)}{c^{6}}$	56
default	$\frac{\frac{a\,c^{6}x^{6}}{6}+b\left(\frac{c^{6}x^{6}\arctan(cx)}{6}-\frac{c^{5}x^{5}}{30}+\frac{c^{3}x^{3}}{18}-\frac{cx}{6}+\frac{\arctan(cx)}{6}\right)}{c^{6}}$	56
parallelrisch	$\frac{15b\arctan(cx)x^{6}c^{6}+15a\,c^{6}x^{6}-3b\,c^{5}x^{5}+5b\,c^{3}x^{3}-15xbc+15b\arctan(cx)}{90c^{6}}$	59
risch	$-\frac{ix^{6}b\ln(icx+1)}{12}+\frac{ix^{6}b\ln(-icx+1)}{12}+\frac{a\,x^{6}}{6}-\frac{b\,x^{5}}{30c}+\frac{b\,x^{3}}{18c^{3}}-\frac{bx}{6c^{5}}+\frac{b\arctan(cx)}{6c^{6}}$	73

input
```
int(x^5*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x^6+b/c^6*(1/6*c^6*x^6*arctan(c*x)-1/30*c^5*x^5+1/18*c^3*x^3-1/6*c*x
+1/6*arctan(c*x))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^5(a+b\arctan(cx))\,dx=\frac{15\,ac^6x^6-3\,bc^5x^5+5\,bc^3x^3-15\,bcx+15\,(bc^6x^6+b)\arctan{(cx)}}{90\,c^6}$$

input
```
integrate(x^5*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*a*c^6*x^6 - 3*b*c^5*x^5 + 5*b*c^3*x^3 - 15*b*c*x + 15*(b*c^6*x^6
+ b)*arctan(c*x))/c^6
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int x^5(a+b\arctan(cx))\,dx = \begin{cases} \frac{ax^6}{6}+\frac{bx^6\operatorname{atan}(cx)}{6}-\frac{bx^5}{30c}+\frac{bx^3}{18c^3}-\frac{bx}{6c^5}+\frac{b\operatorname{atan}(cx)}{6c^6} & \text{for } c\neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*x**6/6 + b*x**6*atan(c*x)/6 - b*x**5/(30*c) + b*x**3/(18*c**3
) - b*x/(6*c**5) + b*atan(c*x)/(6*c**6), Ne(c, 0)), (a*x**6/6, True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^5(a+b\arctan(cx))\,dx = \frac{1}{6}ax^6+\frac{1}{90}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)b$$

input
```
integrate(x^5*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c
^6 - 15*arctan(c*x)/c^7))*b
```

3.1.8 Giac [F]

$$\int x^5(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int x^5(a+b\arctan(cx))\,dx=\frac{\frac{b\,\mathrm{atan}(c\,x)}{6}+\frac{b\,c^3\,x^3}{18}-\frac{b\,c^5\,x^5}{30}-\frac{b\,c\,x}{6}}{c^6}+\frac{a\,x^6}{6}+\frac{b\,x^6\,\mathrm{atan}(c\,x)}{6}$$

input
```
int(x^5*(a + b*atan(c*x)),x)
```

output
```
((b*atan(c*x))/6 + (b*c^3*x^3)/18 - (b*c^5*x^5)/30 - (b*c*x)/6)/c^6 + (a*x
^6)/6 + (b*x^6*atan(c*x))/6
```

3.2 $\int x^4(a+b\arctan(cx))\,dx$

3.2.1 Optimal result . . . 83
3.2.2 Mathematica [A] (verified) . . . 83
3.2.3 Rubi [A] (verified) . . . 84
3.2.4 Maple [A] (verified) . . . 85
3.2.5 Fricas [A] (verification not implemented) . . . 86
3.2.6 Sympy [A] (verification not implemented) . . . 86
3.2.7 Maxima [A] (verification not implemented) . . . 87
3.2.8 Giac [F] . . . 87
3.2.9 Mupad [B] (verification not implemented) . . . 87

3.2.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int x^4(a+b\arctan(cx))\,dx=\frac{bx^2}{10c^3}-\frac{bx^4}{20c}+\frac{1}{5}x^5(a+b\arctan(cx))-\frac{b\log(1+c^2x^2)}{10c^5}$$

output
```
1/10*b*x^2/c^3-1/20*b*x^4/c+1/5*x^5*(a+b*arctan(c*x))-1/10*b*ln(c^2*x^2+1)
/c^5
```

3.2.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int x^4(a+b\arctan(cx))\,dx=\frac{bx^2}{10c^3}-\frac{bx^4}{20c}+\frac{ax^5}{5}+\frac{1}{5}bx^5\arctan(cx)-\frac{b\log(1+c^2x^2)}{10c^5}$$

input
```
Integrate[x^4*(a + b*ArcTan[c*x]),x]
```

output
```
(b*x^2)/(10*c^3) - (b*x^4)/(20*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x])/5 - (b
*Log[1 + c^2*x^2])/(10*c^5)
```

3.2.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5361, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{5}bc\int \frac{x^5}{c^2x^2+1}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\int \frac{x^4}{c^2x^2+1}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\int \left(\frac{x^2}{c^2}+\frac{1}{c^4\,(c^2x^2+1)}-\frac{1}{c^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log\left(c^2x^2+1\right)}{c^6}\right)$$

input
```
Int[x^4*(a + b*ArcTan[c*x]),x]
```

output
```
(x^5*(a + b*ArcTan[c*x]))/5 - (b*c*(-(x^2/c^4) + x^4/(2*c^2) + Log[1 + c^2
*x^2]/c^6))/10
```

3.2.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.2.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

method	result	size
parts	$\frac{a\,x^{5}}{5}+\frac{b\left(\frac{c^{5} x^{5} \arctan\left(c x\right)}{5}-\frac{c^{4} x^{4}}{20}+\frac{c^{2} x^{2}}{10}-\frac{\ln\left(c^{2} x^{2}+1\right)}{10}\right)}{c^{5}}$	54
derivativedivides	$\frac{\frac{a\,c^{5} x^{5}}{5}+b\left(\frac{c^{5} x^{5} \arctan\left(c x\right)}{5}-\frac{c^{4} x^{4}}{20}+\frac{c^{2} x^{2}}{10}-\frac{\ln\left(c^{2} x^{2}+1\right)}{10}\right)}{c^{5}}$	58
default	$\frac{\frac{a\,c^{5} x^{5}}{5}+b\left(\frac{c^{5} x^{5} \arctan\left(c x\right)}{5}-\frac{c^{4} x^{4}}{20}+\frac{c^{2} x^{2}}{10}-\frac{\ln\left(c^{2} x^{2}+1\right)}{10}\right)}{c^{5}}$	58
parallelrisch	$-\frac{-4 b \arctan\left(c x\right) x^{5} c^{5}-4 a\,c^{5} x^{5}+b\,c^{4} x^{4}-2 b\,c^{2} x^{2}+2 b \ln\left(c^{2} x^{2}+1\right)+2 b}{20 c^{5}}$	62
risch	$-\frac{i x^{5} b \ln\left(i c x+1\right)}{10}+\frac{i x^{5} b \ln\left(-i c x+1\right)}{10}+\frac{a\,x^{5}}{5}-\frac{b\,x^{4}}{20 c}+\frac{b\,x^{2}}{10 c^{3}}-\frac{b \ln\left(-c^{2} x^{2}-1\right)}{10 c^{5}}$	73

input
```
int(x^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+b/c^5*(1/5*c^5*x^5*arctan(c*x)-1/20*c^4*x^4+1/10*c^2*x^2-1/10*ln
(c^2*x^2+1))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int x^4(a+b\arctan(cx))\,dx = \frac{4\,bc^5x^5\arctan(cx)+4\,ac^5x^5-bc^4x^4+2\,bc^2x^2-2\,b\log(c^2x^2+1)}{20\,c^5}$$

input
```
integrate(x^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/20*(4*b*c^5*x^5*arctan(c*x) + 4*a*c^5*x^5 - b*c^4*x^4 + 2*b*c^2*x^2 - 2*
b*log(c^2*x^2 + 1))/c^5
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int x^4(a+b\arctan(cx))\,dx = \begin{cases} \frac{ax^5}{5}+\frac{bx^5\operatorname{atan}(cx)}{5}-\frac{bx^4}{20c}+\frac{bx^2}{10c^3}-\frac{b\log\left(x^2+\frac{1}{c^2}\right)}{10c^5} & \text{for } c\neq 0\\ \frac{ax^5}{5} & \text{otherwise}\end{cases}$$

input
```
integrate(x**4*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*x**5/5 + b*x**5*atan(c*x)/5 - b*x**4/(20*c) + b*x**2/(10*c**3
) - b*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a*x**5/5, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int x^4(a+b\arctan(cx))\,dx=\frac{1}{5}\,ax^5+\frac{1}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)b$$

input
```
integrate(x^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2
*x^2 + 1)/c^6))*b
```

3.2.8 Giac [F]

$$\int x^4(a+b\arctan(cx))\,dx=\int(b\arctan\left(cx\right)+a)x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int x^4(a+b\arctan(cx))\,dx=\frac{a\,x^5}{5}-\frac{\frac{b\,\ln\left(c^2\,x^2+1\right)}{10}-\frac{b\,c^2\,x^2}{10}+\frac{b\,c^4\,x^4}{20}}{c^5}+\frac{b\,x^5\,\mathrm{atan}(c\,x)}{5}$$

input
```
int(x^4*(a + b*atan(c*x)),x)
```

output
```
(a*x^5)/5 - ((b*log(c^2*x^2 + 1))/10 - (b*c^2*x^2)/10 + (b*c^4*x^4)/20)/c^
5 + (b*x^5*atan(c*x))/5
```

3.3 $\int x^3(a+b\arctan(cx))\,dx$

3.3.1 Optimal result . . . 88
3.3.2 Mathematica [A] (verified) . . . 88
3.3.3 Rubi [A] (verified) . . . 89
3.3.4 Maple [A] (verified) . . . 90
3.3.5 Fricas [A] (verification not implemented) . . . 90
3.3.6 Sympy [A] (verification not implemented) . . . 91
3.3.7 Maxima [A] (verification not implemented) . . . 91
3.3.8 Giac [F] . . . 91
3.3.9 Mupad [B] (verification not implemented) . . . 92

3.3.1 Optimal result

Integrand size = 12, antiderivative size = 48

$$\int x^3(a+b\arctan(cx))\,dx=\frac{bx}{4c^3}-\frac{bx^3}{12c}-\frac{b\arctan(cx)}{4c^4}+\frac{1}{4}x^4(a+b\arctan(cx))$$

output
```
1/4*b*x/c^3-1/12*b*x^3/c-1/4*b*arctan(c*x)/c^4+1/4*x^4*(a+b*arctan(c*x))
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int x^3(a+b\arctan(cx))\,dx=\frac{bx}{4c^3}-\frac{bx^3}{12c}+\frac{ax^4}{4}-\frac{b\arctan(cx)}{4c^4}+\frac{1}{4}bx^4\arctan(cx)$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x]),x]
```

output
```
(b*x)/(4*c^3) - (b*x^3)/(12*c) + (a*x^4)/4 - (b*ArcTan[c*x])/(4*c^4) + (b*
x^4*ArcTan[c*x])/4
```

3.3.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5361, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\int\frac{x^4}{c^2x^2+1}dx$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\int\left(\frac{x^2}{c^2}+\frac{1}{c^4\left(c^2x^2+1\right)}-\frac{1}{c^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)$$

input
```
Int[x^3*(a + b*ArcTan[c*x]),x]
```

output
```
(x^4*(a + b*ArcTan[c*x]))/4 - (b*c*(-(x/c^4) + x^3/(3*c^2) + ArcTan[c*x]/c
^5))/4
```

3.3.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.3.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

method	result	size
parts	$\frac{a\,x^4}{4}+\frac{b\left(\frac{c^4x^4\arctan(cx)}{4}-\frac{c^3x^3}{12}+\frac{cx}{4}-\frac{\arctan(cx)}{4}\right)}{c^4}$	44
derivativedivides	$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4x^4\arctan(cx)}{4}-\frac{c^3x^3}{12}+\frac{cx}{4}-\frac{\arctan(cx)}{4}\right)}{c^4}$	48
default	$\frac{\frac{a\,c^4x^4}{4}+b\left(\frac{c^4x^4\arctan(cx)}{4}-\frac{c^3x^3}{12}+\frac{cx}{4}-\frac{\arctan(cx)}{4}\right)}{c^4}$	48
parallelrisch	$\frac{3b\arctan(cx)x^4c^4+3a\,c^4x^4-b\,c^3x^3+3xbc-3b\arctan(cx)}{12c^4}$	50
risch	$-\frac{ix^4b\ln(icx+1)}{8}+\frac{ix^4b\ln(-icx+1)}{8}+\frac{a\,x^4}{4}-\frac{b\,x^3}{12c}+\frac{bx}{4c^3}-\frac{b\arctan(cx)}{4c^4}$	64

input
```
int(x^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+b/c^4*(1/4*c^4*x^4*arctan(c*x)-1/12*c^3*x^3+1/4*c*x-1/4*arctan(c
*x))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int x^3(a+b\arctan(cx))\,dx=\frac{3\,ac^4x^4-bc^3x^3+3\,bcx+3\,(bc^4x^4-b)\arctan{(cx)}}{12\,c^4}$$

input
```
integrate(x^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*x^4 - b*c^3*x^3 + 3*b*c*x + 3*(b*c^4*x^4 - b)*arctan(c*x))/c
^4
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int x^3(a+b\arctan(cx))\,dx = \begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{atan}(cx)}{4} - \frac{bx^3}{12c} + \frac{bx}{4c^3} - \frac{b \operatorname{atan}(cx)}{4c^4} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*x**4/4 + b*x**4*atan(c*x)/4 - b*x**3/(12*c) + b*x/(4*c**3) -
b*atan(c*x)/(4*c**4), Ne(c, 0)), (a*x**4/4, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int x^3(a+b\arctan(cx))\,dx = \frac{1}{4}ax^4 + \frac{1}{12}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)b$$

input
```
integrate(x^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*
x)/c^5))*b
```

3.3.8 Giac [F]

$$\int x^3(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int x^3(a+b\arctan(cx))\,dx=\frac{a\,x^4}{4}-\frac{\frac{b\,\mathrm{atan}(c\,x)}{4}+\frac{b\,c^3\,x^3}{12}-\frac{b\,c\,x}{4}}{c^4}+\frac{b\,x^4\,\mathrm{atan}(c\,x)}{4}$$

input
```
int(x^3*(a + b*atan(c*x)),x)
```

output
```
(a*x^4)/4 - ((b*atan(c*x))/4 + (b*c^3*x^3)/12 - (b*c*x)/4)/c^4 + (b*x^4*at
an(c*x))/4
```

3.4 $\int x^2(a+b\arctan(cx))\,dx$

3.4.1 Optimal result . . . 93
3.4.2 Mathematica [A] (verified) . . . 93
3.4.3 Rubi [A] (verified) . . . 94
3.4.4 Maple [A] (verified) . . . 95
3.4.5 Fricas [A] (verification not implemented) . . . 96
3.4.6 Sympy [A] (verification not implemented) . . . 96
3.4.7 Maxima [A] (verification not implemented) . . . 96
3.4.8 Giac [F] . . . 97
3.4.9 Mupad [B] (verification not implemented) . . . 97

3.4.1 Optimal result

Integrand size = 12, antiderivative size = 45

$$\int x^2(a+b\arctan(cx))\,dx=-\frac{bx^2}{6c}+\frac{1}{3}x^3(a+b\arctan(cx))+\frac{b\log(1+c^2x^2)}{6c^3}$$

output
```
-1/6*b*x^2/c+1/3*x^3*(a+b*arctan(c*x))+1/6*b*ln(c^2*x^2+1)/c^3
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int x^2(a+b\arctan(cx))\,dx=-\frac{bx^2}{6c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\arctan(cx)+\frac{b\log(1+c^2x^2)}{6c^3}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x]),x]
```

output
```
-1/6*(b*x^2)/c + (a*x^3)/3 + (b*x^3*ArcTan[c*x])/3 + (b*Log[1 + c^2*x^2])/
(6*c^3)
```

3.4.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5361, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{3}bc\int \frac{x^3}{c^2x^2+1}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int \frac{x^2}{c^2x^2+1}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int \left(\frac{1}{c^2}-\frac{1}{c^2\left(c^2x^2+1\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)$$

input
```
Int[x^2*(a + b*ArcTan[c*x]),x]
```

output
```
(x^3*(a + b*ArcTan[c*x]))/3 - (b*c*(x^2/c^2 - Log[1 + c^2*x^2]/c^4))/6
```

3.4.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.4.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
parts	$\frac{x^3 a}{3}+\frac{b\left(\frac{c^3x^3\arctan(cx)}{3}-\frac{c^2x^2}{6}+\frac{\ln\left(c^2x^2+1\right)}{6}\right)}{c^3}$	46
derivativedivides	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\arctan(cx)}{3}-\frac{c^2x^2}{6}+\frac{\ln\left(c^2x^2+1\right)}{6}\right)}{c^3}$	50
default	$\frac{\frac{a\,c^3x^3}{3}+b\left(\frac{c^3x^3\arctan(cx)}{3}-\frac{c^2x^2}{6}+\frac{\ln\left(c^2x^2+1\right)}{6}\right)}{c^3}$	50
parallelrisch	$\frac{2x^3\arctan(cx)b\,c^3+2a\,c^3x^3-b\,c^2x^2+b\ln(c^2x^2+1)}{6c^3}$	50
risch	$-\frac{ix^3b\ln(icx+1)}{6}+\frac{ix^3b\ln(-icx+1)}{6}+\frac{x^3a}{3}-\frac{b\,x^2}{6c}+\frac{b\ln(-c^2x^2-1)}{6c^3}$	64

input
```
int(x^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*a+b/c^3*(1/3*c^3*x^3*arctan(c*x)-1/6*c^2*x^2+1/6*ln(c^2*x^2+1))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int x^2(a+b\arctan(cx))\,dx = \frac{2\,bc^3x^3\arctan{(cx)}+2\,ac^3x^3-bc^2x^2+b\log{(c^2x^2+1)}}{6\,c^3}$$

input
```
integrate(x^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*b*c^3*x^3*arctan(c*x) + 2*a*c^3*x^3 - b*c^2*x^2 + b*log(c^2*x^2 + 1
))/c^3
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int x^2(a+b\arctan(cx))\,dx = \begin{cases} \frac{ax^3}{3}+\frac{bx^3\,\mathrm{atan}\,(cx)}{3}-\frac{bx^2}{6c}+\frac{b\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} & \text{for } c\neq 0 \\ \frac{ax^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*x**3/3 + b*x**3*atan(c*x)/3 - b*x**2/(6*c) + b*log(x**2 + c**
(-2))/(6*c**3), Ne(c, 0)), (a*x**3/3, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int x^2(a+b\arctan(cx))\,dx = \frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\arctan{(cx)}-c\left(\frac{x^2}{c^2}-\frac{\log{(c^2x^2+1)}}{c^4}\right)\right)b$$

input
```
integrate(x^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b
```

3.4.8 Giac [F]

$$\int x^2(a+b\arctan(cx))\,dx=\int (b\arctan(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^2(a+b\arctan(cx))\,dx=\frac{a\,x^3}{3}+\frac{b\,x^3\,\mathrm{atan}(c\,x)}{3}+\frac{b\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}-\frac{b\,x^2}{6\,c}$$

input
```
int(x^2*(a + b*atan(c*x)),x)
```

output
```
(a*x^3)/3 + (b*x^3*atan(c*x))/3 + (b*log(c^2*x^2 + 1))/(6*c^3) - (b*x^2)/(
6*c)
```

3.5 $\int x(a+b\arctan(cx))\,dx$

3.5.1 Optimal result 98
3.5.2 Mathematica [A] (verified) 98
3.5.3 Rubi [A] (verified) 99
3.5.4 Maple [A] (verified) 100
3.5.5 Fricas [A] (verification not implemented) 100
3.5.6 Sympy [A] (verification not implemented) 101
3.5.7 Maxima [A] (verification not implemented) 101
3.5.8 Giac [F] 101
3.5.9 Mupad [B] (verification not implemented) 102

3.5.1 Optimal result

Integrand size = 10, antiderivative size = 37

$$\int x(a+b\arctan(cx))\,dx=-\frac{bx}{2c}+\frac{b\arctan(cx)}{2c^2}+\frac{1}{2}x^2(a+b\arctan(cx))$$

output
```
-1/2*b*x/c+1/2*b*arctan(c*x)/c^2+1/2*x^2*(a+b*arctan(c*x))
```

3.5.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x(a+b\arctan(cx))\,dx=-\frac{bx}{2c}+\frac{ax^2}{2}+\frac{b\arctan(cx)}{2c^2}+\frac{1}{2}bx^2\arctan(cx)$$

input
```
Integrate[x*(a + b*ArcTan[c*x]),x]
```

output
```
-1/2*(b*x)/c + (a*x^2)/2 + (b*ArcTan[c*x])/(2*c^2) + (b*x^2*ArcTan[c*x])/2
```

3.5.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5361, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int\frac{x^2}{c^2x^2+1}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)$$

input
```
Int[x*(a + b*ArcTan[c*x]),x]
```

output
```
(x^2*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2
```

3.5.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.5.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
parts	$\frac{a\,x^2}{2}+\frac{b\left(\frac{c^2x^2\arctan(cx)}{2}-\frac{cx}{2}+\frac{\arctan(cx)}{2}\right)}{c^2}$	36
parallelrisch	$\frac{\arctan(cx)b\,c^2x^2+c^2x^2a-xbc+b\arctan(cx)}{2c^2}$	38
derivativedivides	$\frac{\frac{c^2x^2a}{2}+b\left(\frac{c^2x^2\arctan(cx)}{2}-\frac{cx}{2}+\frac{\arctan(cx)}{2}\right)}{c^2}$	40
default	$\frac{\frac{c^2x^2a}{2}+b\left(\frac{c^2x^2\arctan(cx)}{2}-\frac{cx}{2}+\frac{\arctan(cx)}{2}\right)}{c^2}$	40
risch	$-\frac{ix^2b\ln(icx+1)}{4}+\frac{ix^2b\ln(-icx+1)}{4}+\frac{a\,x^2}{2}-\frac{bx}{2c}+\frac{b\arctan(cx)}{2c^2}$	55

input
```
int(x*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arctan(c*x)-1/2*c*x+1/2*arctan(c*x))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x(a+b\arctan(cx))\,dx=\frac{ac^2x^2-bcx+(bc^2x^2+b)\arctan(cx)}{2\,c^2}$$

input
```
integrate(x*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/2*(a*c^2*x^2 - b*c*x + (b*c^2*x^2 + b)*arctan(c*x))/c^2
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x(a+b\arctan(cx))\,dx = \begin{cases} \frac{ax^2}{2} + \frac{bx^2\operatorname{atan}(cx)}{2} - \frac{bx}{2c} + \frac{b\operatorname{atan}(cx)}{2c^2} & \text{for } c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*x**2/2 + b*x**2*atan(c*x)/2 - b*x/(2*c) + b*atan(c*x)/(2*c**2
), Ne(c, 0)), (a*x**2/2, True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x(a+b\arctan(cx))\,dx = \frac{1}{2}ax^2 + \frac{1}{2}\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)b$$

input
```
integrate(x*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b
```

3.5.8 Giac [F]

$$\int x(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x(a+b\arctan(cx))\,dx=\frac{a\,x^2}{2}+\frac{b\,\mathrm{atan}(c\,x)}{2\,c^2}+\frac{b\,x^2\,\mathrm{atan}(c\,x)}{2}-\frac{b\,x}{2\,c}$$

input
```
int(x*(a + b*atan(c*x)),x)
```

output
```
(a*x^2)/2 + (b*atan(c*x))/(2*c^2) + (b*x^2*atan(c*x))/2 - (b*x)/(2*c)
```

3.6 $\int (a+b\arctan(cx))\,dx$

3.6.1 Optimal result 103
3.6.2 Mathematica [A] (verified) 103
3.6.3 Rubi [A] (verified) 104
3.6.4 Maple [A] (verified) 104
3.6.5 Fricas [A] (verification not implemented) 105
3.6.6 Sympy [A] (verification not implemented) 105
3.6.7 Maxima [A] (verification not implemented) 105
3.6.8 Giac [A] (verification not implemented) 106
3.6.9 Mupad [B] (verification not implemented) 106

3.6.1 Optimal result

Integrand size = 8, antiderivative size = 29

$$\int (a+b\arctan(cx))\,dx = ax+bx\arctan(cx)-\frac{b\log\left(1+c^2x^2\right)}{2c}$$

output
```
a*x+b*x*arctan(c*x)-1/2*b*ln(c^2*x^2+1)/c
```

3.6.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a+b\arctan(cx))\,dx = ax+bx\arctan(cx)-\frac{b\log\left(1+c^2x^2\right)}{2c}$$

input
```
Integrate[a + b*ArcTan[c*x],x]
```

output
```
a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

3.6.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\arctan(cx))\,dx$$

$$\Big\downarrow \; 2009$$

$$ax + bx\arctan(cx) - \frac{b\log(c^2x^2+1)}{2c}$$

input
```
Int[a + b*ArcTan[c*x],x]
```

output
```
a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

3.6.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.6.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
default	$ax + bx\arctan(cx) - \frac{b\ln(c^2x^2+1)}{2c}$	28
parts	$ax + bx\arctan(cx) - \frac{b\ln(c^2x^2+1)}{2c}$	28
parallelrisch	$-\frac{b(-2cx\arctan(cx)+\ln(c^2x^2+1))}{2c} + ax$	30
derivativedivides	$\frac{cxa+b\left(cx\arctan(cx)-\frac{\ln(c^2x^2+1)}{2}\right)}{c}$	32
risch	$ax - \frac{ibx\ln(icx+1)}{2} + \frac{ibx\ln(-icx+1)}{2} - \frac{b\ln(-c^2x^2-1)}{2c}$	48

input
```
int(a+b*arctan(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arctan(c*x)-1/2*b*ln(c^2*x^2+1)/c
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int (a+b\arctan(cx))\,dx=\frac{2\,bcx\arctan(cx)+2\,acx-b\log(c^2x^2+1)}{2\,c}$$

input
```
integrate(a+b*arctan(c*x),x, algorithm="fricas")
```

output
```
1/2*(2*b*c*x*arctan(c*x) + 2*a*c*x - b*log(c^2*x^2 + 1))/c
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int (a+b\arctan(cx))\,dx=ax+b\left(\begin{cases} x\,\mathrm{atan}\,(cx)-\frac{\log(c^2x^2+1)}{2c} & \text{for } c\neq 0\\ 0 & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*atan(c*x),x)
```

output
```
a*x + b*Piecewise((x*atan(c*x) - log(c**2*x**2 + 1)/(2*c), Ne(c, 0)), (0,
True))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int (a+b\arctan(cx))\,dx=ax+\frac{(2\,cx\arctan(cx)-\log(c^2x^2+1))b}{2\,c}$$

input
```
integrate(a+b*arctan(c*x),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b/c
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int (a+b\arctan(cx))\,dx = ax + \frac{(2\,cx\arctan{(cx)} - \log{(c^2x^2+1)})b}{2\,c}$$

input
```
integrate(a+b*arctan(c*x),x, algorithm="giac")
```

output
```
a*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b/c
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int (a+b\arctan(cx))\,dx = a\,x - \frac{b\,\ln{(c^2\,x^2+1)}}{2\,c} + b\,x\,\mathrm{atan}(c\,x)$$

input
```
int(a + b*atan(c*x),x)
```

output
```
a*x - (b*log(c^2*x^2 + 1))/(2*c) + b*x*atan(c*x)
```

3.7 $\int \frac{a+b\arctan(cx)}{x}\,dx$

3.7.1 Optimal result 107
3.7.2 Mathematica [A] (verified) 107
3.7.3 Rubi [A] (verified) 108
3.7.4 Maple [A] (verified) 109
3.7.5 Fricas [F] 109
3.7.6 Sympy [F] 109
3.7.7 Maxima [F] 110
3.7.8 Giac [F] 110
3.7.9 Mupad [B] (verification not implemented) 110

3.7.1 Optimal result

Integrand size = 12, antiderivative size = 35

$$\int \frac{a+b\arctan(cx)}{x}\,dx = a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)$$

output
```
a*ln(x)+1/2*I*b*polylog(2,-I*c*x)-1/2*I*b*polylog(2,I*c*x)
```

3.7.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x}\,dx = a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)$$

input
```
Integrate[(a + b*ArcTan[c*x])/x,x]
```

output
```
a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

3.7.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x}\,dx$$

$$\Big\downarrow \text{ 5355}$$

$$\frac{1}{2}ib\int \frac{\log(1-icx)}{x}dx-\frac{1}{2}ib\int \frac{\log(icx+1)}{x}dx+a\log(x)$$

$$\Big\downarrow \text{ 2838}$$

$$a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)$$

input
```
Int[(a + b*ArcTan[c*x])/x,x]
```

output
```
a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

3.7.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

3.7.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{ib\,\mathrm{dilog}(-icx+1)}{2}+a\ln(-icx)+\frac{ib\,\mathrm{dilog}(icx+1)}{2}$
parts	$a\ln(x)+b\Big(\ln(cx)\arctan(cx)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(-icx+1)}{2}+\frac{i\,\mathrm{dilog}(icx+1)}{2}-\frac{i\,\mathrm{dilog}(}{}$
derivativedivides	$a\ln(cx)+b\Big(\ln(cx)\arctan(cx)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(-icx+1)}{2}+\frac{i\,\mathrm{dilog}(icx+1)}{2}-\frac{i\,\mathrm{dilog}}{}$
default	$a\ln(cx)+b\Big(\ln(cx)\arctan(cx)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(-icx+1)}{2}+\frac{i\,\mathrm{dilog}(icx+1)}{2}-\frac{i\,\mathrm{dilog}}{}$

input
```
int((a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*b*dilog(1-I*c*x)+a*ln(-I*c*x)+1/2*I*b*dilog(1+I*c*x)
```

3.7.5 Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x}\,dx=\int \frac{b\arctan(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/x, x)
```

3.7.6 Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x}\,dx=\int \frac{a+b\,\mathrm{atan}(cx)}{x}\,dx$$

input
```
integrate((a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))/x, x)
```

3.7.7 Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x}\,dx = \int \frac{b\arctan(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
b*integrate(arctan(c*x)/x, x) + a*log(x)
```

3.7.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x}\,dx = \int \frac{b\arctan(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.7.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{a+b\arctan(cx)}{x}\,dx = a\ln(x) - \frac{b\left(\mathrm{Li}_2(1-c\,x\,1\mathrm{i}) - \mathrm{Li}_2(1+c\,x\,1\mathrm{i})\right)1\mathrm{i}}{2}$$

input
```
int((a + b*atan(c*x))/x,x)
```

output
```
a*log(x) - (b*(dilog(1 - c*x*1i) - dilog(c*x*1i + 1))*1i)/2
```

3.8 $\int \frac{a+b\arctan(cx)}{x^2}\,dx$

3.8.1 Optimal result 111
3.8.2 Mathematica [A] (verified) 111
3.8.3 Rubi [A] (verified) 112
3.8.4 Maple [A] (verified) 113
3.8.5 Fricas [A] (verification not implemented) 114
3.8.6 Sympy [A] (verification not implemented) 114
3.8.7 Maxima [A] (verification not implemented) 114
3.8.8 Giac [F] 115
3.8.9 Mupad [B] (verification not implemented) 115

3.8.1 Optimal result

Integrand size = 12, antiderivative size = 35

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx = -\frac{a+b\arctan(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1+c^2x^2\right)$$

output
```
(-a-b*arctan(c*x))/x+b*c*ln(x)-1/2*b*c*ln(c^2*x^2+1)
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx = -\frac{a}{x}-\frac{b\arctan(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1+c^2x^2\right)$$

input
```
Integrate[(a + b*ArcTan[c*x])/x^2,x]
```

output
```
-(a/x) - (b*ArcTan[c*x])/x + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2
```

3.8.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5361, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx$$

$$\Big\downarrow \text{5361}$$

$$bc\int \frac{1}{x\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{x}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}bc\int \frac{1}{x^2\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{x}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}$$

input
```
Int[(a + b*ArcTan[c*x])/x^2,x]
```

output
```
-((a + b*ArcTan[c*x])/x) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2
```

3.8.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.8.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

method	result	size
parallelrisch	$\frac{2bc\ln(x)x-bc\ln(c^2x^2+1)x-2b\arctan(cx)-2a}{2x}$	39
parts	$-\frac{a}{x}+bc\left(-\frac{\arctan(cx)}{cx}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)$	40
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\arctan(cx)}{cx}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)\right)$	44
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\arctan(cx)}{cx}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)\right)$	44
risch	$\frac{ib\ln(icx+1)}{2x}-\frac{-2bc\ln(x)x+bc\ln(-c^2x^2-1)x+ib\ln(-icx+1)+2a}{2x}$	60

input
```
int((a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*b*c*ln(x)*x-b*c*ln(c^2*x^2+1)*x-2*b*arctan(c*x)-2*a)/x
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{a + b\arctan(cx)}{x^2}\,dx = -\frac{bcx\log\left(c^2x^2+1\right) - 2\,bcx\log\left(x\right) + 2\,b\arctan\left(cx\right) + 2\,a}{2\,x}$$

input
```
integrate((a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(b*c*x*log(c^2*x^2 + 1) - 2*b*c*x*log(x) + 2*b*arctan(c*x) + 2*a)/x
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{a + b\arctan(cx)}{x^2}\,dx = \begin{cases} -\frac{a}{x} + bc\log\left(x\right) - \frac{bc\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{b\,\mathrm{atan}\left(cx\right)}{x} & \text{for}\ c \neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atan(c*x))/x**2,x)
```

output
```
Piecewise((-a/x + b*c*log(x) - b*c*log(x**2 + c**(-2))/2 - b*atan(c*x)/x,
Ne(c, 0)), (-a/x, True))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{a + b\arctan(cx)}{x^2}\,dx = -\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan\left(cx\right)}{x}\right)b - \frac{a}{x}$$

input
```
integrate((a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b - a/x
```

3.8.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx = \int \frac{b\arctan(cx)+a}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx = b\,c\,\ln(x) - \frac{a}{x} - \frac{b\,\mathrm{atan}(c\,x)}{x} - \frac{b\,c\,\ln(c^2\,x^2+1)}{2}$$

input
```
int((a + b*atan(c*x))/x^2,x)
```

output
```
b*c*log(x) - a/x - (b*atan(c*x))/x - (b*c*log(c^2*x^2 + 1))/2
```

3.9 $\int \frac{a+b\arctan(cx)}{x^3}\,dx$

3.9.1 Optimal result 116
3.9.2 Mathematica [C] (verified) 116
3.9.3 Rubi [A] (verified) 117
3.9.4 Maple [A] (verified) 118
3.9.5 Fricas [A] (verification not implemented) 118
3.9.6 Sympy [A] (verification not implemented) 119
3.9.7 Maxima [A] (verification not implemented) 119
3.9.8 Giac [F] 119
3.9.9 Mupad [B] (verification not implemented) 120

3.9.1 Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx=-\frac{bc}{2x}-\frac{1}{2}bc^2\arctan(cx)-\frac{a+b\arctan(cx)}{2x^2}$$

output
```
-1/2*b*c/x-1/2*b*c^2*arctan(c*x)+1/2*(-a-b*arctan(c*x))/x^2
```

3.9.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{b\arctan(cx)}{2x^2}-\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x}$$

input
```
Integrate[(a + b*ArcTan[c*x])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*ArcTan[c*x])/(2*x^2) - (b*c*Hypergeometric2F1[-1/2, 1, 1/2
, -(c^2*x^2)])/(2*x)
```

3.9.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5361, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}bc\int \frac{1}{x^2\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{2x^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}$$

input
```
Int[(a + b*ArcTan[c*x])/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c*x])/x^2 + (b*c*(-x^(-1) - c*ArcTan[c*x]))/2
```

3.9.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.9.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$-\frac{\arctan(cx) b\, c^{2} x^{2}-c^{2} x^{2} a+x b c+b \arctan(cx)+a}{2 x^{2}}$	39
parts	$-\frac{a}{2x^{2}}+b\, c^{2}\left(-\frac{\arctan(cx)}{2c^{2}x^{2}}-\frac{\arctan(cx)}{2}-\frac{1}{2cx}\right)$	40
derivativedivides	$c^{2}\left(-\frac{a}{2c^{2}x^{2}}+b\left(-\frac{\arctan(cx)}{2c^{2}x^{2}}-\frac{\arctan(cx)}{2}-\frac{1}{2cx}\right)\right)$	44
default	$c^{2}\left(-\frac{a}{2c^{2}x^{2}}+b\left(-\frac{\arctan(cx)}{2c^{2}x^{2}}-\frac{\arctan(cx)}{2}-\frac{1}{2cx}\right)\right)$	44
risch	$\frac{ib\ln(icx+1)}{4x^{2}}-\frac{-ib\, c^{2}\ln(-cx+i)x^{2}+ib\, c^{2}\ln(-cx-i)x^{2}+ib\ln(-icx+1)+2xbc+2a}{4x^{2}}$	79

input
```
int((a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctan(c*x)*b*c^2*x^2-c^2*x^2*a+x*b*c+b*arctan(c*x)+a)/x^2
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx=-\frac{bcx+(bc^2x^2+b)\arctan(cx)+a}{2\,x^2}$$

input
```
integrate((a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
-1/2*(b*c*x + (b*c^2*x^2 + b)*arctan(c*x) + a)/x^2
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{a + b\arctan(cx)}{x^3}\,dx = -\frac{a}{2x^2} - \frac{bc^2\operatorname{atan}(cx)}{2} - \frac{bc}{2x} - \frac{b\operatorname{atan}(cx)}{2x^2}$$

input
```
integrate((a+b*atan(c*x))/x**3,x)
```

output
```
-a/(2*x**2) - b*c**2*atan(c*x)/2 - b*c/(2*x) - b*atan(c*x)/(2*x**2)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{a + b\arctan(cx)}{x^3}\,dx = -\frac{1}{2}\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)b - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b - 1/2*a/x^2
```

3.9.8 Giac [F]

$$\int \frac{a + b\arctan(cx)}{x^3}\,dx = \int \frac{b\arctan(cx) + a}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx = -\frac{\frac{a}{2}+\frac{b\,\mathrm{atan}(c\,x)}{2}+\frac{b\,c\,x}{2}}{x^2}-\frac{b\,c\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,\sqrt{c^2}}{2}$$

input
```
int((a + b*atan(c*x))/x^3,x)
```

output
```
- (a/2 + (b*atan(c*x))/2 + (b*c*x)/2)/x^2 - (b*c*atan((c^2*x)/(c^2)^(1/2))
*(c^2)^(1/2))/2
```

3.10 $\int \frac{a+b\arctan(cx)}{x^4}\,dx$

3.10.1 Optimal result 121
3.10.2 Mathematica [A] (verified) 121
3.10.3 Rubi [A] (verified) 122
3.10.4 Maple [A] (verified) 123
3.10.5 Fricas [A] (verification not implemented) 124
3.10.6 Sympy [A] (verification not implemented) 124
3.10.7 Maxima [A] (verification not implemented) 124
3.10.8 Giac [F] 125
3.10.9 Mupad [B] (verification not implemented) 125

3.10.1 Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = -\frac{bc}{6x^2}-\frac{a+b\arctan(cx)}{3x^3}-\frac{1}{3}bc^3\log(x)+\frac{1}{6}bc^3\log\left(1+c^2x^2\right)$$

output
```
-1/6*b*c/x^2+1/3*(-a-b*arctan(c*x))/x^3-1/3*b*c^3*ln(x)+1/6*b*c^3*ln(c^2*x
^2+1)
```

3.10.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{b\arctan(cx)}{3x^3}+\frac{1}{6}bc\left(-\frac{1}{x^2}-2c^2\log(x)+c^2\log\left(1+c^2x^2\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*ArcTan[c*x])/(3*x^3) + (b*c*(-x^(-2) - 2*c^2*Log[x] + c^2*
Log[1 + c^2*x^2]))/6
```

3.10.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5361, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}bc\int \frac{1}{x^3\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{3x^3}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{6}bc\int \frac{1}{x^4\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{3x^3}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{6}bc\int \left(\frac{c^4}{c^2x^2+1}-\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\arctan(cx)}{3x^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}$$

input
```
Int[(a + b*ArcTan[c*x])/x^4,x]
```

output
```
-1/3*(a + b*ArcTan[c*x])/x^3 + (b*c*(-x^(-2) - c^2*Log[x^2] + c^2*Log[1 +
c^2*x^2]))/6
```

3.10.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.10.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
parts	$-\frac{a}{3x^3}+b\,c^3\left(-\frac{\arctan(cx)}{3c^3x^3}+\frac{\ln(c^2x^2+1)}{6}-\frac{1}{6c^2x^2}-\frac{\ln(cx)}{3}\right)$	52
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\arctan(cx)}{3c^3x^3}+\frac{\ln(c^2x^2+1)}{6}-\frac{1}{6c^2x^2}-\frac{\ln(cx)}{3}\right)\right)$	56
default	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\arctan(cx)}{3c^3x^3}+\frac{\ln(c^2x^2+1)}{6}-\frac{1}{6c^2x^2}-\frac{\ln(cx)}{3}\right)\right)$	56
parallelrisch	$-\frac{2b\,c^3\ln(x)x^3-b\,c^3\ln(c^2x^2+1)x^3-b\,c^3x^3+xbc+2b\arctan(cx)+2a}{6x^3}$	60
risch	$\frac{ib\ln(icx+1)}{6x^3}-\frac{2b\,c^3\ln(x)x^3-b\,c^3\ln(c^2x^2+1)x^3+ib\ln(-icx+1)+xbc+2a}{6x^3}$	72

input
```
int((a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arctan(c*x)+1/6*ln(c^2*x^2+1)-1/6/c^2/x^2-1
/3*ln(c*x))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \frac{bc^3x^3\log\left(c^2x^2+1\right)-2\,bc^3x^3\log\left(x\right)-bcx-2\,b\arctan\left(cx\right)-2\,a}{6\,x^3}$$

input
```
integrate((a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/6*(b*c^3*x^3*log(c^2*x^2 + 1) - 2*b*c^3*x^3*log(x) - b*c*x - 2*b*arctan(
c*x) - 2*a)/x^3
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \begin{cases} -\frac{a}{3x^3}-\frac{bc^3\log\left(x\right)}{3}+\frac{bc^3\log\left(x^2+\frac{1}{c^2}\right)}{6}-\frac{bc}{6x^2}-\frac{b\,\mathrm{atan}\left(cx\right)}{3x^3} & \text{for}\ c\neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atan(c*x))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3) - b*c**3*log(x)/3 + b*c**3*log(x**2 + c**(-2))/6 -
b*c/(6*x**2) - b*atan(c*x)/(3*x**3), Ne(c, 0)), (-a/(3*x**3), True))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\,\arctan\left(cx\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
b - 1/3*a/x^3
```

3.10.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \int \frac{b\arctan(cx)+a}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \frac{b\,c^3\,\ln\left(c^2\,x^2+1\right)}{6} - \frac{\frac{a}{3}+\frac{b\,\mathrm{atan}(c\,x)}{3}+\frac{b\,c\,x}{6}}{x^3} - \frac{b\,c^3\,\ln\left(x\right)}{3}$$

input
```
int((a + b*atan(c*x))/x^4,x)
```

output
```
(b*c^3*log(c^2*x^2 + 1))/6 - (a/3 + (b*atan(c*x))/3 + (b*c*x)/6)/x^3 - (b*
c^3*log(x))/3
```

3.11 $\int \frac{a+b\arctan(cx)}{x^5}\,dx$

3.11.1 Optimal result 126
3.11.2 Mathematica [C] (verified) 126
3.11.3 Rubi [A] (verified) 127
3.11.4 Maple [A] (verified) 128
3.11.5 Fricas [A] (verification not implemented) 128
3.11.6 Sympy [A] (verification not implemented) 129
3.11.7 Maxima [A] (verification not implemented) 129
3.11.8 Giac [F] 129
3.11.9 Mupad [B] (verification not implemented) 130

3.11.1 Optimal result

Integrand size = 12, antiderivative size = 48

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx=-\frac{bc}{12x^3}+\frac{bc^3}{4x}+\frac{1}{4}bc^4\arctan(cx)-\frac{a+b\arctan(cx)}{4x^4}$$

output
```
-1/12*b*c/x^3+1/4*b*c^3/x+1/4*b*c^4*arctan(c*x)+1/4*(-a-b*arctan(c*x))/x^4
```

3.11.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx=-\frac{a}{4x^4}-\frac{b\arctan(cx)}{4x^4}-\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3}$$

input
```
Integrate[(a + b*ArcTan[c*x])/x^5,x]
```

output
```
-1/4*a/x^4 - (b*ArcTan[c*x])/(4*x^4) - (b*c*Hypergeometric2F1[-3/2, 1, -1/
2, -(c^2*x^2)])/(12*x^3)
```

3.11.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5361, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx$$

$$\Big\downarrow \ 5361$$

$$\frac{1}{4}bc\int \frac{1}{x^4\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{4x^4}$$

$$\Big\downarrow \ 264$$

$$\frac{1}{4}bc\left(c^2\left(-\int \frac{1}{x^2\left(c^2x^2+1\right)}dx\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan(cx)}{4x^4}$$

$$\Big\downarrow \ 264$$

$$\frac{1}{4}bc\left(-\left(c^2\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan(cx)}{4x^4}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{4}bc\left(-\left(c^2\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan(cx)}{4x^4}$$

input
```
Int[(a + b*ArcTan[c*x])/x^5,x]
```

output
```
-1/4*(a + b*ArcTan[c*x])/x^4 + (b*c*(-1/3*1/x^3 - c^2*(-x^(-1) - c*ArcTan[
c*x])))/4
```

3.11.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.11.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{3b\arctan(cx)x^4c^4+3bc^3x^3-xbc-3b\arctan(cx)-3a}{12x^4}$	44
parts	$-\frac{a}{4x^4}+bc^4\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{1}{12c^3x^3}+\frac{1}{4cx}+\frac{\arctan(cx)}{4}\right)$	48
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{1}{12c^3x^3}+\frac{1}{4cx}+\frac{\arctan(cx)}{4}\right)\right)$	52
default	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{1}{12c^3x^3}+\frac{1}{4cx}+\frac{\arctan(cx)}{4}\right)\right)$	52
risch	$\frac{ib\ln(icx+1)}{8x^4}-\frac{3ibc^4\ln(-cx+i)x^4-3ibc^4\ln(-cx-i)x^4-6bc^3x^3+3ib\ln(-icx+1)+2xbc+6a}{24x^4}$	88

input
```
int((a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*b*arctan(c*x)*x^4*c^4+3*b*c^3*x^3-x*b*c-3*b*arctan(c*x)-3*a)/x^4
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx=\frac{3\,bc^3x^3-bcx+3\,(bc^4x^4-b)\arctan\,(cx)-3\,a}{12\,x^4}$$

input
```
integrate((a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/12*(3*b*c^3*x^3 - b*c*x + 3*(b*c^4*x^4 - b)*arctan(c*x) - 3*a)/x^4
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = -\frac{a}{4x^4}+\frac{bc^4\operatorname{atan}(cx)}{4}+\frac{bc^3}{4x}-\frac{bc}{12x^3}-\frac{b\operatorname{atan}(cx)}{4x^4}$$

input
```
integrate((a+b*atan(c*x))/x**5,x)
```

output
```
-a/(4*x**4) + b*c**4*atan(c*x)/4 + b*c**3/(4*x) - b*c/(12*x**3) - b*atan(c
*x)/(4*x**4)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = \frac{1}{12}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan(cx)}{x^4}\right)b-\frac{a}{4\,x^4}$$

input
```
integrate((a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b -
 1/4*a/x^4
```

3.11.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = \int \frac{b\arctan(cx)+a}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = \frac{b\,c^4\,\mathrm{atan}(c\,x)}{4} - \frac{-b\,c^3\,x^3 + \frac{b\,c\,x}{3} + a}{4\,x^4} - \frac{b\,\mathrm{atan}(c\,x)}{4\,x^4}$$

input
```
int((a + b*atan(c*x))/x^5,x)
```

output
```
(b*c^4*atan(c*x))/4 - (a - b*c^3*x^3 + (b*c*x)/3)/(4*x^4) - (b*atan(c*x))/
(4*x^4)
```

3.12 $\int \frac{a+b\arctan(cx)}{x^6}\,dx$

3.12.1 Optimal result . . . 131
3.12.2 Mathematica [A] (verified) . . . 131
3.12.3 Rubi [A] (verified) . . . 132
3.12.4 Maple [A] (verified) . . . 133
3.12.5 Fricas [A] (verification not implemented) . . . 134
3.12.6 Sympy [A] (verification not implemented) . . . 134
3.12.7 Maxima [A] (verification not implemented) . . . 134
3.12.8 Giac [F] . . . 135
3.12.9 Mupad [B] (verification not implemented) . . . 135

3.12.1 Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx=-\frac{bc}{20x^4}+\frac{bc^3}{10x^2}-\frac{a+b\arctan(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1+c^2x^2\right)$$

output
```
-1/20*b*c/x^4+1/10*b*c^3/x^2+1/5*(-a-b*arctan(c*x))/x^5+1/5*b*c^5*ln(x)-1/
10*b*c^5*ln(c^2*x^2+1)
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx=-\frac{a}{5x^5}-\frac{bc}{20x^4}+\frac{bc^3}{10x^2}-\frac{b\arctan(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1+c^2x^2\right)$$

input
```
Integrate[(a + b*ArcTan[c*x])/x^6,x]
```

output
```
-1/5*a/x^5 - (b*c)/(20*x^4) + (b*c^3)/(10*x^2) - (b*ArcTan[c*x])/(5*x^5) +
 (b*c^5*Log[x])/5 - (b*c^5*Log[1 + c^2*x^2])/10
```

3.12.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5361, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^6}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{5}bc\int \frac{1}{x^5\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{5x^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{10}bc\int \frac{1}{x^6\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{5x^5}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{10}bc\int \left(-\frac{c^6}{c^2x^2+1}+\frac{c^4}{x^2}-\frac{c^2}{x^4}+\frac{1}{x^6}\right)dx^2-\frac{a+b\arctan(cx)}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}bc\left(c^4\log\left(x^2\right)+\frac{c^2}{x^2}-c^4\log\left(c^2x^2+1\right)-\frac{1}{2x^4}\right)-\frac{a+b\arctan(cx)}{5x^5}$$

input
```
Int[(a + b*ArcTan[c*x])/x^6,x]
```

output
```
-1/5*(a + b*ArcTan[c*x])/x^5 + (b*c*(-1/2*1/x^4 + c^2/x^2 + c^4*Log[x^2] -
 c^4*Log[1 + c^2*x^2]))/10
```

3.12.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.12.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

method	result	size
parts	$-\frac{a}{5x^5}+b\,c^5\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{1}{20c^4x^4}+\frac{\ln(cx)}{5}+\frac{1}{10c^2x^2}-\frac{\ln(c^2x^2+1)}{10}\right)$	60
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{1}{20c^4x^4}+\frac{\ln(cx)}{5}+\frac{1}{10c^2x^2}-\frac{\ln(c^2x^2+1)}{10}\right)\right)$	64
default	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\arctan(cx)}{5c^5x^5}-\frac{1}{20c^4x^4}+\frac{\ln(cx)}{5}+\frac{1}{10c^2x^2}-\frac{\ln(c^2x^2+1)}{10}\right)\right)$	64
parallelrisch	$\frac{4b\,c^5\ln(x)x^5-2b\,c^5\ln(c^2x^2+1)x^5-2b\,c^5x^5+2b\,c^3x^3-xbc-4b\arctan(cx)-4a}{20x^5}$	70
risch	$\frac{ib\ln(icx+1)}{10x^5}-\frac{-4b\,c^5\ln(x)x^5+2b\,c^5\ln(-c^2x^2-1)x^5-2b\,c^3x^3+2ib\ln(-icx+1)+xbc+4a}{20x^5}$	82

input
```
int((a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arctan(c*x)-1/20/c^4/x^4+1/5*ln(c*x)+1/10/c
^2/x^2-1/10*ln(c^2*x^2+1))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx$$
$$= -\frac{2\,bc^5x^5\log\left(c^2x^2+1\right)-4\,bc^5x^5\log\left(x\right)-2\,bc^3x^3+bcx+4\,b\arctan\left(cx\right)+4\,a}{20\,x^5}$$

input
```
integrate((a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/20*(2*b*c^5*x^5*log(c^2*x^2 + 1) - 4*b*c^5*x^5*log(x) - 2*b*c^3*x^3 + b
*c*x + 4*b*arctan(c*x) + 4*a)/x^5
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx$$
$$= \begin{cases} -\frac{a}{5x^5}+\frac{bc^5\log\left(x\right)}{5}-\frac{bc^5\log\left(x^2+\frac{1}{c^2}\right)}{10}+\frac{bc^3}{10x^2}-\frac{bc}{20x^4}-\frac{b\,\mathrm{atan}\left(cx\right)}{5x^5} & \text{for}\ c\neq 0\\ -\frac{a}{5x^5} & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*atan(c*x))/x**6,x)
```

output
```
Piecewise((-a/(5*x**5) + b*c**5*log(x)/5 - b*c**5*log(x**2 + c**(-2))/10 +
 b*c**3/(10*x**2) - b*c/(20*x**4) - b*atan(c*x)/(5*x**5), Ne(c, 0)), (-a/(
5*x**5), True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx$$
$$= -\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan\left(cx\right)}{x^5}\right)b-\frac{a}{5\,x^5}$$

input
```
integrate((a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
 4*arctan(c*x)/x^5)*b - 1/5*a/x^5
```

3.12.8 Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx=\int \frac{b\arctan(cx)+a}{x^6}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
sage0*x
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx=\frac{b\,c^5\,\ln(x)}{5}-\frac{b\,\mathrm{atan}(c\,x)}{5\,x^5}-\frac{b\,c^5\,\ln(c^2\,x^2+1)}{10}-\frac{-\frac{b\,c^3\,x^3}{2}+\frac{b\,c\,x}{4}+a}{5\,x^5}$$

input
```
int((a + b*atan(c*x))/x^6,x)
```

output
```
(b*c^5*log(x))/5 - (b*atan(c*x))/(5*x^5) - (b*c^5*log(c^2*x^2 + 1))/10 - (
a - (b*c^3*x^3)/2 + (b*c*x)/4)/(5*x^5)
```

3.13 $\int x^5(a+b\arctan(cx))^2\,dx$

3.13.1 Optimal result 136
3.13.2 Mathematica [A] (verified) 136
3.13.3 Rubi [A] (verified) 137
3.13.4 Maple [A] (verified) 140
3.13.5 Fricas [A] (verification not implemented) 141
3.13.6 Sympy [A] (verification not implemented) 141
3.13.7 Maxima [A] (verification not implemented) 142
3.13.8 Giac [F] 142
3.13.9 Mupad [B] (verification not implemented) 143

3.13.1 Optimal result

Integrand size = 14, antiderivative size = 144

$$\int x^5(a+b\arctan(cx))^2\,dx=-\frac{abx}{3c^5}-\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}-\frac{b^2x\arctan(cx)}{3c^5}+\frac{bx^3(a+b\arctan(cx))}{9c^3}-\frac{bx^5(a+b\arctan(cx))}{15c}+\frac{(a+b\arctan(cx))^2}{6c^6}+\frac{1}{6}x^6(a+b\arctan(cx))^2+\frac{23b^2\log(1+c^2x^2)}{90c^6}$$

output
```
-1/3*a*b*x/c^5-4/45*b^2*x^2/c^4+1/60*b^2*x^4/c^2-1/3*b^2*x*arctan(c*x)/c^5
+1/9*b*x^3*(a+b*arctan(c*x))/c^3-1/15*b*x^5*(a+b*arctan(c*x))/c+1/6*(a+b*a
rctan(c*x))^2/c^6+1/6*x^6*(a+b*arctan(c*x))^2+23/90*b^2*ln(c^2*x^2+1)/c^6
```

3.13.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96

$$\int x^5(a+b\arctan(cx))^2\,dx=\frac{cx(30a^2c^5x^5+b^2cx(-16+3c^2x^2)-4ab(15-5c^2x^2+3c^4x^4))+4b(bcx(-15+5c^2x^2-3c^4x^4)+15a(1+}{180c^6}$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x])^2,x]
```

output
```
(c*x*(30*a^2*c^5*x^5 + b^2*c*x*(-16 + 3*c^2*x^2) - 4*a*b*(15 - 5*c^2*x^2 +
 3*c^4*x^4)) + 4*b*(b*c*x*(-15 + 5*c^2*x^2 - 3*c^4*x^4) + 15*a*(1 + c^6*x^
6))*ArcTan[c*x] + 30*b^2*(1 + c^6*x^6)*ArcTan[c*x]^2 + 46*b^2*Log[1 + c^2*
x^2])/(180*c^6)
```

3.13.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.38, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5361, 5451, 5361, 243, 49, 2009, 5451, 5361, 243, 49, 2009, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\int\frac{x^6(a+b\arctan(cx))}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\int x^4(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{5}bc\int\frac{x^5}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\int\frac{x^4}{c^2x^2+1}dx^2}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\int\left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx^2}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5451

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\int x^2(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}}{c^2}\right)$$

$\downarrow$ 5361

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{3}bc\int\frac{x^3}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx)}{c^2x^2+1}}{c^2}}{c^2}\right.$$

$\downarrow$ 243

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int\frac{x^2}{c^2x^2+1}dx^2}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx}{c^2x^2+1}}{c^2}}{c^2}\right.$$

$\downarrow$ 49

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int\left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2}{c^2}-\int x^2(}{c^2}\right.$$

$\downarrow$ 2009

$$\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2}-\frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\int\frac{x^2(a+b\,a}{c^2x}}{c^2}\right.$$

$\downarrow$ 5451

3.13. $\int x^5(a+b\arctan(cx))^2\,dx$

$$\frac{1}{6}x^6(a+b\arctan(cx))^2 - \frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx)) - \frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2} - \frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx)) - \frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2} - \frac{\int(a+b\arctan \ldots}{c^2}}{c^2}\right.$$

$\downarrow$ 2009

$$\frac{1}{6}x^6(a+b\arctan(cx))^2 - \frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx)) - \frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2} - \frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx)) - \frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2} - \frac{ax+bx\,\text{arct}\ldots}{\ldots}}{c^2}\right.$$

$\downarrow$ 5419

$$\frac{1}{6}x^6(a+b\arctan(cx))^2 - \frac{1}{3}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx)) - \frac{1}{10}bc\left(-\frac{x^2}{c^4}+\frac{x^4}{2c^2}+\frac{\log(c^2x^2+1)}{c^6}\right)}{c^2} - \frac{\frac{\frac{1}{3}x^3(a+b\arctan(cx)) - \frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2} - \frac{ax+bx\,\text{arct}\ldots}{\ldots}}{c^2}\right.$$

input
```
Int[x^5*(a + b*ArcTan[c*x])^2,x]
```

output
```
(x^6*(a + b*ArcTan[c*x])^2)/6 - (b*c*(((x^5*(a + b*ArcTan[c*x]))/5 - (b*c*
(-(x^2/c^4) + x^4/(2*c^2) + Log[1 + c^2*x^2]/c^6))/10)/c^2 - (((x^3*(a + b
*ArcTan[c*x]))/3 - (b*c*(x^2/c^2 - Log[1 + c^2*x^2]/c^4))/6)/c^2 - (-1/2*(
a + b*ArcTan[c*x])^2/(b*c^3) + (a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2
])/(2*c))/c^2)/c^2)/c^2))/3
```

3.13.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.13.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

method	result
parts	$\frac{x^6 a^2}{6} + \frac{b^2 \left(\frac{c^6 x^6 \arctan(cx)^2}{6} - \frac{c^5 x^5 \arctan(cx)}{15} + \frac{c^3 x^3 \arctan(cx)}{9} - \frac{cx \arctan(cx)}{3} + \frac{\arctan(cx)^2}{6} + \frac{c^4 x^4}{60} - \frac{4c^2 x^2}{45} + \frac{23 \ln\left(c^2 x^2 + 1 \cdots\right.}{90} \cdots \right.}{c^6}$
derivativedivides	$\frac{\frac{a^2 c^6 x^6}{6} + b^2 \left(\frac{c^6 x^6 \arctan(cx)^2}{6} - \frac{c^5 x^5 \arctan(cx)}{15} + \frac{c^3 x^3 \arctan(cx)}{9} - \frac{cx \arctan(cx)}{3} + \frac{\arctan(cx)^2}{6} + \frac{c^4 x^4}{60} - \frac{4c^2 x^2}{45} + \frac{23 \ln\left(c^2 x^2 + 1 \cdots\right.}{90} \cdots \right.}{c^6}$
default	$\frac{\frac{a^2 c^6 x^6}{6} + b^2 \left(\frac{c^6 x^6 \arctan(cx)^2}{6} - \frac{c^5 x^5 \arctan(cx)}{15} + \frac{c^3 x^3 \arctan(cx)}{9} - \frac{cx \arctan(cx)}{3} + \frac{\arctan(cx)^2}{6} + \frac{c^4 x^4}{60} - \frac{4c^2 x^2}{45} + \frac{23 \ln\left(c^2 x^2 + 1 \cdots\right.}{90} \cdots \right.}{c^6}$
parallelrisch	$\frac{30b^2 \arctan(cx)^2 x^6 c^6 + 60ab \arctan(cx) x^6 c^6 + 30a^2 c^6 x^6 - 12b^2 \arctan(cx) x^5 c^5 - 12ab\, c^5 x^5 + 3b^2 c^4 x^4 + 20b^2 \arctan(cx) x^3 c \cdots}{180c^6}$
risch	$-\frac{b^2 (c^6 x^6 + 1) \ln(icx + 1)^2}{24c^6} - \frac{ib\left(30a\, c^6 x^6 + 15ib\, c^6 x^6 \ln(-icx + 1) - 6b\, c^5 x^5 + 10b\, c^3 x^3 - 30xbc + 15ib \ln(-icx + 1)\right) \ln(icx \cdots}{180c^6}$

input
```
int(x^5*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a^2+b^2/c^6*(1/6*c^6*x^6*arctan(c*x)^2-1/15*c^5*x^5*arctan(c*x)+1/
9*c^3*x^3*arctan(c*x)-1/3*c*x*arctan(c*x)+1/6*arctan(c*x)^2+1/60*c^4*x^4-4
/45*c^2*x^2+23/90*ln(c^2*x^2+1))+2*a*b/c^6*(1/6*c^6*x^6*arctan(c*x)-1/30*c
^5*x^5+1/18*c^3*x^3-1/6*c*x+1/6*arctan(c*x))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

$$\int x^5(a+b\arctan(cx))^2\,dx$$
$$= \frac{30\,a^2c^6x^6 - 12\,abc^5x^5 + 3\,b^2c^4x^4 + 20\,abc^3x^3 - 16\,b^2c^2x^2 - 60\,abcx + 30\,(b^2c^6x^6 + b^2)\arctan{(cx)}^2 + 46\,b^2}{180\,c^6}$$

input
```
integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/180*(30*a^2*c^6*x^6 - 12*a*b*c^5*x^5 + 3*b^2*c^4*x^4 + 20*a*b*c^3*x^3 -
16*b^2*c^2*x^2 - 60*a*b*c*x + 30*(b^2*c^6*x^6 + b^2)*arctan(c*x)^2 + 46*b^
2*log(c^2*x^2 + 1) + 4*(15*a*b*c^6*x^6 - 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 1
5*b^2*c*x + 15*a*b)*arctan(c*x))/c^6
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.38

$$\int x^5(a+b\arctan(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2x^6}{6} + \frac{abx^6\operatorname{atan}{(cx)}}{3} - \frac{abx^5}{15c} + \frac{abx^3}{9c^3} - \frac{abx}{3c^5} + \frac{ab\operatorname{atan}{(cx)}}{3c^6} + \frac{b^2x^6\operatorname{atan}^2{(cx)}}{6} - \frac{b^2x^5\operatorname{atan}{(cx)}}{15c} + \frac{b^2x^4}{60c^2} + \frac{b^2x^3\operatorname{atan}{(cx)}}{9c^3} - \frac{4b^2}{45} \\ \frac{a^2x^6}{6} \end{cases}$$

input
```
integrate(x**5*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*x**6/6 + a*b*x**6*atan(c*x)/3 - a*b*x**5/(15*c) + a*b*x**3
/(9*c**3) - a*b*x/(3*c**5) + a*b*atan(c*x)/(3*c**6) + b**2*x**6*atan(c*x)*
*2/6 - b**2*x**5*atan(c*x)/(15*c) + b**2*x**4/(60*c**2) + b**2*x**3*atan(c
*x)/(9*c**3) - 4*b**2*x**2/(45*c**4) - b**2*x*atan(c*x)/(3*c**5) + 23*b**2
*log(x**2 + c**(-2))/(90*c**6) + b**2*atan(c*x)**2/(6*c**6), Ne(c, 0)), (a
**2*x**6/6, True))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.13

$$\int x^5(a+b\arctan(cx))^2\,dx = \frac{1}{6}\,b^2x^6\arctan\left(cx\right)^2 + \frac{1}{6}\,a^2x^6 + \frac{1}{45}\left(15\,x^6\arctan\left(cx\right) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\arctan\left(cx\right)}{c^7}\right)\right)ab - \frac{1}{180}\left(4\,c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\arctan\left(cx\right)}{c^7}\right)\arctan\left(cx\right) - \frac{3\,c^4x^4 - 16\,c^2x^2 - 30\arctan\left(cx\right)^2 +}{c^6}\right.$$

input
```
integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6*arctan(c*x)^2 + 1/6*a^2*x^6 + 1/45*(15*x^6*arctan(c*x) - c*((3
*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a*b - 1/180*(4*c*(
(3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)*arctan(c*x) - (3*
c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c^2*x^2 + 1))/c^6)*b^2
```

3.13.8 Giac [F]

$$\int x^5(a+b\arctan(cx))^2\,dx = \int \left(b\arctan\left(cx\right)+a\right)^2x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.19

$$\int x^5(a+b\arctan(cx))^2\,dx$$

$$= \frac{30\,b^2\,\mathrm{atan}(c\,x)^2+46\,b^2\,\ln\left(c^2\,x^2+1\right)+30\,a^2\,c^6\,x^6-16\,b^2\,c^2\,x^2+3\,b^2\,c^4\,x^4+60\,a\,b\,\mathrm{atan}(c\,x)+20\,b^2\,c^3\,x^3}{}$$

input
```
int(x^5*(a + b*atan(c*x))^2,x)
```

output
```
(30*b^2*atan(c*x)^2 + 46*b^2*log(c^2*x^2 + 1) + 30*a^2*c^6*x^6 - 16*b^2*c^
2*x^2 + 3*b^2*c^4*x^4 + 60*a*b*atan(c*x) + 20*b^2*c^3*x^3*atan(c*x) - 12*b
^2*c^5*x^5*atan(c*x) - 60*b^2*c*x*atan(c*x) + 30*b^2*c^6*x^6*atan(c*x)^2 +
 20*a*b*c^3*x^3 - 12*a*b*c^5*x^5 - 60*a*b*c*x + 60*a*b*c^6*x^6*atan(c*x))/
(180*c^6)
```

3.14 $\int x^4(a+b\arctan(cx))^2\,dx$

3.14.1 Optimal result . . . 144
3.14.2 Mathematica [A] (verified) . . . 144
3.14.3 Rubi [A] (verified) . . . 145
3.14.4 Maple [A] (verified) . . . 149
3.14.5 Fricas [F] . . . 150
3.14.6 Sympy [F] . . . 150
3.14.7 Maxima [F] . . . 150
3.14.8 Giac [F] . . . 151
3.14.9 Mupad [F(-1)] . . . 151

3.14.1 Optimal result

Integrand size = 14, antiderivative size = 170

$$\int x^4(a+b\arctan(cx))^2\,dx = -\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{3b^2\arctan(cx)}{10c^5}+\frac{bx^2(a+b\arctan(cx))}{5c^3} - \frac{bx^4(a+b\arctan(cx))}{10c}+\frac{i(a+b\arctan(cx))^2}{5c^5} + \frac{1}{5}x^5(a+b\arctan(cx))^2+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^5} + \frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}$$

output
```
-3/10*b^2*x/c^4+1/30*b^2*x^3/c^2+3/10*b^2*arctan(c*x)/c^5+1/5*b*x^2*(a+b*a
rctan(c*x))/c^3-1/10*b*x^4*(a+b*arctan(c*x))/c+1/5*I*(a+b*arctan(c*x))^2/c
^5+1/5*x^5*(a+b*arctan(c*x))^2+2/5*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5
+1/5*I*b^2*polylog(2,1-2/(1+I*c*x))/c^5
```

3.14.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int x^4(a+b\arctan(cx))^2\,dx$$
$$= \frac{9ab-9b^2cx+6abc^2x^2+b^2c^3x^3-3abc^4x^4+6a^2c^5x^5+6b^2(-i+c^5x^5)\arctan(cx)^2-3b\arctan(cx)\left(-4a\right.\cdots}{\cdots}$$

input
```
Integrate[x^4*(a + b*ArcTan[c*x])^2,x]
```

output
```
(9*a*b - 9*b^2*c*x + 6*a*b*c^2*x^2 + b^2*c^3*x^3 - 3*a*b*c^4*x^4 + 6*a^2*c
^5*x^5 + 6*b^2*(-I + c^5*x^5)*ArcTan[c*x]^2 - 3*b*ArcTan[c*x]*(-4*a*c^5*x^
5 + b*(-3 - 2*c^2*x^2 + c^4*x^4) - 4*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - 6
*a*b*Log[1 + c^2*x^2] - (6*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*
c^5)
```

3.14.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {5361, 5451, 5361, 254, 2009, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\arctan(cx))}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\int x^3(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\int\frac{x^4}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\int\left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \ 5451$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}}{c^2}\right)$$

$$\Big\downarrow \ 5361$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int\frac{x^2}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}}{c^2}\right)$$

$$\Big\downarrow \ 262$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx}{c^2x^2+1}}{c^2}}{c^2}\right.$$

$$\Big\downarrow \ 216$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}d}{c^2}}{c^2}\right.$$

$$\Big\downarrow \ 5455$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}}{c}}{c^2}\right.$$

$$\Big\downarrow \ 5379$$

3.14. $\int x^4(a+b\arctan(cx))^2\,dx$

$$\frac{1}{5}x^5(a+b\arctan(cx))^2 - \frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx)) - \frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5} - \frac{x}{c^4} + \frac{x^3}{3c^2}\right)}{c^2} - \frac{\frac{1}{2}x^2(a+b\arctan(cx)) - \frac{1}{2}bc\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)}{c^2} - \frac{-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\,\text{ar}}{c}}{}}{c^2}\right.$$

$\Big\downarrow$ 2849

$$\frac{1}{5}x^5(a+b\arctan(cx))^2 - \frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx)) - \frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5} - \frac{x}{c^4} + \frac{x^3}{3c^2}\right)}{c^2} - \frac{\frac{1}{2}x^2(a+b\arctan(cx)) - \frac{1}{2}bc\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)}{c^2} - \frac{-\frac{\frac{ib\int \frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{}{i}}{c}}{}}{c^2}\right.$$

$\Big\downarrow$ 2752

$$\frac{1}{5}x^5(a+b\arctan(cx))^2 - \frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx)) - \frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5} - \frac{x}{c^4} + \frac{x^3}{3c^2}\right)}{c^2} - \frac{\frac{1}{2}x^2(a+b\arctan(cx)) - \frac{1}{2}bc\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)}{c^2} - \frac{-\frac{i(a+b\arctan(cx))^2}{2bc^2} - }{c^2}\right.$$

input
```
Int[x^4*(a + b*ArcTan[c*x])^2,x]
```

output
```
(x^5*(a + b*ArcTan[c*x])^2)/5 - (2*b*c*(((x^4*(a + b*ArcTan[c*x]))/4 - (b*
c*(-(x/c^4) + x^3/(3*c^2) + ArcTan[c*x]/c^5))/4)/c^2 - (((x^2*(a + b*ArcTa
n[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2)/c^2 - (((-1/2*I)*(a + b*Ar
cTan[c*x])^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/
2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/c)/c)/c^2)/c^2))/5
```

3.14.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.14. $\int x^4(a+b\arctan(cx))^2\,dx$

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.14.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.56

method	result
parts	$\frac{a^2x^5}{5} + \frac{b^2\left(\frac{c^5x^5\arctan(cx)^2}{5} - \frac{c^4x^4\arctan(cx)}{10} + \frac{c^2x^2\arctan(cx)}{5} - \frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{5} + \frac{c^3x^3}{30} - \frac{3cx}{10} + \frac{3\arctan(cx)}{10} - \frac{i\left(\ln(\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2c^5x^5}{5} + b^2\left(\frac{c^5x^5\arctan(cx)^2}{5} - \frac{c^4x^4\arctan(cx)}{10} + \frac{c^2x^2\arctan(cx)}{5} - \frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{5} + \frac{c^3x^3}{30} - \frac{3cx}{10} + \frac{3\arctan(cx)}{10} - \frac{i\left(\ln(\right.}{}\right.}{}$
default	$\frac{\frac{a^2c^5x^5}{5} + b^2\left(\frac{c^5x^5\arctan(cx)^2}{5} - \frac{c^4x^4\arctan(cx)}{10} + \frac{c^2x^2\arctan(cx)}{5} - \frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{5} + \frac{c^3x^3}{30} - \frac{3cx}{10} + \frac{3\arctan(cx)}{10} - \frac{i\left(\ln(\right.}{}\right.}{}$
risch	$\frac{ib^2\ln(-icx+1)x^2}{10c^3} + \frac{ib^2\ln(icx+1)x^4}{20c} + \frac{137ab}{150c^5} - \frac{ab\,x^4}{10c} + \frac{413ib^2}{2250c^5} + \frac{ia^2}{5c^5} - \frac{b^2\ln(icx+1)^2x^5}{20} - \frac{b^2\ln(-icx+1)^2}{20}$

input
```
int(x^4*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+b^2/c^5*(1/5*c^5*x^5*arctan(c*x)^2-1/10*c^4*x^4*arctan(c*x)+1/
5*c^2*x^2*arctan(c*x)-1/5*arctan(c*x)*ln(c^2*x^2+1)+1/30*c^3*x^3-3/10*c*x+
3/10*arctan(c*x)-1/10*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/
2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+1/10*I*(ln(c*x+I)*ln(c^2*x^2+1)
-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))))+2*a*b/
c^5*(1/5*c^5*x^5*arctan(c*x)-1/20*c^4*x^4+1/10*c^2*x^2-1/10*ln(c^2*x^2+1))
```

3.14.5 Fricas [F]

$$\int x^4(a+b\arctan(cx))^2\,dx=\int(b\arctan(cx)+a)^2x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^4*arctan(c*x)^2 + 2*a*b*x^4*arctan(c*x) + a^2*x^4, x)
```

3.14.6 Sympy [F]

$$\int x^4(a+b\arctan(cx))^2\,dx=\int x^4(a+b\,\mathrm{atan}(cx))^2\,dx$$

input
```
integrate(x**4*(a+b*atan(c*x))**2,x)
```

output
```
Integral(x**4*(a + b*atan(c*x))**2, x)
```

3.14.7 Maxima [F]

$$\int x^4(a+b\arctan(cx))^2\,dx=\int(b\arctan(cx)+a)^2x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*x^5 + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c
^2*x^2 + 1)/c^6))*a*b + 1/80*(4*x^5*arctan(c*x)^2 - x^5*log(c^2*x^2 + 1)^2
 + 80*integrate(1/80*(4*c^2*x^6*log(c^2*x^2 + 1) - 8*c*x^5*arctan(c*x) + 6
0*(c^2*x^6 + x^4)*arctan(c*x)^2 + 5*(c^2*x^6 + x^4)*log(c^2*x^2 + 1)^2)/(c
^2*x^2 + 1), x))*b^2
```

3.14.8 Giac [F]

$$\int x^4(a+b\arctan(cx))^2\,dx=\int(b\arctan(cx)+a)^2x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int x^4(a+b\arctan(cx))^2\,dx=\int x^4\,(a+b\,\mathrm{atan}(c\,x))^2\,dx$$

input
```
int(x^4*(a + b*atan(c*x))^2,x)
```

output
```
int(x^4*(a + b*atan(c*x))^2, x)
```

3.15 $\int x^3(a+b\arctan(cx))^2\,dx$

3.15.1 Optimal result 152
3.15.2 Mathematica [A] (verified) 152
3.15.3 Rubi [A] (verified) 153
3.15.4 Maple [A] (verified) 155
3.15.5 Fricas [A] (verification not implemented) 156
3.15.6 Sympy [A] (verification not implemented) 156
3.15.7 Maxima [A] (verification not implemented) 157
3.15.8 Giac [F] 157
3.15.9 Mupad [B] (verification not implemented) 157

3.15.1 Optimal result

Integrand size = 14, antiderivative size = 112

$$\int x^3(a+b\arctan(cx))^2\,dx=\frac{abx}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2x\arctan(cx)}{2c^3}-\frac{bx^3(a+b\arctan(cx))}{6c}-\frac{(a+b\arctan(cx))^2}{4c^4}+\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{b^2\log(1+c^2x^2)}{3c^4}$$

output
```
1/2*a*b*x/c^3+1/12*b^2*x^2/c^2+1/2*b^2*x*arctan(c*x)/c^3-1/6*b*x^3*(a+b*ar
ctan(c*x))/c-1/4*(a+b*arctan(c*x))^2/c^4+1/4*x^4*(a+b*arctan(c*x))^2-1/3*b
^2*ln(c^2*x^2+1)/c^4
```

3.15.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int x^3(a+b\arctan(cx))^2\,dx=\frac{cx(6ab+b^2cx-2abc^2x^2+3a^2c^3x^3)-2b(bcx(-3+c^2x^2)+a(3-3c^4x^4))\arctan(cx)+3b^2(-1+c^4x^4)\,a}{12c^4}$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(c*x*(6*a*b + b^2*c*x - 2*a*b*c^2*x^2 + 3*a^2*c^3*x^3) - 2*b*(b*c*x*(-3 +
c^2*x^2) + a*(3 - 3*c^4*x^4))*ArcTan[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTan[c*
x]^2 - 4*b^2*Log[1 + c^2*x^2])/(12*c^4)
```

3.15.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5361, 5451, 5361, 243, 49, 2009, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\int x^2(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{3}bc\int\frac{x^3}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int\frac{x^2}{c^2x^2+1}dx^2}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int\left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-$$
$$\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-$$
$$\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\frac{\int(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-$$
$$\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}}{c^2}\right)$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^2-$$
$$\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}}{c^2}\right)$$

input
```
Int[x^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(x^4*(a + b*ArcTan[c*x])^2)/4 - (b*c*(((x^3*(a + b*ArcTan[c*x]))/3 - (b*c*
(x^2/c^2 - Log[1 + c^2*x^2]/c^4))/6)/c^2 - (-1/2*(a + b*ArcTan[c*x])^2/(b*
c^3) + (a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/c^2)/c^2))/2
```

3.15.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.15.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05

method	result
parts	$\frac{a^2x^4}{4}+\frac{b^2\left(\frac{c^4x^4\arctan(cx)^2}{4}-\frac{c^3x^3\arctan(cx)}{6}+\frac{cx\arctan(cx)}{2}-\frac{\arctan(cx)^2}{4}+\frac{c^2x^2}{12}-\frac{\ln(c^2x^2+1)}{3}\right)}{c^4}+\frac{2ab\left(\frac{c^4x^4\arctan(cx}{4}\right.}{}$
derivativedivides	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(\frac{c^4x^4\arctan(cx)^2}{4}-\frac{c^3x^3\arctan(cx)}{6}+\frac{cx\arctan(cx)}{2}-\frac{\arctan(cx)^2}{4}+\frac{c^2x^2}{12}-\frac{\ln(c^2x^2+1)}{3}\right)+2ab\left(\frac{c^4x^4\arctan(cx)}{4}-\right.}{c^4}$
default	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(\frac{c^4x^4\arctan(cx)^2}{4}-\frac{c^3x^3\arctan(cx)}{6}+\frac{cx\arctan(cx)}{2}-\frac{\arctan(cx)^2}{4}+\frac{c^2x^2}{12}-\frac{\ln(c^2x^2+1)}{3}\right)+2ab\left(\frac{c^4x^4\arctan(cx)}{4}-\right.}{c^4}$
parallelrisch	$-\frac{-3x^4\arctan(cx)^2b^2c^4-6x^4\arctan(cx)abc^4-3a^2c^4x^4+2b^2\arctan(cx)x^3c^3+2abc^3x^3-b^2c^2x^2-6b^2\arctan(cx)xc-6a}{12c^4}$
risch	$-\frac{b^2(c^4x^4-1)\ln(icx+1)^2}{16c^4}-\frac{ib(6ac^4x^4+3ibc^4x^4\ln(-icx+1)-2bc^3x^3+6xbc-3ib\ln(-icx+1))\ln(icx+1)}{24c^4}-\frac{b^2x^4\ln}{}$

input
```
int(x^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+b^2/c^4*(1/4*c^4*x^4*arctan(c*x)^2-1/6*c^3*x^3*arctan(c*x)+1/2
*c*x*arctan(c*x)-1/4*arctan(c*x)^2+1/12*c^2*x^2-1/3*ln(c^2*x^2+1))+2*a*b/c
^4*(1/4*c^4*x^4*arctan(c*x)-1/12*c^3*x^3+1/4*c*x-1/4*arctan(c*x))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{3\,a^2c^4x^4-2\,abc^3x^3+b^2c^2x^2+6\,abcx+3\left(b^2c^4x^4-b^2\right)\arctan\left(cx\right)^2-4\,b^2\log\left(c^2x^2+1\right)+2\left(3\,abc^4x^4-b\right.}{12\,c^4}$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/12*(3*a^2*c^4*x^4 - 2*a*b*c^3*x^3 + b^2*c^2*x^2 + 6*a*b*c*x + 3*(b^2*c^4
*x^4 - b^2)*arctan(c*x)^2 - 4*b^2*log(c^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 - b^
2*c^3*x^3 + 3*b^2*c*x - 3*a*b)*arctan(c*x))/c^4
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.38

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2x^4}{4}+\frac{abx^4\operatorname{atan}\left(cx\right)}{2}-\frac{abx^3}{6c}+\frac{abx}{2c^3}-\frac{ab\operatorname{atan}\left(cx\right)}{2c^4}+\frac{b^2x^4\operatorname{atan}^2\left(cx\right)}{4}-\frac{b^2x^3\operatorname{atan}\left(cx\right)}{6c}+\frac{b^2x^2}{12c^2}+\frac{b^2x\operatorname{atan}\left(cx\right)}{2c^3}-\frac{b^2\log\left(x^2+\frac{1}{c}\right.}{3c^4} \\ \frac{a^2x^4}{4}\end{cases}$$

input
```
integrate(x**3*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*x**4/4 + a*b*x**4*atan(c*x)/2 - a*b*x**3/(6*c) + a*b*x/(2*
c**3) - a*b*atan(c*x)/(2*c**4) + b**2*x**4*atan(c*x)**2/4 - b**2*x**3*atan
(c*x)/(6*c) + b**2*x**2/(12*c**2) + b**2*x*atan(c*x)/(2*c**3) - b**2*log(x
**2 + c**(-2))/(3*c**4) - b**2*atan(c*x)**2/(4*c**4), Ne(c, 0)), (a**2*x**
4/4, True))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{1}{4}\,b^2x^4\arctan\left(cx\right)^2+\frac{1}{4}\,a^2x^4+\frac{1}{6}\,\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)ab$$
$$-\frac{1}{12}\,\left(2\,c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\arctan\left(cx\right)-\frac{c^2x^2+3\,\arctan\left(cx\right)^2-4\,\log\left(c^2x^2+1\right)}{c^4}\right)b^2$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arctan(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arctan(c*x) - c*((c^2
*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4
 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2
*x^2 + 1))/c^4)*b^2
```

3.15.8 Giac [F]

$$\int x^3(a+b\arctan(cx))^2\,dx=\int\left(b\arctan\left(cx\right)+a\right)^2x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.20

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{3\,a^2\,c^4\,x^4-4\,b^2\,\ln\left(c^2\,x^2+1\right)-3\,b^2\,\mathrm{atan}(c\,x)^2+b^2\,c^2\,x^2-6\,a\,b\,\mathrm{atan}(c\,x)-2\,b^2\,c^3\,x^3\,\mathrm{atan}(c\,x)+6\,b^2\,c\,x\,a}{12\,c^4}$$

input
```
int(x^3*(a + b*atan(c*x))^2,x)
```

output
```
(3*a^2*c^4*x^4 - 4*b^2*log(c^2*x^2 + 1) - 3*b^2*atan(c*x)^2 + b^2*c^2*x^2
- 6*a*b*atan(c*x) - 2*b^2*c^3*x^3*atan(c*x) + 6*b^2*c*x*atan(c*x) + 3*b^2*
c^4*x^4*atan(c*x)^2 - 2*a*b*c^3*x^3 + 6*a*b*c*x + 6*a*b*c^4*x^4*atan(c*x))
/(12*c^4)
```

3.15. $\int x^3(a+b\arctan(cx))^2\,dx$

3.16 $\int x^2(a+b\arctan(cx))^2\,dx$

3.16.1 Optimal result 159
3.16.2 Mathematica [A] (verified) 159
3.16.3 Rubi [A] (verified) 160
3.16.4 Maple [A] (verified) 163
3.16.5 Fricas [F] 164
3.16.6 Sympy [F] 164
3.16.7 Maxima [F] 164
3.16.8 Giac [F] 165
3.16.9 Mupad [F(-1)] 165

3.16.1 Optimal result

Integrand size = 14, antiderivative size = 138

$$\int x^2(a+b\arctan(cx))^2\,dx = \frac{b^2x}{3c^2}-\frac{b^2\arctan(cx)}{3c^3}-\frac{bx^2(a+b\arctan(cx))}{3c} - \frac{i(a+b\arctan(cx))^2}{3c^3}+\frac{1}{3}x^3(a+b\arctan(cx))^2 - \frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}$$

output
```
1/3*b^2*x/c^2-1/3*b^2*arctan(c*x)/c^3-1/3*b*x^2*(a+b*arctan(c*x))/c-1/3*I*
(a+b*arctan(c*x))^2/c^3+1/3*x^3*(a+b*arctan(c*x))^2-2/3*b*(a+b*arctan(c*x)
)*ln(2/(1+I*c*x))/c^3-1/3*I*b^2*polylog(2,1-2/(1+I*c*x))/c^3
```

3.16.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.95

$$\int x^2(a+b\arctan(cx))^2\,dx = \frac{b^2cx-abc^2x^2+a^2c^3x^3+b^2(i+c^3x^3)\arctan(cx)^2-b\arctan(cx)\left(b+bc^2x^2-2ac^3x^3+2b\log\left(1+e^{2i\,\mathrm{arct}}\right.\right.}{3c^3}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(b^2*c*x - a*b*c^2*x^2 + a^2*c^3*x^3 + b^2*(I + c^3*x^3)*ArcTan[c*x]^2 - b
*ArcTan[c*x]*(b + b*c^2*x^2 - 2*a*c^3*x^3 + 2*b*Log[1 + E^((2*I)*ArcTan[c*
x])]) + a*b*Log[1 + c^2*x^2] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(
3*c^3)
```

3.16.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5361, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int\frac{x^2}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5455

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}}{c^2}\right)$$

$\downarrow$ 5379

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}}{c^2}\right)$$

$\downarrow$ 2849

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}}{c^2}\right)$$

$\downarrow$ 2752

$$\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}}{c}}{c^2}\right)$$

input
```
Int[x^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(x^3*(a + b*ArcTan[c*x])^2)/3 - (2*b*c*(((x^2*(a + b*ArcTan[c*x]))/2 - (b*
c*(x/c^2 - ArcTan[c*x]/c^3))/2)/c^2 - (((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b
*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2,
1 - 2/(1 + I*c*x)])/c)/c)/c^2))/3
```

3.16. $\int x^2(a+b\arctan(cx))^2\,dx$

3.16.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.16.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.72

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\frac{c^3x^3\arctan(cx)^2}{3}-\frac{c^2x^2\arctan(cx)}{3}+\frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{cx}{3}-\frac{\arctan(cx)}{3}+\frac{i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i)}{2}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{c^3x^3\arctan(cx)^2}{3}-\frac{c^2x^2\arctan(cx)}{3}+\frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{cx}{3}-\frac{\arctan(cx)}{3}+\frac{i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i)}{2}\right.}{}\right.}{}$
default	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{c^3x^3\arctan(cx)^2}{3}-\frac{c^2x^2\arctan(cx)}{3}+\frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{cx}{3}-\frac{\arctan(cx)}{3}+\frac{i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i)}{2}\right.}{}\right.}{}$
risch	$\frac{ib^2\ln(-icx+1)^2}{12c^3}+\frac{b^2\ln(icx+1)\ln(-icx+1)x^3}{6}-\frac{2ib^2\ln\left(c^2x^2+1\right)}{9c^3}+\frac{5ib^2\ln(-icx+1)}{36c^3}-\frac{ib^2\operatorname{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{3c^3}-ib$

input
```
int(x^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+b^2/c^3*(1/3*c^3*x^3*arctan(c*x)^2-1/3*c^2*x^2*arctan(c*x)+1/3
*arctan(c*x)*ln(c^2*x^2+1)+1/3*c*x-1/3*arctan(c*x)+1/6*I*(ln(c*x-I)*ln(c^2
*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I))
)-1/6*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c
*x+I)*ln(1/2*I*(c*x-I))))+2*a*b/c^3*(1/3*c^3*x^3*arctan(c*x)-1/6*c^2*x^2+1
/6*ln(c^2*x^2+1))
```

3.16.5 Fricas [F]

$$\int x^2(a+b\arctan(cx))^2\,dx=\int(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2, x)
```

3.16.6 Sympy [F]

$$\int x^2(a+b\arctan(cx))^2\,dx=\int x^2(a+b\operatorname{atan}(cx))^2\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))**2,x)
```

output
```
Integral(x**2*(a + b*atan(c*x))**2, x)
```

3.16.7 Maxima [F]

$$\int x^2(a+b\arctan(cx))^2\,dx=\int(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))
*a*b + 1/48*(4*x^3*arctan(c*x)^2 - x^3*log(c^2*x^2 + 1)^2 + 48*integrate(1
/48*(4*c^2*x^4*log(c^2*x^2 + 1) - 8*c*x^3*arctan(c*x) + 36*(c^2*x^4 + x^2)
*arctan(c*x)^2 + 3*(c^2*x^4 + x^2)*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x))*
b^2
```

3.16.8 Giac [F]

$$\int x^2(a+b\arctan(cx))^2\,dx = \int (b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arctan(cx))^2\,dx = \int x^2\,(a+b\,\mathrm{atan}(c\,x))^2\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^2,x)
```

output
```
int(x^2*(a + b*atan(c*x))^2, x)
```

3.17 $\int x(a+b\arctan(cx))^2\,dx$

3.17.1 Optimal result 166
3.17.2 Mathematica [A] (verified) 166
3.17.3 Rubi [A] (verified) 167
3.17.4 Maple [A] (verified) 168
3.17.5 Fricas [A] (verification not implemented) 169
3.17.6 Sympy [A] (verification not implemented) 169
3.17.7 Maxima [A] (verification not implemented) 170
3.17.8 Giac [F] 170
3.17.9 Mupad [B] (verification not implemented) 170

3.17.1 Optimal result

Integrand size = 12, antiderivative size = 76

$$\int x(a+b\arctan(cx))^2\,dx=-\frac{abx}{c}-\frac{b^2x\arctan(cx)}{c}+\frac{(a+b\arctan(cx))^2}{2c^2}+\frac{1}{2}x^2(a+b\arctan(cx))^2+\frac{b^2\log(1+c^2x^2)}{2c^2}$$

output
```
-a*b*x/c-b^2*x*arctan(c*x)/c+1/2*(a+b*arctan(c*x))^2/c^2+1/2*x^2*(a+b*arct
an(c*x))^2+1/2*b^2*ln(c^2*x^2+1)/c^2
```

3.17.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int x(a+b\arctan(cx))^2\,dx=\frac{acx(-2b+acx)+2b(a-bcx+ac^2x^2)\arctan(cx)+b^2(1+c^2x^2)\arctan(cx)^2+b^2\log(1+c^2x^2)}{2c^2}$$

input
```
Integrate[x*(a + b*ArcTan[c*x])^2,x]
```

output
```
(a*c*x*(-2*b + a*c*x) + 2*b*(a - b*c*x + a*c^2*x^2)*ArcTan[c*x] + b^2*(1 +
 c^2*x^2)*ArcTan[c*x]^2 + b^2*Log[1 + c^2*x^2])/(2*c^2)
```

3.17.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5361, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{\int(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)$$

input
```
Int[x*(a + b*ArcTan[c*x])^2,x]
```

output
```
(x^2*(a + b*ArcTan[c*x])^2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) +
(a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/c^2)
```

3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.17.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

method	result
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\frac{c^2x^2\arctan(cx)^2}{2}+\frac{\arctan(cx)^2}{2}-cx\arctan(cx)+\frac{\ln\left(c^2x^2+1\right)}{2}\right)}{c^2}+x^2ab\arctan\left(cx\right)-\frac{abx}{c}+\frac{ab\operatorname{arct}}{c}$
derivativedivides	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{c^2x^2\arctan(cx)^2}{2}+\frac{\arctan(cx)^2}{2}-cx\arctan(cx)+\frac{\ln\left(c^2x^2+1\right)}{2}\right)+2ab\left(\frac{c^2x^2\arctan(cx)}{2}-\frac{cx}{2}+\frac{\arctan(cx)}{2}\right)}{c^2}$
default	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{c^2x^2\arctan(cx)^2}{2}+\frac{\arctan(cx)^2}{2}-cx\arctan(cx)+\frac{\ln\left(c^2x^2+1\right)}{2}\right)+2ab\left(\frac{c^2x^2\arctan(cx)}{2}-\frac{cx}{2}+\frac{\arctan(cx)}{2}\right)}{c^2}$
parallelrisch	$\frac{b^2\arctan(cx)^2x^2c^2+2ab\arctan(cx)x^2c^2+c^2x^2a^2-2b^2\arctan(cx)xc-2abcx+b^2\arctan(cx)^2+b^2\ln(c^2x^2+1)+2ab\operatorname{arctan}}{2c^2}$
risch	$-\frac{b^2\left(c^2x^2+1\right)\ln(icx+1)^2}{8c^2}-\frac{ib\left(2c^2x^2a+ib\,c^2x^2\ln(-icx+1)-2xbc+ib\ln(-icx+1)\right)\ln(icx+1)}{4c^2}+\frac{iab\,x^2\ln(-icx+1)}{2}$

input
```
int(x*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+b^2/c^2*(1/2*c^2*x^2*arctan(c*x)^2+1/2*arctan(c*x)^2-c*x*arcta
n(c*x)+1/2*ln(c^2*x^2+1))+x^2*a*b*arctan(c*x)-a*b*x/c+1/c^2*a*b*arctan(c*x
)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int x(a+b\arctan(cx))^2\,dx$$
$$=\frac{a^2c^2x^2-2\,abcx+(b^2c^2x^2+b^2)\arctan{(cx)}^2+b^2\log{(c^2x^2+1)}+2\,(abc^2x^2-b^2cx+ab)\arctan{(cx)}}{2\,c^2}$$

input
```
integrate(x*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/2*(a^2*c^2*x^2 - 2*a*b*c*x + (b^2*c^2*x^2 + b^2)*arctan(c*x)^2 + b^2*log
(c^2*x^2 + 1) + 2*(a*b*c^2*x^2 - b^2*c*x + a*b)*arctan(c*x))/c^2
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.41

$$\int x(a+b\arctan(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2x^2}{2}+abx^2\operatorname{atan}{(cx)}-\frac{abx}{c}+\frac{ab\operatorname{atan}{(cx)}}{c^2}+\frac{b^2x^2\operatorname{atan}^2{(cx)}}{2}-\frac{b^2x\operatorname{atan}{(cx)}}{c}+\frac{b^2\log{\left(x^2+\frac{1}{c^2}\right)}}{2c^2}+\frac{b^2\operatorname{atan}^2{(cx)}}{2c^2} & \text{for } c \neq \\ \frac{a^2x^2}{2} & \text{otherw}\end{cases}$$

input
```
integrate(x*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*x**2/2 + a*b*x**2*atan(c*x) - a*b*x/c + a*b*atan(c*x)/c**2
 + b**2*x**2*atan(c*x)**2/2 - b**2*x*atan(c*x)/c + b**2*log(x**2 + c**(-2)
)/(2*c**2) + b**2*atan(c*x)**2/(2*c**2), Ne(c, 0)), (a**2*x**2/2, True))
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int x(a+b\arctan(cx))^2\,dx$$
$$=\frac{1}{2}\,b^2x^2\arctan\left(cx\right)^2+\frac{1}{2}\,a^2x^2+\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)ab$$
$$-\frac{1}{2}\left(2\,c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\arctan\left(cx\right)+\frac{\arctan\left(cx\right)^2-\log\left(c^2x^2+1\right)}{c^2}\right)b^2$$

input
```
integrate(x*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*arctan(c*x)^2 + 1/2*a^2*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - ar
ctan(c*x)/c^3))*a*b - 1/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (ar
ctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2
```

3.17.8 Giac [F]

$$\int x(a+b\arctan(cx))^2\,dx=\int\left(b\arctan\left(cx\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int x(a+b\arctan(cx))^2\,dx$$
$$=\frac{\frac{b^2\,\mathrm{atan}(c\,x)^2}{2}+\frac{b^2\,\ln\left(c^2\,x^2+1\right)}{2}-c\left(x\,\mathrm{atan}(c\,x)\,b^2+a\,x\,b\right)+a\,b\,\mathrm{atan}(c\,x)}{c^2}$$
$$+\frac{a^2\,x^2}{2}+\frac{b^2\,x^2\,\mathrm{atan}(c\,x)^2}{2}+a\,b\,x^2\,\mathrm{atan}(c\,x)$$

input
```
int(x*(a + b*atan(c*x))^2,x)
```

output
```
((b^2*atan(c*x)^2)/2 + (b^2*log(c^2*x^2 + 1))/2 - c*(b^2*x*atan(c*x) + a*b
*x) + a*b*atan(c*x))/c^2 + (a^2*x^2)/2 + (b^2*x^2*atan(c*x)^2)/2 + a*b*x^2
*atan(c*x)
```

3.17. $\int x(a+b\arctan(cx))^2\,dx$

3.18 $\int (a+b\arctan(cx))^2 \, dx$

3.18.1 Optimal result 172
3.18.2 Mathematica [A] (verified) 172
3.18.3 Rubi [A] (verified) 173
3.18.4 Maple [A] (verified) 175
3.18.5 Fricas [F] 175
3.18.6 Sympy [F] 175
3.18.7 Maxima [F] 176
3.18.8 Giac [F] 176
3.18.9 Mupad [F(-1)] 176

3.18.1 Optimal result

Integrand size = 10, antiderivative size = 83

$$\int (a+b\arctan(cx))^2 \, dx = \frac{i(a+b\arctan(cx))^2}{c} + x(a+b\arctan(cx))^2 + \frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} + \frac{ib^2 \operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}$$

output
```
I*(a+b*arctan(c*x))^2/c+x*(a+b*arctan(c*x))^2+2*b*(a+b*arctan(c*x))*ln(2/(
1+I*c*x))/c+I*b^2*polylog(2,1-2/(1+I*c*x))/c
```

3.18.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int (a+b\arctan(cx))^2 \, dx$$
$$= \frac{b^2(-i+cx)\arctan(cx)^2 + 2b\arctan(cx)\left(acx + b\log\left(1+e^{2i\arctan(cx)}\right)\right) + a(acx - b\log(1+c^2x^2)) - ib^2}{c}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2,x]
```

output
```
(b^2*(-I + c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(a*c*x + b*Log[1 + E^((2*I
)*ArcTan[c*x])]) + a*(a*c*x - b*Log[1 + c^2*x^2]) - I*b^2*PolyLog[2, -E^((
2*I)*ArcTan[c*x])])/c
```

3.18.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \ 5345$$

$$x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx$$

$$\Big\downarrow \ 5455$$

$$x(a+b\arctan(cx))^2-2bc\left(-\frac{\int \frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)$$

$$\Big\downarrow \ 5379$$

$$x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int \frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)$$

$$\Big\downarrow \ 2849$$

$$x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int \frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)$$

$$\Big\downarrow \ 2752$$

$$x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}}{c}\right)$$

input
```
Int[(a + b*ArcTan[c*x])^2,x]
```

output
```
x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c^2)
- (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/
(1 + I*c*x)])/c)/c)
```

3.18.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.18.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{cx\,a^2 - i\arctan(cx)^2 b^2 + \arctan(cx)^2 b^2 cx + 2\arctan(cx)\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)b^2 - i\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)b^2 + 2abcx\arctan(}{c}$
default	$\frac{cx\,a^2 - i\arctan(cx)^2 b^2 + \arctan(cx)^2 b^2 cx + 2\arctan(cx)\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)b^2 - i\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)b^2 + 2abcx\arctan(}{c}$
parts	$a^2x + b^2\arctan(cx)^2 x - \frac{ib^2\arctan(cx)^2}{c} - \frac{ib^2\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)}{c} + \frac{2b^2\arctan(cx)\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)}{c}$
risch	$\frac{ib^2\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{c} - \frac{ba\ln(icx+1)}{c} + \frac{b^2\ln(icx+1)\ln(-icx+1)x}{2} + \frac{ib^2\ln(icx+1)\ln(-icx+1)}{2c} + \frac{2ab}{c} - b^2$

input
```
int((a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*x*a^2-I*arctan(c*x)^2*b^2+arctan(c*x)^2*b^2*c*x+2*arctan(c*x)*ln(1+
(1+I*c*x)^2/(c^2*x^2+1))*b^2-I*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))*b^2+2*a
*b*c*x*arctan(c*x)-a*b*ln(c^2*x^2+1))
```

3.18.5 Fricas [F]

$$\int (a+b\arctan(cx))^2\,dx = \int (b\arctan(cx)+a)^2\,dx$$

input
```
integrate((a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2, x)
```

3.18.6 Sympy [F]

$$\int (a+b\arctan(cx))^2\,dx = \int (a+b\,\mathrm{atan}(cx))^2\,dx$$

input
```
integrate((a+b*atan(c*x))**2,x)
```

output
```
Integral((a + b*atan(c*x))**2, x)
```

3.18.7 Maxima [F]

$$\int (a + b \arctan(cx))^2 \, dx = \int (b \arctan(cx) + a)^2 \, dx$$

input
```
integrate((a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/16*(4*x*arctan(c*x)^2 + 192*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^
2 + 1), x) + 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x
) + 64*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - x*log(c
^2*x^2 + 1)^2 + 4*arctan(c*x)^3/c - 128*c*integrate(1/16*x*arctan(c*x)/(c^
2*x^2 + 1), x) + 16*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x))*b
^2 + a^2*x + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b/c
```

3.18.8 Giac [F]

$$\int (a + b \arctan(cx))^2 \, dx = \int (b \arctan(cx) + a)^2 \, dx$$

input
```
integrate((a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int (a + b \arctan(cx))^2 \, dx = \int (a + b \operatorname{atan}(c x))^2 \, dx$$

input
```
int((a + b*atan(c*x))^2,x)
```

output
```
int((a + b*atan(c*x))^2, x)
```

3.19 $\int \frac{(a+b\arctan(cx))^2}{x}\,dx$

3.19.1 Optimal result 177
3.19.2 Mathematica [A] (verified) 178
3.19.3 Rubi [A] (verified) 178
3.19.4 Maple [C] (warning: unable to verify) 180
3.19.5 Fricas [F] 181
3.19.6 Sympy [F] 181
3.19.7 Maxima [F] 181
3.19.8 Giac [F] 182
3.19.9 Mupad [F(-1)] 182

3.19.1 Optimal result

Integrand size = 14, antiderivative size = 132

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x}\,dx &= 2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\ &\quad - ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\ &\quad + ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\ &\quad - \frac{1}{2}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\ &\quad + \frac{1}{2}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))-I*b*(a+b*arctan(c*x))*polyl
og(2,1-2/(1+I*c*x))+I*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^
2*polylog(3,1-2/(1+I*c*x))+1/2*b^2*polylog(3,-1+2/(1+I*c*x))
```

3.19.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = a^2\log(cx)+iab(\operatorname{PolyLog}(2,-icx)-\operatorname{PolyLog}(2,icx)) + b^2\left(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx)^3+\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - \arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + i\arctan(cx)\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + i\arctan(cx)\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx)}\right)-\frac{1}{2}\operatorname{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/x,x]
```

output
```
a^2*Log[c*x] + I*a*b*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + b^2*((-1
/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E^((-2*I)*Ar
cTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]
*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, -E^((2*I)*A
rcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3, -E^((2*I)
*ArcTan[c*x])]/2)
```

3.19.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5357, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx$$

$$\Big\downarrow \text{ 5357}$$

$$2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-4bc\int \frac{(a+b\arctan(cx))\operatorname{arctanh}\left(1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx$$

$$\Big\downarrow \text{ 5523}$$

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-$$
$$4bc\left(\frac{1}{2}\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{1}{2}\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)$$

$\big\downarrow$ 5529

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-$$
$$4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+\frac{1}{2}\left(\frac{1}{2}ib\int\frac{\text{PolyLog}(}{c^2}\right.\right.$$

$\big\downarrow$ 7164

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-$$
$$4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}+\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{4c}\right)+\frac{1}{2}\left(-\frac{i\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right.}{2c}\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])^2/x,x]
```

output
```
2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] - 4*b*c*((((I/2)*(a + b
*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c + (b*PolyLog[3, 1 - 2/(1 +
I*c*x)])/(4*c))/2 + (((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 +
I*c*x)])/c - (b*PolyLog[3, -1 + 2/(1 + I*c*x)])/(4*c))/2)
```

3.19.3.1 Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.19.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 1002, normalized size of antiderivative = 7.59

Expression too large to display

input
```
int((a+b*arctan(c*x))^2/x,x)
```

output
```
a^2*ln(c*x)+b^2*(ln(c*x)*arctan(c*x)^2+I*arctan(c*x)*polylog(2,-(1+I*c*x)^
2/(c^2*x^2+1))-1/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-arctan(c*x)^2*ln((1
+I*c*x)^2/(c^2*x^2+1)-1)+arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2
*I*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,(1+I*c*x
)/(c^2*x^2+1)^(1/2))+arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*a
rctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*c*x)/
(c^2*x^2+1)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*
x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))-csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))^2-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*
x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3-csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+
I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))^2+csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))
^3+1)*arctan(c*x)^2)+2*a*b*(ln(c*x)*arctan(c*x)+1/2*I*ln(c*x)*ln(1+I*c*x)-
1/2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2*I*dilog(1-I*c*x))
```

3.19.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx=\int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x, x)
```

3.19.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx=\int \frac{\left(a+b\operatorname{atan}\left(cx\right)\right)^2}{x}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x,x)
```

output
```
Integral((a + b*atan(c*x))**2/x, x)
```

3.19.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx=\int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x)^2 + b^2*log(c^2*x^2 + 1)^2
 + 32*a*b*arctan(c*x))/x, x)
```

3.19.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x}\,dx$$

input
```
int((a + b*atan(c*x))^2/x,x)
```

output
```
int((a + b*atan(c*x))^2/x, x)
```

3.20 $\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx$

3.20.1 Optimal result 183
3.20.2 Mathematica [A] (verified) 183
3.20.3 Rubi [A] (verified) 184
3.20.4 Maple [B] (verified) 185
3.20.5 Fricas [F] 186
3.20.6 Sympy [F] 187
3.20.7 Maxima [F] 187
3.20.8 Giac [F] 187
3.20.9 Mupad [F(-1)] 188

3.20.1 Optimal result

Integrand size = 14, antiderivative size = 82

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx = -ic(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^2}{x} + 2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) - ib^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)$$

output
```
-I*c*(a+b*arctan(c*x))^2-(a+b*arctan(c*x))^2/x+2*b*c*(a+b*arctan(c*x))*ln(
2-2/(1-I*c*x))-I*b^2*c*polylog(2,-1+2/(1-I*c*x))
```

3.20.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx = \frac{b^2(-1-icx)\arctan(cx)^2+2b\arctan(cx)\left(-a+bcx\log\left(1-e^{2i\arctan(cx)}\right)\right)-a(a-2bcx\log(cx)+bcx\log}{x}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/x^2,x]
```

output
```
(b^2*(-1 - I*c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(-a + b*c*x*Log[1 - E^((
2*I)*ArcTan[c*x])]) - a*(a - 2*b*c*x*Log[c*x] + b*c*x*Log[1 + c^2*x^2]) -
I*b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x
```

3.20.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5361, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx$$

$$\Big\downarrow \text{ 5361}$$

$$2bc\int \frac{a+b\arctan(cx)}{x\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))^2}{x}$$

$$\Big\downarrow \text{ 5459}$$

$$-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)$$

$$\Big\downarrow \text{ 5403}$$

$$-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)$$

$$\Big\downarrow \text{ 2897}$$

$$-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)$$

input
```
Int[(a + b*ArcTan[c*x])^2/x^2,x]
```

output
```
-((a + b*ArcTan[c*x])^2/x) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b + I
*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2/(
1 - I*c*x)])/2))
```

3.20.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.20.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(78) = 156.

Time = 2.35 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.29

method	result
parts	$-\frac{a^2}{x}+b^2c\left(-\frac{\arctan(cx)^2}{cx}+2\ln(cx)\arctan(cx)-\arctan(cx)\ln(c^2x^2+1)+i\ln(cx)\ln(\right.$
derivativedivides	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\arctan(cx)^2}{cx}+2\ln(cx)\arctan(cx)-\arctan(cx)\ln(c^2x^2+1)+i\ln(cx)\right.\right.$
default	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\arctan(cx)^2}{cx}+2\ln(cx)\arctan(cx)-\arctan(cx)\ln(c^2x^2+1)+i\ln(cx)\right.\right.$

input
```
int((a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+b^2*c*(-1/c/x*arctan(c*x)^2+2*ln(c*x)*arctan(c*x)-arctan(c*x)*ln(c^
2*x^2+1)+I*ln(c*x)*ln(1+I*c*x)-I*ln(c*x)*ln(1-I*c*x)+I*dilog(1+I*c*x)-I*di
log(1-I*c*x)-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(
c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+1/2*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*l
n(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))))+2*a*b*c*(-1/
c/x*arctan(c*x)+ln(c*x)-1/2*ln(c^2*x^2+1))
```

3.20.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx=\int \frac{(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x^2, x)
```

3.20.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx=\int \frac{(a+b\,\mathrm{atan}\,(cx))^2}{x^2}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x**2,x)
```

output
```
Integral((a + b*atan(c*x))**2/x**2, x)
```

3.20.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx=\int \frac{(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
-(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b + 1/16*(4*(c*arct
an(c*x)^3 + 4*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x
) - 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + 32*c*
integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) + 48*integrate(1/16*arcta
n(c*x)^2/(c^2*x^4 + x^2), x) + 4*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^
4 + x^2), x))*x - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^2/x - a^2/x
```

3.20.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx=\int \frac{(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/x^2,x)
```

output
```
int((a + b*atan(c*x))^2/x^2, x)
```

3.21 $\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx$

3.21.1 Optimal result 189
3.21.2 Mathematica [A] (verified) 189
3.21.3 Rubi [A] (verified) 190
3.21.4 Maple [A] (verified) 192
3.21.5 Fricas [A] (verification not implemented) 193
3.21.6 Sympy [A] (verification not implemented) 193
3.21.7 Maxima [A] (verification not implemented) 194
3.21.8 Giac [F] 194
3.21.9 Mupad [B] (verification not implemented) 194

3.21.1 Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{bc(a+b\arctan(cx))}{x} - \frac{1}{2}c^2(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^2}{2x^2} + b^2c^2\log(x) - \frac{1}{2}b^2c^2\log\left(1+c^2x^2\right)$$

output
```
-b*c*(a+b*arctan(c*x))/x-1/2*c^2*(a+b*arctan(c*x))^2-1/2*(a+b*arctan(c*x))
^2/x^2+b^2*c^2*ln(x)-1/2*b^2*c^2*ln(c^2*x^2+1)
```

3.21.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{a^2+2abcx+2b(a+bcx+ac^2x^2)\arctan(cx)+b^2(1+c^2x^2)\arctan(cx)^2-2b^2c^2x^2\log(x)+b^2c^2x^2\log(}{2x^2}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/x^3,x]
```

output
```
-1/2*(a^2 + 2*a*b*c*x + 2*b*(a + b*c*x + a*c^2*x^2)*ArcTan[c*x] + b^2*(1 +
 c^2*x^2)*ArcTan[c*x]^2 - 2*b^2*c^2*x^2*Log[x] + b^2*c^2*x^2*Log[1 + c^2*x
^2])/x^2
```

3.21.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5361, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{5361}$$

$$bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5453}$$

$$bc\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5361}$$

$$bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

$$\Big\downarrow \text{5419}$$

$$bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}$$

input
```
Int[(a + b*ArcTan[c*x])^2/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c*x])^2/x^2 + b*c*(-((a + b*ArcTan[c*x])/x) - (c*(a + b
*ArcTan[c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2)
```

3.21.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.21.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.24

method	result
parts	$-\frac{a^2}{2x^2}+b^2c^2\left(-\frac{\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{cx}-\frac{\arctan(cx)^2}{2}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)-\frac{ab\arctan(cx)}{x^2}-$
derivativedivides	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{cx}-\frac{\arctan(cx)^2}{2}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)-\frac{ab\arctan(cx)}{c^2x^2}\right.$
default	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{cx}-\frac{\arctan(cx)^2}{2}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)-\frac{ab\arctan(cx)}{c^2x^2}\right.$
parallelrisch	$\frac{-b^2\arctan(cx)^2x^2c^2+2b^2c^2\ln(x)x^2-b^2c^2\ln(c^2x^2+1)x^2-2ab\arctan(cx)x^2c^2+c^2x^2a^2-2b^2\arctan(cx)xc-2abcx-b^2 a}{2x^2}$
risch	$\frac{b^2(c^2x^2+1)\ln(icx+1)^2}{8x^2}+\frac{ib(ibc^2x^2\ln(-icx+1)+2xbc+2a+ib\ln(-icx+1))\ln(icx+1)}{4x^2}-\frac{-4i\ln((-3ibc-ac)x-3b+}{}$

input
```
int((a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2+b^2*c^2*(-1/2/c^2/x^2*arctan(c*x)^2-1/c/x*arctan(c*x)-1/2*arc
tan(c*x)^2-1/2*ln(c^2*x^2+1)+ln(c*x))-a*b/x^2*arctan(c*x)-a*b*c^2*arctan(c
*x)-a*b*c/x
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx =$$
$$-\frac{b^2c^2x^2\log\left(c^2x^2+1\right)-2\,b^2c^2x^2\log\left(x\right)+2\,abcx+\left(b^2c^2x^2+b^2\right)\arctan\left(cx\right)^2+a^2+2\left(abc^2x^2+b^2cx+\right.}{2\,x^2}$$

input
```
integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
-1/2*(b^2*c^2*x^2*log(c^2*x^2 + 1) - 2*b^2*c^2*x^2*log(x) + 2*a*b*c*x + (b
^2*c^2*x^2 + b^2)*arctan(c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + b^2*c*x + a*b)*ar
ctan(c*x))/x^2
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.51

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx$$
$$=\begin{cases} -\frac{a^2}{2x^2}-abc^2\operatorname{atan}\left(cx\right)-\frac{abc}{x}-\frac{ab\operatorname{atan}\left(cx\right)}{x^2}+b^2c^2\log\left(x\right)-\frac{b^2c^2\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{b^2c^2\operatorname{atan}^2\left(cx\right)}{2}-\frac{b^2c\operatorname{atan}\left(cx\right)}{x}-\frac{b^2\operatorname{at}}{} \\ -\frac{a^2}{2x^2} \end{cases}$$

input
```
integrate((a+b*atan(c*x))**2/x**3,x)
```

output
```
Piecewise((-a**2/(2*x**2) - a*b*c**2*atan(c*x) - a*b*c/x - a*b*atan(c*x)/x
**2 + b**2*c**2*log(x) - b**2*c**2*log(x**2 + c**(-2))/2 - b**2*c**2*atan(
c*x)**2/2 - b**2*c*atan(c*x)/x - b**2*atan(c*x)**2/(2*x**2), Ne(c, 0)), (-
a**2/(2*x**2), True))
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = -\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)ab + \frac{1}{2}\left(\left(\arctan(cx)^2-\log(c^2x^2+1)+2\log(x)\right)c^2-2\left(c\arctan(cx)+\frac{1}{x}\right)c\arctan(cx)\right)b^2 - \frac{b^2\arctan(cx)^2}{2x^2}-\frac{a^2}{2x^2}$$

input
```
integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b + 1/2*((arctan(c*x)^2 - l
og(c^2*x^2 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c*x))*b
^2 - 1/2*b^2*arctan(c*x)^2/x^2 - 1/2*a^2/x^2
```

3.21.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.21.9 Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.77

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = b^2c^2\ln(x)-\frac{a^2}{2x^2}-\frac{b^2c^2\operatorname{atan}(cx)^2}{2}-\frac{b^2c^2\ln(cx+1\mathrm{i})}{2} - \frac{b^2c^2\ln(1+cx\,1\mathrm{i})}{2}-\frac{b^2\operatorname{atan}(cx)^2}{2x^2}-\frac{abc}{x}-\frac{ab\operatorname{atan}(cx)}{x^2} - \frac{b^2c\operatorname{atan}(cx)}{x}-\frac{abc^2\ln(cx+1\mathrm{i})\,1\mathrm{i}}{2}+\frac{abc^2\ln(1+cx\,1\mathrm{i})\,1\mathrm{i}}{2}$$

input
```
int((a + b*atan(c*x))^2/x^3,x)
```

output
```
b^2*c^2*log(x) - a^2/(2*x^2) - (b^2*c^2*atan(c*x)^2)/2 - (b^2*c^2*log(c*x
+ 1i))/2 - (b^2*c^2*log(c*x*1i + 1))/2 - (b^2*atan(c*x)^2)/(2*x^2) - (a*b*
c)/x - (a*b*atan(c*x))/x^2 - (a*b*c^2*log(c*x + 1i)*1i)/2 + (a*b*c^2*log(c
*x*1i + 1)*1i)/2 - (b^2*c*atan(c*x))/x
```

3.21. $\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx$

3.22 $\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx$

3.22.1 Optimal result 196
3.22.2 Mathematica [A] (verified) 196
3.22.3 Rubi [A] (verified) 197
3.22.4 Maple [B] (verified) 200
3.22.5 Fricas [F] 200
3.22.6 Sympy [F] 201
3.22.7 Maxima [F] 201
3.22.8 Giac [F] 201
3.22.9 Mupad [F(-1)] 202

3.22.1 Optimal result

Integrand size = 14, antiderivative size = 140

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = -\frac{b^2c^2}{3x}-\frac{1}{3}b^2c^3\arctan(cx)-\frac{bc(a+b\arctan(cx))}{3x^2}+\frac{1}{3}ic^3(a+b\arctan(cx))^2-\frac{(a+b\arctan(cx))^2}{3x^3}-\frac{2}{3}bc^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)+\frac{1}{3}ib^2c^3\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)$$

output
```
-1/3*b^2*c^2/x-1/3*b^2*c^3*arctan(c*x)-1/3*b*c*(a+b*arctan(c*x))/x^2+1/3*I
*c^3*(a+b*arctan(c*x))^2-1/3*(a+b*arctan(c*x))^2/x^3-2/3*b*c^3*(a+b*arctan
(c*x))*ln(2-2/(1-I*c*x))+1/3*I*b^2*c^3*polylog(2,-1+2/(1-I*c*x))
```

3.22.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = -\frac{a^2+abcx+b^2c^2x^2+b^2(1-ic^3x^3)\arctan(cx)^2+b\arctan(cx)\left(2a+bcx+bc^3x^3+2bc^3x^3\log\left(1-e^{2i a\right.}\right.}{3x^3}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/x^4,x]
```

output
```
-1/3*(a^2 + a*b*c*x + b^2*c^2*x^2 + b^2*(1 - I*c^3*x^3)*ArcTan[c*x]^2 + b*
ArcTan[c*x]*(2*a + b*c*x + b*c^3*x^3 + 2*b*c^3*x^3*Log[1 - E^((2*I)*ArcTan
[c*x])]) + 2*a*b*c^3*x^3*Log[c*x] - a*b*c^3*x^3*Log[1 + c^2*x^2] - I*b^2*c
^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x^3
```

3.22.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5361, 5453, 5361, 264, 216, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^4}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{2}{3}bc\left(\int \frac{a+b\arctan(cx)}{x^3}dx-c^2\int \frac{a+b\arctan(cx)}{x\left(c^2x^2+1\right)}dx\right)-\frac{(a+b\arctan(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x\left(c^2x^2+1\right)}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x\left(c^2x^2+1\right)}dx\right)+\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x\left(c^2x^2+1\right)}dx\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{(a+b\arctan(cx))^2}{3x^3}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)$$

$$\Big\downarrow \text{5403}$$

$$-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)\right)-\right.$$

$$\Big\downarrow \text{2897}$$

$$-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])^2/x^4,x]
```

output
```
-1/3*(a + b*ArcTan[c*x])^2/x^3 + (2*b*c*(-1/2*(a + b*ArcTan[c*x])/x^2 + (b
*c*(-x^(-1) - c*ArcTan[c*x]))/2 - c^2*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b
+ I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 +
2/(1 - I*c*x)])/2))))/3
```

3.22. $\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx$

3.22.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.22.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(122) = 244.

Time = 2.91 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.21

method	result
parts	$-\frac{a^2}{3x^3}+b^2c^3\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)\ln(c^2x^2+1)}{3}-\frac{\arctan(cx)}{3c^2x^2}-\frac{2\ln(cx)\arctan(cx)}{3}+\frac{i\left(\ln(cx-i)\ln(\right.}{}\right.$
derivativedivides	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)\ln(c^2x^2+1)}{3}-\frac{\arctan(cx)}{3c^2x^2}-\frac{2\ln(cx)\arctan(cx)}{3}+\frac{i\left(\ln(cx-\right.}{}\right.\right.$
default	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)\ln(c^2x^2+1)}{3}-\frac{\arctan(cx)}{3c^2x^2}-\frac{2\ln(cx)\arctan(cx)}{3}+\frac{i\left(\ln(cx-\right.}{}\right.\right.$

input
```
int((a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3+b^2*c^3*(-1/3/c^3/x^3*arctan(c*x)^2+1/3*arctan(c*x)*ln(c^2*x^
2+1)-1/3/c^2/x^2*arctan(c*x)-2/3*ln(c*x)*arctan(c*x)+1/6*I*(ln(c*x-I)*ln(c
^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I
)))-1/6*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln
(c*x+I)*ln(1/2*I*(c*x-I)))-1/3/c/x-1/3*arctan(c*x)-1/3*I*ln(c*x)*ln(1+I*c*
x)+1/3*I*ln(c*x)*ln(1-I*c*x)-1/3*I*dilog(1+I*c*x)+1/3*I*dilog(1-I*c*x))+2*
a*b*c^3*(-1/3/c^3/x^3*arctan(c*x)+1/6*ln(c^2*x^2+1)-1/6/c^2/x^2-1/3*ln(c*x
))
```

3.22.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx=\int \frac{(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x^4, x)
```

3.22.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(a+b\operatorname{atan}(cx))^2}{x^4}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x**4,x)
```

output
```
Integral((a + b*atan(c*x))**2/x**4, x)
```

3.22.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
a*b + 1/48*(48*x^3*integrate(-1/48*(4*c^2*x^2*log(c^2*x^2 + 1) - 8*c*x*arc
tan(c*x) - 36*(c^2*x^2 + 1)*arctan(c*x)^2 - 3*(c^2*x^2 + 1)*log(c^2*x^2 +
1)^2)/(c^2*x^6 + x^4), x) - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^2/x^3
- 1/3*a^2/x^3
```

3.22.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

output
```
sage0*x
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x^4}\,dx$$

input
```
int((a + b*atan(c*x))^2/x^4,x)
```

output
```
int((a + b*atan(c*x))^2/x^4, x)
```

3.23 $\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$

3.23.1 Optimal result 203
3.23.2 Mathematica [A] (verified) 203
3.23.3 Rubi [A] (verified) 204
3.23.4 Maple [A] (verified) 207
3.23.5 Fricas [A] (verification not implemented) 208
3.23.6 Sympy [A] (verification not implemented) 208
3.23.7 Maxima [A] (verification not implemented) 209
3.23.8 Giac [F] 209
3.23.9 Mupad [B] (verification not implemented) 209

3.23.1 Optimal result

Integrand size = 14, antiderivative size = 116

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = -\frac{b^2c^2}{12x^2}-\frac{bc(a+b\arctan(cx))}{6x^3}+\frac{bc^3(a+b\arctan(cx))}{2x}+\frac{1}{4}c^4(a+b\arctan(cx))^2-\frac{(a+b\arctan(cx))^2}{4x^4}-\frac{2}{3}b^2c^4\log(x)+\frac{1}{3}b^2c^4\log\left(1+c^2x^2\right)$$

output
```
-1/12*b^2*c^2/x^2-1/6*b*c*(a+b*arctan(c*x))/x^3+1/2*b*c^3*(a+b*arctan(c*x)
)/x+1/4*c^4*(a+b*arctan(c*x))^2-1/4*(a+b*arctan(c*x))^2/x^4-2/3*b^2*c^4*ln
(x)+1/3*b^2*c^4*ln(c^2*x^2+1)
```

3.23.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = \frac{-3a^2-2abcx-b^2c^2x^2+6abc^3x^3+2b(bcx(-1+3c^2x^2)+3a(-1+c^4x^4))\arctan(cx)+3b^2(-1+c^4x^4)\,a}{12x^4}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/x^5,x]
```

output
```
(-3*a^2 - 2*a*b*c*x - b^2*c^2*x^2 + 6*a*b*c^3*x^3 + 2*b*(b*c*x*(-1 + 3*c^2
*x^2) + 3*a*(-1 + c^4*x^4))*ArcTan[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTan[c*x]
^2 - 8*b^2*c^4*x^4*Log[x] + 4*b^2*c^4*x^4*Log[1 + c^2*x^2])/(12*x^4)
```

3.23.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5361, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^5}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}bc\int \frac{a+b\arctan(cx)}{x^4(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{2}bc\left(\int \frac{a+b\arctan(cx)}{x^4}dx-c^2\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx\right)-\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx\right)+\frac{1}{3}bc\int \frac{1}{x^3(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{3x^3}\right)-\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx\right)+\frac{1}{6}bc\int \frac{1}{x^4(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{3x^3}\right)-\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx\right)+\frac{1}{6}bc\int \left(\frac{c^4}{c^2x^2+1}-\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\arctan(cx)}{3x^3}\right)-\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx\right)-\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)\right)-\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \textbf{5453}$$

$$\frac{1}{2}bc\left(-\left(c^2\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)\right)-\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\right.\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \textbf{5361}$$

$$\frac{1}{2}bc\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \textbf{243}$$

$$\frac{1}{2}bc\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+b\arctan(cx)}{3x^3}\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \textbf{47}$$

$$\frac{1}{2}bc\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+b}{}\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \textbf{14}$$

$$\frac{1}{2}bc\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+b\,a}{}\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

$$\Big\downarrow \textbf{16}$$

$$\frac{1}{2}bc\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)\right)-\frac{a+b\arctan}{3x^3}\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

3.23. $\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{2}bc\left(-\left(c^2\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc(\log(x^2)-\log(c^2x^2+1))\right)\right)-\frac{a+b\arctan(cx)}{3x^3}+\right.$$
$$\frac{(a+b\arctan(cx))^2}{4x^4}$$

input
```
Int[(a + b*ArcTan[c*x])^2/x^5,x]
```

output
```
-1/4*(a + b*ArcTan[c*x])^2/x^4 + (b*c*(-1/3*(a + b*ArcTan[c*x])/x^3 - c^2*
(-((a + b*ArcTan[c*x])/x) - (c*(a + b*ArcTan[c*x])^2)/(2*b) + (b*c*(Log[x^
2] - Log[1 + c^2*x^2]))/2) + (b*c*(-x^(-2) - c^2*Log[x^2] + c^2*Log[1 + c^
2*x^2]))/6))/2
```

3.23.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.23.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

method	result
parts	$-\frac{a^2}{4x^4}+b^2c^4\left(-\frac{\arctan(cx)^2}{4c^4x^4}-\frac{\arctan(cx)}{6c^3x^3}+\frac{\arctan(cx)}{2cx}+\frac{\arctan(cx)^2}{4}-\frac{1}{12c^2x^2}-\frac{2\ln(cx)}{3}+\frac{\ln(c^2x^2+1}{3}\right.$
derivativedivides	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\arctan(cx)^2}{4c^4x^4}-\frac{\arctan(cx)}{6c^3x^3}+\frac{\arctan(cx)}{2cx}+\frac{\arctan(cx)^2}{4}-\frac{1}{12c^2x^2}-\frac{2\ln(cx)}{3}+\frac{\ln(c^2}{3}\right.\right.$
default	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\arctan(cx)^2}{4c^4x^4}-\frac{\arctan(cx)}{6c^3x^3}+\frac{\arctan(cx)}{2cx}+\frac{\arctan(cx)^2}{4}-\frac{1}{12c^2x^2}-\frac{2\ln(cx)}{3}+\frac{\ln(c^2}{3}\right.\right.$
parallelrisch	$-\frac{-3x^4\arctan(cx)^2b^2c^4+8b^2c^4\ln(x)x^4-4b^2c^4\ln(c^2x^2+1)x^4-6x^4\arctan(cx)ab\,c^4-b^2c^4x^4-6b^2\arctan(cx)x^3c^3-6ab}{12x^4}$
risch	$-\frac{b^2(c^4x^4-1)\ln(icx+1)^2}{16x^4}+\frac{ib(-3ib\,c^4x^4\ln(-icx+1)-6b\,c^3x^3+2xbc+6a+3ib\ln(-icx+1))\ln(icx+1)}{24x^4}-\frac{12i\ln((-4}{}$

input
```
int((a+b*arctan(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4*a^2+b^2*c^4*(-1/4/c^4/x^4*arctan(c*x)^2-1/6/c^3/x^3*arctan(c*x)+1
/2/c/x*arctan(c*x)+1/4*arctan(c*x)^2-1/12/c^2/x^2-2/3*ln(c*x)+1/3*ln(c^2*x
^2+1))+2*a*b*c^4*(-1/4/c^4/x^4*arctan(c*x)-1/12/c^3/x^3+1/4/c/x+1/4*arctan
(c*x))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = \frac{4\,b^2c^4x^4\log\left(c^2x^2+1\right)-8\,b^2c^4x^4\log\left(x\right)+6\,abc^3x^3-b^2c^2x^2-2\,abcx+3\left(b^2c^4x^4-b^2\right)\arctan\left(cx\right)^2-3\,a^2}{12\,x^4}$$

input
```
integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="fricas")
```

output
```
1/12*(4*b^2*c^4*x^4*log(c^2*x^2 + 1) - 8*b^2*c^4*x^4*log(x) + 6*a*b*c^3*x^
3 - b^2*c^2*x^2 - 2*a*b*c*x + 3*(b^2*c^4*x^4 - b^2)*arctan(c*x)^2 - 3*a^2
+ 2*(3*a*b*c^4*x^4 + 3*b^2*c^3*x^3 - b^2*c*x - 3*a*b)*arctan(c*x))/x^4
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = \begin{cases} -\frac{a^2}{4x^4}+\frac{abc^4\operatorname{atan}(cx)}{2}+\frac{abc^3}{2x}-\frac{abc}{6x^3}-\frac{ab\operatorname{atan}(cx)}{2x^4}-\frac{2b^2c^4\log(x)}{3}+\frac{b^2c^4\log\left(x^2+\frac{1}{c^2}\right)}{3}+\frac{b^2c^4\operatorname{atan}^2(cx)}{4}+\frac{b^2c^3\operatorname{atan}(cx)}{2x}- \\ -\frac{a^2}{4x^4} \end{cases}$$

input
```
integrate((a+b*atan(c*x))**2/x**5,x)
```

output
```
Piecewise((-a**2/(4*x**4) + a*b*c**4*atan(c*x)/2 + a*b*c**3/(2*x) - a*b*c/
(6*x**3) - a*b*atan(c*x)/(2*x**4) - 2*b**2*c**4*log(x)/3 + b**2*c**4*log(x
**2 + c**(-2))/3 + b**2*c**4*atan(c*x)**2/4 + b**2*c**3*atan(c*x)/(2*x) -
b**2*c**2/(12*x**2) - b**2*c*atan(c*x)/(6*x**3) - b**2*atan(c*x)**2/(4*x**
4), Ne(c, 0)), (-a**2/(4*x**4), True))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = \frac{1}{6}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan\left(cx\right)}{x^4}\right)ab$$
$$+\frac{1}{12}\left(2\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c\arctan\left(cx\right)-\frac{\left(3\,c^2x^2\arctan\left(cx\right)^2-4\,c^2x^2\log\left(c^2x^2+1\right)+8\,c^2\cdots\right.}{x^2}\right.$$
$$-\frac{b^2\arctan\left(cx\right)^2}{4\,x^4}-\frac{a^2}{4\,x^4}$$

input
```
integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="maxima")
```

output
```
1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*a*b
+ 1/12*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c*arctan(c*x) - (3*c^2
*x^2*arctan(c*x)^2 - 4*c^2*x^2*log(c^2*x^2 + 1) + 8*c^2*x^2*log(x) + 1)*c^
2/x^2)*b^2 - 1/4*b^2*arctan(c*x)^2/x^4 - 1/4*a^2/x^4
```

3.23.8 Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="giac")
```

output
```
sage0*x
```

3.23.9 Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$$
$$=\frac{b^2\,c^4\,\mathrm{atan}(c\,x)^2}{4}-\frac{2\,b^2\,c^4\,\ln\left(x\right)}{3}$$
$$-\frac{\frac{b^2\,\mathrm{atan}(c\,x)^2}{4}+\frac{a^2}{4}+x\left(\frac{c\,\mathrm{atan}(c\,x)\,b^2}{6}+\frac{a\,c\,b}{6}\right)-x^3\left(\frac{b^2\,c^3\,\mathrm{atan}(c\,x)}{2}+\frac{a\,b\,c^3}{2}\right)+\frac{b^2\,c^2\,x^2}{12}+\frac{a\,b\,\mathrm{atan}(c\,x)}{2}}{x^4}$$
$$+\frac{b^2\,c^4\,\ln\left(c\,x+1\mathrm{i}\right)}{3}+\frac{b^2\,c^4\,\ln\left(1+c\,x\,1\mathrm{i}\right)}{3}+\frac{a\,b\,c^4\,\ln\left(c\,x+1\mathrm{i}\right)\,1\mathrm{i}}{4}-\frac{a\,b\,c^4\,\ln\left(1+c\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{4}$$

input
```
int((a + b*atan(c*x))^2/x^5,x)
```

output
```
(b^2*c^4*atan(c*x)^2)/4 - (2*b^2*c^4*log(x))/3 - ((b^2*atan(c*x)^2)/4 + a^
2/4 + x*((b^2*c*atan(c*x))/6 + (a*b*c)/6) - x^3*((b^2*c^3*atan(c*x))/2 + (
a*b*c^3)/2) + (b^2*c^2*x^2)/12 + (a*b*atan(c*x))/2)/x^4 + (b^2*c^4*log(c*x
 + 1i))/3 + (b^2*c^4*log(c*x*1i + 1))/3 + (a*b*c^4*log(c*x + 1i)*1i)/4 - (
a*b*c^4*log(c*x*1i + 1)*1i)/4
```

3.23. $\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$

3.24 $\int x^5(a+b\arctan(cx))^3\,dx$

3.24.1 Optimal result . . . 211
3.24.2 Mathematica [A] (verified) . . . 212
3.24.3 Rubi [B] (verified) . . . 212
3.24.4 Maple [A] (verified) . . . 219
3.24.5 Fricas [F] . . . 219
3.24.6 Sympy [F] . . . 220
3.24.7 Maxima [F] . . . 220
3.24.8 Giac [F] . . . 221
3.24.9 Mupad [F(-1)] . . . 221

3.24.1 Optimal result

Integrand size = 14, antiderivative size = 255

$$\begin{aligned}\int x^5(a+b\arctan(cx))^3\,dx = {} & \frac{19b^3x}{60c^5}-\frac{b^3x^3}{60c^3}-\frac{19b^3\arctan(cx)}{60c^6}-\frac{4b^2x^2(a+b\arctan(cx))}{15c^4} \\ & +\frac{b^2x^4(a+b\arctan(cx))}{20c^2}-\frac{23ib(a+b\arctan(cx))^2}{30c^6} \\ & -\frac{bx(a+b\arctan(cx))^2}{2c^5}+\frac{bx^3(a+b\arctan(cx))^2}{6c^3} \\ & -\frac{bx^5(a+b\arctan(cx))^2}{10c}+\frac{(a+b\arctan(cx))^3}{6c^6} \\ & +\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{23b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{15c^6} \\ & -\frac{23ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{30c^6}\end{aligned}$$

output
```
19/60*b^3*x/c^5-1/60*b^3*x^3/c^3-19/60*b^3*arctan(c*x)/c^6-4/15*b^2*x^2*(a
+b*arctan(c*x))/c^4+1/20*b^2*x^4*(a+b*arctan(c*x))/c^2-23/30*I*b*(a+b*arct
an(c*x))^2/c^6-1/2*b*x*(a+b*arctan(c*x))^2/c^5+1/6*b*x^3*(a+b*arctan(c*x))
^2/c^3-1/10*b*x^5*(a+b*arctan(c*x))^2/c+1/6*(a+b*arctan(c*x))^3/c^6+1/6*x^
6*(a+b*arctan(c*x))^3-23/15*b^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^6-23/3
0*I*b^3*polylog(2,1-2/(1+I*c*x))/c^6
```

3.24.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.14

$$\int x^5(a+b\arctan(cx))^3\,dx$$
$$=\frac{-19ab^2-30a^2bcx+19b^3cx-16ab^2c^2x^2+10a^2bc^3x^3-b^3c^3x^3+3ab^2c^4x^4-6a^2bc^5x^5+10a^3c^6x^6+2b^2(b}{}$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x])^3,x]
```

output
```
(-19*a*b^2 - 30*a^2*b*c*x + 19*b^3*c*x - 16*a*b^2*c^2*x^2 + 10*a^2*b*c^3*x
^3 - b^3*c^3*x^3 + 3*a*b^2*c^4*x^4 - 6*a^2*b*c^5*x^5 + 10*a^3*c^6*x^6 + 2*
b^2*(b*(23*I - 15*c*x + 5*c^3*x^3 - 3*c^5*x^5) + 15*a*(1 + c^6*x^6))*ArcTa
n[c*x]^2 + 10*b^3*(1 + c^6*x^6)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(b^2*(-19 -
16*c^2*x^2 + 3*c^4*x^4) - 4*a*b*c*x*(15 - 5*c^2*x^2 + 3*c^4*x^4) + 30*a^2*
(1 + c^6*x^6) - 92*b^2*Log[1 + E^((2*I)*ArcTan[c*x])]) + 46*a*b^2*Log[1 +
c^2*x^2] + (46*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(60*c^6)
```

3.24.3 Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 525 vs. 2(255) = 510.

Time = 2.90 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.214, Rules used = {5361, 5451, 5361, 5451, 5361, 254, 2009, 5451, 5345, 5361, 262, 216, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+b\arctan(cx))^3\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\int\frac{x^6(a+b\arctan(cx))^2}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\int x^4(a+b\arctan(cx))^2dx}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5361

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5451

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\int x^3(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{\int x^2(a+b\arctan(cx))^2dx}{c^2}-\frac{\int\frac{x^2(a+b\,\mathrm{a}}{c^2}}{c^2}\right.$$

$\downarrow$ 5361

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\int\frac{x^4}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{}{}}{c}\right.$$

$\downarrow$ 254

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\int\left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{3}x^3(a+}{}\right.$$

$\downarrow$ 2009

3.24. $\int x^5(a+b\arctan(cx))^3\,dx$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{3}x^3(a+b\,\mathrm{arcta}\cdots}{\cdots}\cdots\right.$$

$\Big\downarrow$ 5451

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}}{c^2}\cdots\right.}{c^2}\cdots\right.$$

$\Big\downarrow$ 5345

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}}{c^2}\cdots\right.}{c^2}\cdots\right.$$

$\Big\downarrow$ 5361

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int\frac{x^2}{c^2x^2+1}dx}{c^2}-\int\frac{x(a\cdots}{\cdots}}{c^2}\cdots\right.}{c^2}\cdots\right.$$

$\Big\downarrow$ 262

3.24. $\int x^5(a+b\arctan(cx))^3\,dx$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3- \frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right)}{c^2}}{c^2}\right.}{c^2}\right.$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3- \frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-}{c^2}\right.}{c^2}\right.$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3- \frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-}{c^2}\right.}{c^2}\right.$$

$$\Big\downarrow \text{5455}$$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3- \frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-}{c^2}\right.}{c^2}\right.$$

$$\Big\downarrow \text{5379}$$

3.24. $\int x^5(a+b\arctan(cx))^3\,dx$

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\right.}{c^2}\right.$$

$\downarrow$ 2849

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\right.}{c^2}\right.$$

$\downarrow$ 2752

$$\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{1}{2}bc\left(\frac{\frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{2}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))-\frac{1}{4}bc\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)}{c^2}-\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\right.}{c^2}\right.$$

input
```
Int[x^5*(a + b*ArcTan[c*x])^3,x]
```

3.24. $\int x^5(a+b\arctan(cx))^3\,dx$

output
```
(x^6*(a + b*ArcTan[c*x])^3)/6 - (b*c*(((x^5*(a + b*ArcTan[c*x])^2)/5 - (2*
b*c*(((x^4*(a + b*ArcTan[c*x]))/4 - (b*c*(-(x/c^4) + x^3/(3*c^2) + ArcTan[
c*x]/c^5))/4)/c^2 - (((x^2*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c
*x]/c^3))/2)/c^2 - (((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c^2) - (((a + b*Ar
cTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])
/c)/c)/c^2)/c^2))/5)/c^2 - (((x^3*(a + b*ArcTan[c*x])^2)/3 - (2*b*c*(((x^2
*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2)/c^2 - (((-1/2
*I)*(a + b*ArcTan[c*x])^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*
x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/c)/c)/c^2))/3)/c^2 - (-1/
3*(a + b*ArcTan[c*x])^3/(b*c^3) + (x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2
*I)*(a + b*ArcTan[c*x])^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*
x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/c)/c))/c^2)/c^2)/c^2))/2
```

3.24.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.24. $\int x^5(a+b\arctan(cx))^3\,dx$

3.24.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{\frac{a^3c^6x^6}{6}+b^3\left(\frac{c^6x^6\arctan(cx)^3}{6}-\frac{c^5x^5\arctan(cx)^2}{10}+\frac{c^3x^3\arctan(cx)^2}{6}-\frac{\arctan(cx)^2cx}{2}+\frac{\arctan(cx)^3}{6}+\frac{c^4x^4\arctan(cx)}{20}-\frac{4c^2x^2\,\text{ar}}{1}\right.}{}$
default	$\frac{\frac{a^3c^6x^6}{6}+b^3\left(\frac{c^6x^6\arctan(cx)^3}{6}-\frac{c^5x^5\arctan(cx)^2}{10}+\frac{c^3x^3\arctan(cx)^2}{6}-\frac{\arctan(cx)^2cx}{2}+\frac{\arctan(cx)^3}{6}+\frac{c^4x^4\arctan(cx)}{20}-\frac{4c^2x^2\,\text{ar}}{1}\right.}{}$
parts	$\frac{a^3x^6}{6}+\frac{b^3\left(\frac{c^6x^6\arctan(cx)^3}{6}-\frac{c^5x^5\arctan(cx)^2}{10}+\frac{c^3x^3\arctan(cx)^2}{6}-\frac{\arctan(cx)^2cx}{2}+\frac{\arctan(cx)^3}{6}+\frac{c^4x^4\arctan(cx)}{20}-\frac{4c^2x^2\,\text{a}}{}\right.}{}$
risch	Expression too large to display

input
```
int(x^5*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^6*(1/6*a^3*c^6*x^6+b^3*(1/6*c^6*x^6*arctan(c*x)^3-1/10*c^5*x^5*arctan(
c*x)^2+1/6*c^3*x^3*arctan(c*x)^2-1/2*arctan(c*x)^2*c*x+1/6*arctan(c*x)^3+1
/20*c^4*x^4*arctan(c*x)-4/15*c^2*x^2*arctan(c*x)+23/30*arctan(c*x)*ln(c^2*
x^2+1)-1/60*c^3*x^3+19/60*c*x-19/60*arctan(c*x)+23/60*I*(ln(c*x-I)*ln(c^2*
x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))
-23/60*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(
c*x+I)*ln(1/2*I*(c*x-I))))+3*a*b^2*(1/6*c^6*x^6*arctan(c*x)^2-1/15*c^5*x^5
*arctan(c*x)+1/9*c^3*x^3*arctan(c*x)-1/3*c*x*arctan(c*x)+1/6*arctan(c*x)^2
+1/60*c^4*x^4-4/45*c^2*x^2+23/90*ln(c^2*x^2+1))+3*a^2*b*(1/6*c^6*x^6*arcta
n(c*x)-1/30*c^5*x^5+1/18*c^3*x^3-1/6*c*x+1/6*arctan(c*x)))
```

3.24.5 Fricas [F]

$$\int x^5(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^5*arctan(c*x)^3 + 3*a*b^2*x^5*arctan(c*x)^2 + 3*a^2*b*x^5*a
rctan(c*x) + a^3*x^5, x)
```

3.24.6 Sympy [F]

$$\int x^5(a+b\arctan(cx))^3\,dx=\int x^5(a+b\,\mathrm{atan}\,(cx))^3\,dx$$

input
```
integrate(x**5*(a+b*atan(c*x))**3,x)
```

output
```
Integral(x**5*(a + b*atan(c*x))**3, x)
```

3.24.7 Maxima [F]

$$\int x^5(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
1/2*a*b^2*x^6*arctan(c*x)^2 + 1/6*a^3*x^6 + 1/30*(15*x^6*arctan(c*x) - c*(
(3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a^2*b - 1/60*(4*
c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)*arctan(c*x) -
(3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c^2*x^2 + 1))/c^6)*a*b
^2 + 1/480*(20*(5760*c^7*integrate(1/480*x^7*arctan(c*x)^3/(c^7*x^2 + c^5)
, x) - 1440*c^6*integrate(1/480*x^6*arctan(c*x)^2/(c^7*x^2 + c^5), x) - 36
0*c^6*integrate(1/480*x^6*log(c^2*x^2 + 1)^2/(c^7*x^2 + c^5), x) - 288*c^6
*integrate(1/480*x^6*log(c^2*x^2 + 1)/(c^7*x^2 + c^5), x) + 5760*c^5*integ
rate(1/480*x^5*arctan(c*x)^3/(c^7*x^2 + c^5), x) + 576*c^5*integrate(1/480
*x^5*arctan(c*x)/(c^7*x^2 + c^5), x) + 480*c^4*integrate(1/480*x^4*log(c^2
*x^2 + 1)/(c^7*x^2 + c^5), x) - 960*c^3*integrate(1/480*x^3*arctan(c*x)/(c
^7*x^2 + c^5), x) - 1440*c^2*integrate(1/480*x^2*log(c^2*x^2 + 1)/(c^7*x^2
 + c^5), x) + 2880*c*integrate(1/480*x*arctan(c*x)/(c^7*x^2 + c^5), x) - a
rctan(c*x)^3/c^6 - 360*integrate(1/480*log(c^2*x^2 + 1)^2/(c^7*x^2 + c^5),
 x))*c^6 + 40*(c^6*x^6 + 1)*arctan(c*x)^3 - 4*(3*c^5*x^5 - 5*c^3*x^3 + 15*
c*x)*arctan(c*x)^2 + (3*c^5*x^5 - 5*c^3*x^3 + 15*c*x)*log(c^2*x^2 + 1)^2)*
b^3/c^6
```

3.24.8 Giac [F]

$$\int x^5(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int x^5(a+b\arctan(cx))^3\,dx=\int x^5\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

input
```
int(x^5*(a + b*atan(c*x))^3,x)
```

output
```
int(x^5*(a + b*atan(c*x))^3, x)
```

3.25 $\int x^4(a+b\arctan(cx))^3\,dx$

3.25.1 Optimal result 222
3.25.2 Mathematica [A] (verified) 223
3.25.3 Rubi [A] (verified) 223
3.25.4 Maple [C] (warning: unable to verify) 229
3.25.5 Fricas [F] 229
3.25.6 Sympy [F] 230
3.25.7 Maxima [F] 230
3.25.8 Giac [F] 230
3.25.9 Mupad [F(-1)] 231

3.25.1 Optimal result

Integrand size = 14, antiderivative size = 271

$$\begin{aligned}\int x^4(a+b\arctan(cx))^3\,dx = &-\frac{9ab^2x}{10c^4}-\frac{b^3x^2}{20c^3}-\frac{9b^3x\arctan(cx)}{10c^4}\\&+\frac{b^2x^3(a+b\arctan(cx))}{10c^2}+\frac{9b(a+b\arctan(cx))^2}{20c^5}\\&+\frac{3bx^2(a+b\arctan(cx))^2}{10c^3}-\frac{3bx^4(a+b\arctan(cx))^2}{20c}\\&+\frac{i(a+b\arctan(cx))^3}{5c^5}+\frac{1}{5}x^5(a+b\arctan(cx))^3\\&+\frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{5c^5}+\frac{b^3\log(1+c^2x^2)}{2c^5}\\&+\frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}\\&+\frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{10c^5}\end{aligned}$$

output
```
-9/10*a*b^2*x/c^4-1/20*b^3*x^2/c^3-9/10*b^3*x*arctan(c*x)/c^4+1/10*b^2*x^3
*(a+b*arctan(c*x))/c^2+9/20*b*(a+b*arctan(c*x))^2/c^5+3/10*b*x^2*(a+b*arct
an(c*x))^2/c^3-3/20*b*x^4*(a+b*arctan(c*x))^2/c+1/5*I*(a+b*arctan(c*x))^3/
c^5+1/5*x^5*(a+b*arctan(c*x))^3+3/5*b*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/
c^5+1/2*b^3*ln(c^2*x^2+1)/c^5+3/5*I*b^2*(a+b*arctan(c*x))*polylog(2,1-2/(1
+I*c*x))/c^5+3/10*b^3*polylog(3,1-2/(1+I*c*x))/c^5
```

3.25.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.46

$$\int x^4(a+b\arctan(cx))^3\,dx$$
$$=\frac{-b^3-18ab^2cx+6a^2bc^2x^2-b^3c^2x^2+2ab^2c^3x^3-3a^2bc^4x^4+4a^3c^5x^5+18ab^2\arctan(cx)-18b^3cx\arctan}{}$$

input
```
Integrate[x^4*(a + b*ArcTan[c*x])^3,x]
```

output
```
(-b^3 - 18*a*b^2*c*x + 6*a^2*b*c^2*x^2 - b^3*c^2*x^2 + 2*a*b^2*c^3*x^3 - 3
*a^2*b*c^4*x^4 + 4*a^3*c^5*x^5 + 18*a*b^2*ArcTan[c*x] - 18*b^3*c*x*ArcTan[
c*x] + 12*a*b^2*c^2*x^2*ArcTan[c*x] + 2*b^3*c^3*x^3*ArcTan[c*x] - 6*a*b^2*
c^4*x^4*ArcTan[c*x] + 12*a^2*b*c^5*x^5*ArcTan[c*x] - (12*I)*a*b^2*ArcTan[c
*x]^2 + 9*b^3*ArcTan[c*x]^2 + 6*b^3*c^2*x^2*ArcTan[c*x]^2 - 3*b^3*c^4*x^4*
ArcTan[c*x]^2 + 12*a*b^2*c^5*x^5*ArcTan[c*x]^2 - (4*I)*b^3*ArcTan[c*x]^3 +
 4*b^3*c^5*x^5*ArcTan[c*x]^3 + 24*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTa
n[c*x])] + 12*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*a^2*b*L
og[1 + c^2*x^2] + 10*b^3*Log[1 + c^2*x^2] - (12*I)*b^2*(a + b*ArcTan[c*x])
*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 6*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*
x])])/(20*c^5)
```

3.25.3 Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.37, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {5361, 5451, 5361, 5451, 5361, 243, 49, 2009, 5451, 2009, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\arctan(cx))^3\,dx$$
$$\Big\downarrow \text{5361}$$
$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\int\frac{x^5(a+b\arctan(cx))^2}{c^2x^2+1}dx$$
$$\Big\downarrow \text{5451}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\int x^3(a+b\arctan(cx))^2dx}{c^2}-\frac{\int \frac{x^3(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5361

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\int \frac{x^4(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{\int \frac{x^3(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5451

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\int x^2(a+b\arctan(cx))dx}{c^2}-\frac{\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{\int x(a+b\arctan(cx))^2dx}{c^2}-\frac{\int \frac{x(a+b\arct}{c^2x^2}}{c^2}}{c^2}\right.$$

$\downarrow$ 5361

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{3}bc\int \frac{x^3}{c^2x^2+1}dx}{c^2}-\frac{\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-b}{c^2}\right.$$

$\downarrow$ 243

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int \frac{x^2}{c^2x^2+1}dx^2}{c^2}-\frac{\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-}{c}\right.$$

$\downarrow$ 49

3.25. $\int x^4(a+b\arctan(cx))^3\,dx$

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int\left(\frac{1}{c^2}-\frac{1}{c^2\left(c^2x^2+1\right)}\right)dx^2}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{2}x^2(a+b\,\mathrm{a}\cdots}{\cdots}\cdots\right.$$

$\big\downarrow$ 2009

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{1}{2}x^2(a+b\arctan\cdots}{\cdots}\cdots\right.$$

$\big\downarrow$ 5451

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\frac{\int(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}}{c^2}\right)}{c^2}-\cdots\right.$$

$\big\downarrow$ 2009

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan(\cdots}{c^2x^2+1}\cdots}{c^2}}{c^2}\cdots\right.}{c^2}\cdots\right.$$

$\big\downarrow$ 5419

3.25. $\int x^4(a+b\arctan(cx))^3\,dx$

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-$$

$$\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(c}{2bc^3}}{c^2}\right.}{c^2}\right.$$

$\downarrow$ 5455

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-$$

$$\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(c}{2bc^3}}{c^2}\right.}{c^2}\right.$$

$\downarrow$ 5379

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-$$

$$\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(c}{2bc^3}}{c^2}\right.}{c^2}\right.$$

$\downarrow$ 5529

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-$$

$$\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(c}{2bc^3}}{c^2}\right.}{c^2}\right.$$

$\downarrow$ 7164

3.25. $\int x^4(a+b\arctan(cx))^3\,dx$

$$\frac{1}{5}x^5(a+b\arctan(cx))^3-$$

$$\frac{3}{5}bc\left(\frac{\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)}{c^2}-\frac{\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(c}{2bc^3}}{c^2}\right.}{c^2}\right.$$

input
```
Int[x^4*(a + b*ArcTan[c*x])^3,x]
```

output
```
(x^5*(a + b*ArcTan[c*x])^3)/5 - (3*b*c*(((x^4*(a + b*ArcTan[c*x])^2)/4 - (
b*c*(((x^3*(a + b*ArcTan[c*x]))/3 - (b*c*(x^2/c^2 - Log[1 + c^2*x^2]/c^4))
/6)/c^2 - (-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) + (a*x + b*x*ArcTan[c*x] - (
b*Log[1 + c^2*x^2])/(2*c))/c^2)/c^2))/2)/c^2 - (((x^2*(a + b*ArcTan[c*x])^
2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) + (a*x + b*x*ArcTan[c*x] -
(b*Log[1 + c^2*x^2])/(2*c))/c^2))/c^2 - (((-1/3*I)*(a + b*ArcTan[c*x])^3)/
(b*c^2) - (((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c - 2*b*(((-1/2*I)*(
a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/
(1 + I*c*x)])/(4*c)))/c)/c^2)/c^2))/5
```

3.25.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.25. $\int x^4(a+b\arctan(cx))^3\,dx$

3.25.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.28 (sec) , antiderivative size = 1185, normalized size of antiderivative = 4.37

Expression too large to display

input
```
int(x^4*(a+b*arctan(c*x))^3,x)
```

output
```
1/c^5*(1/5*a^3*c^5*x^5+b^3*(1/5*c^5*x^5*arctan(c*x)^3-3/20*c^4*x^4*arctan(
c*x)^2+3/10*c^2*x^2*arctan(c*x)^2-3/10*arctan(c*x)^2*ln(c^2*x^2+1)+3/5*arc
tan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-1/20*I*(3*Pi*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1))^3*arctan(c*x)^2-6*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(
I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*arctan(c*x)^2+3*Pi*csgn(I*(1+I*c*x)^2/(c^2*
x^2+1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2-3*Pi*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2
*arctan(c*x)^2+3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))^2*arctan(c*x)^2-3*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*c
sgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)
^2+3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*ar
ctan(c*x)^2-3*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)
^2/(c^2*x^2+1))^2)*arctan(c*x)^2+6*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*
csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2-3*Pi*csgn(I*(1+(1+I*
c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2+4*arctan(c*x)^3+2*I*arctan(c*x)*c^3
*x^3-I*c^2*x^2-20*arctan(c*x)-18*I*arctan(c*x)*c*x+12*I*ln(2)*arctan(c*x)^
2+9*I*arctan(c*x)^2-I)-ln(1+(1+I*c*x)^2/(c^2*x^2+1))-3/5*I*arctan(c*x)*pol
ylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/10*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1)))
+3*a*b^2*(1/5*c^5*x^5*arctan(c*x)^2-1/10*c^4*x^4*arctan(c*x)+1/5*c^2*x^...
```

3.25.5 Fricas [F]

$$\int x^4(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^4*arctan(c*x)^3 + 3*a*b^2*x^4*arctan(c*x)^2 + 3*a^2*b*x^4*a
rctan(c*x) + a^3*x^4, x)
```

3.25.6 Sympy [F]

$$\int x^4(a+b\arctan(cx))^3\,dx=\int x^4(a+b\,\mathrm{atan}\,(cx))^3\;dx$$

input
```
integrate(x**4*(a+b*atan(c*x))**3,x)
```

output
```
Integral(x**4*(a + b*atan(c*x))**3, x)
```

3.25.7 Maxima [F]

$$\int x^4(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
1/40*b^3*x^5*arctan(c*x)^3 - 3/160*b^3*x^5*arctan(c*x)*log(c^2*x^2 + 1)^2
+ 1/5*a^3*x^5 + 3/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log
(c^2*x^2 + 1)/c^6))*a^2*b + integrate(1/160*(12*b^3*c^2*x^6*arctan(c*x)*lo
g(c^2*x^2 + 1) + 140*(b^3*c^2*x^6 + b^3*x^4)*arctan(c*x)^3 + 12*(40*a*b^2*
c^2*x^6 - b^3*c*x^5 + 40*a*b^2*x^4)*arctan(c*x)^2 + 3*(b^3*c*x^5 + 5*(b^3*
c^2*x^6 + b^3*x^4)*arctan(c*x))*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x)
```

3.25.8 Giac [F]

$$\int x^4(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x^4\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int x^4(a+b\arctan(cx))^3\,dx=\int x^4\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

input
```
int(x^4*(a + b*atan(c*x))^3,x)
```

output
```
int(x^4*(a + b*atan(c*x))^3, x)
```

3.26 $\int x^3(a+b\arctan(cx))^3\,dx$

3.26.1 Optimal result . . . 232
3.26.2 Mathematica [A] (verified) . . . 233
3.26.3 Rubi [A] (verified) . . . 233
3.26.4 Maple [A] (verified) . . . 238
3.26.5 Fricas [F] . . . 239
3.26.6 Sympy [F] . . . 239
3.26.7 Maxima [F] . . . 240
3.26.8 Giac [F] . . . 240
3.26.9 Mupad [F(-1)] . . . 241

3.26.1 Optimal result

Integrand size = 14, antiderivative size = 194

$$\begin{aligned}\int x^3(a+b\arctan(cx))^3\,dx = &-\frac{b^3x}{4c^3}+\frac{b^3\arctan(cx)}{4c^4}+\frac{b^2x^2(a+b\arctan(cx))}{4c^2}\\&+\frac{ib(a+b\arctan(cx))^2}{c^4}+\frac{3bx(a+b\arctan(cx))^2}{4c^3}\\&-\frac{bx^3(a+b\arctan(cx))^2}{4c}-\frac{(a+b\arctan(cx))^3}{4c^4}\\&+\frac{1}{4}x^4(a+b\arctan(cx))^3+\frac{2b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4}\\&+\frac{ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4}\end{aligned}$$

output
```
-1/4*b^3*x/c^3+1/4*b^3*arctan(c*x)/c^4+1/4*b^2*x^2*(a+b*arctan(c*x))/c^2+I
*b*(a+b*arctan(c*x))^2/c^4+3/4*b*x*(a+b*arctan(c*x))^2/c^3-1/4*b*x^3*(a+b*
arctan(c*x))^2/c-1/4*(a+b*arctan(c*x))^3/c^4+1/4*x^4*(a+b*arctan(c*x))^3+2
*b^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4+I*b^3*polylog(2,1-2/(1+I*c*x))/
c^4
```

3.26.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.16

$$\int x^3(a+b\arctan(cx))^3\,dx$$
$$=\frac{ab^2+3a^2bcx-b^3cx+ab^2c^2x^2-a^2bc^3x^3+a^3c^4x^4-b^2(b(4i-3cx+c^3x^3)+a(3-3c^4x^4))\arctan(cx)^2+}{}$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x])^3,x]
```

output
```
(a*b^2 + 3*a^2*b*c*x - b^3*c*x + a*b^2*c^2*x^2 - a^2*b*c^3*x^3 + a^3*c^4*x
^4 - b^2*(b*(4*I - 3*c*x + c^3*x^3) + a*(3 - 3*c^4*x^4))*ArcTan[c*x]^2 + b
^3*(-1 + c^4*x^4)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(-2*a*b*c*x*(-3 + c^2*x^2)
 + b^2*(1 + c^2*x^2) + 3*a^2*(-1 + c^4*x^4) + 8*b^2*Log[1 + E^((2*I)*ArcTa
n[c*x])]) - 4*a*b^2*Log[1 + c^2*x^2] - (4*I)*b^3*PolyLog[2, -E^((2*I)*ArcT
an[c*x])])/(4*c^4)
```

3.26.3 Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.58, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5361, 5451, 5361, 5451, 5345, 5361, 262, 216, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\arctan(cx))^3\,dx$$

$$\Big\downarrow \ 5361$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\int\frac{x^4(a+b\arctan(cx))^2}{c^2x^2+1}dx$$

$$\Big\downarrow \ 5451$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\int x^2(a+b\arctan(cx))^2dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \ 5361$$

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$\downarrow$ 5451

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{\int(a+b\arctan(cx))^2dx}{c^2}-\frac{\int\frac{(a+b\arctan(cx}{c^2x^2+1}}{c^2}}{c^2}\right.$$

$\downarrow$ 5345

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\frac{x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx)}{c^2x^2+1}}{c^2}}{c^2}\right.$$

$\downarrow$ 5361

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int\frac{x^2}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{x(a+b\arctan(cx))^2-2bc\int}{c^2}\right.$$

$\downarrow$ 262

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{x(a+b\arctan(cx)}{}\right.$$

$\downarrow$ 216

3.26. $\int x^3(a+b\arctan(cx))^3\,dx$

$$\frac{1}{4}x^4(a+b\arctan(cx))^3 - \frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2} - \frac{x(a+b\arctan(cx))^2-}{}\right.$$

$\Big\downarrow$ 5419

$$\frac{1}{4}x^4(a+b\arctan(cx))^3 - \frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)}{c^2} - \frac{x(a+b\arctan(cx))^2-}{}\right.$$

$\Big\downarrow$ 5455

$$\frac{1}{4}x^4(a+b\arctan(cx))^3 - \frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}}{c^2}\right)}{c^2} - \frac{-}{}\right.$$

$\Big\downarrow$ 5379

$$\frac{1}{4}x^4(a+b\arctan(cx))^3 - \frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-}{c^2}\right.}{c^2}\right.$$

$\Big\downarrow$ 2849

3.26. $\int x^3(a+b\arctan(cx))^3\,dx$

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{-\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(\cdots}{c}}{c}\cdots}{c^2}\cdots\right.}{c^2}\cdots\right.$$

$\Big\downarrow$ 2752

$$\frac{1}{4}x^4(a+b\arctan(cx))^3-\frac{3}{4}bc\left(\frac{\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\cdots}{c}}{c^2}\cdots\right.}{c^2}\cdots\right.$$

input
```
Int[x^3*(a + b*ArcTan[c*x])^3,x]
```

output
```
(x^4*(a + b*ArcTan[c*x])^3)/4 - (3*b*c*(((x^3*(a + b*ArcTan[c*x])^2)/3 - (
2*b*c*(((x^2*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2)/c
^2 - (((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log
[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/c)/c)/c^2))/3
)/c^2 - (-1/3*(a + b*ArcTan[c*x])^3/(b*c^3) + (x*(a + b*ArcTan[c*x])^2 - 2
*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log
[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/c)/c))/c^2)/c
^2))/4
```

3.26. $\int x^3(a+b\arctan(cx))^3\,dx$

3.26.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.26.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.75

method	result
derivativedivides	$\frac{a^3c^4x^4}{4}+b^3\left(\frac{c^4x^4\arctan(cx)^3}{4}-\frac{c^3x^3\arctan(cx)^2}{4}+\frac{3\arctan(cx)^2cx}{4}-\frac{\arctan(cx)^3}{4}+\frac{c^2x^2\arctan(cx)}{4}-\arctan(cx)\ln(c^2x^2+1)-\right.$
default	$\frac{a^3c^4x^4}{4}+b^3\left(\frac{c^4x^4\arctan(cx)^3}{4}-\frac{c^3x^3\arctan(cx)^2}{4}+\frac{3\arctan(cx)^2cx}{4}-\frac{\arctan(cx)^3}{4}+\frac{c^2x^2\arctan(cx)}{4}-\arctan(cx)\ln(c^2x^2+1)-\right.$
parts	$\frac{a^3x^4}{4}+\frac{b^3\left(\frac{c^4x^4\arctan(cx)^3}{4}-\frac{c^3x^3\arctan(cx)^2}{4}+\frac{3\arctan(cx)^2cx}{4}-\frac{\arctan(cx)^3}{4}+\frac{c^2x^2\arctan(cx)}{4}-\arctan(cx)\ln(c^2x^2+1)-\right.}{}$
risch	Expression too large to display

input
```
int(x^3*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/4*a^3*c^4*x^4+b^3*(1/4*c^4*x^4*arctan(c*x)^3-1/4*c^3*x^3*arctan(c
*x)^2+3/4*arctan(c*x)^2*c*x-1/4*arctan(c*x)^3+1/4*c^2*x^2*arctan(c*x)-arct
an(c*x)*ln(c^2*x^2+1)-1/4*c*x+1/4*arctan(c*x)-1/2*I*(ln(c*x-I)*ln(c^2*x^2+
1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+1/2
*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)
*ln(1/2*I*(c*x-I))))+3*a*b^2*(1/4*c^4*x^4*arctan(c*x)^2-1/6*c^3*x^3*arctan
(c*x)+1/2*c*x*arctan(c*x)-1/4*arctan(c*x)^2+1/12*c^2*x^2-1/3*ln(c^2*x^2+1)
)+3*a^2*b*(1/4*c^4*x^4*arctan(c*x)-1/12*c^3*x^3+1/4*c*x-1/4*arctan(c*x)))
```

3.26.5 Fricas [F]

$$\int x^3(a+b\arctan(cx))^3\,dx=\int (b\arctan{(cx)}+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arctan(c*x)^3 + 3*a*b^2*x^3*arctan(c*x)^2 + 3*a^2*b*x^3*a
rctan(c*x) + a^3*x^3, x)
```

3.26.6 Sympy [F]

$$\int x^3(a+b\arctan(cx))^3\,dx=\int x^3(a+b\operatorname{atan}{(cx)})^3\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))**3,x)
```

output
```
Integral(x**3*(a + b*atan(c*x))**3, x)
```

3.26.7 Maxima [F]

$$\int x^3(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
3/4*a*b^2*x^4*arctan(c*x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arctan(c*x) - c*((c
^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a^2*b - 1/4*(2*c*((c^2*x^3 - 3*x)/
c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(
c^2*x^2 + 1))/c^4)*a*b^2 + 1/64*(4*(512*c^5*integrate(1/64*x^5*arctan(c*x)
^3/(c^5*x^2 + c^3), x) - 192*c^4*integrate(1/64*x^4*arctan(c*x)^2/(c^5*x^2
 + c^3), x) - 48*c^4*integrate(1/64*x^4*log(c^2*x^2 + 1)^2/(c^5*x^2 + c^3)
, x) - 64*c^4*integrate(1/64*x^4*log(c^2*x^2 + 1)/(c^5*x^2 + c^3), x) + 51
2*c^3*integrate(1/64*x^3*arctan(c*x)^3/(c^5*x^2 + c^3), x) + 128*c^3*integ
rate(1/64*x^3*arctan(c*x)/(c^5*x^2 + c^3), x) + 192*c^2*integrate(1/64*x^2
*log(c^2*x^2 + 1)/(c^5*x^2 + c^3), x) - 384*c*integrate(1/64*x*arctan(c*x)
/(c^5*x^2 + c^3), x) + arctan(c*x)^3/c^4 + 48*integrate(1/64*log(c^2*x^2 +
 1)^2/(c^5*x^2 + c^3), x))*c^4 + 8*(c^4*x^4 - 1)*arctan(c*x)^3 - 4*(c^3*x^
3 - 3*c*x)*arctan(c*x)^2 + (c^3*x^3 - 3*c*x)*log(c^2*x^2 + 1)^2)*b^3/c^4
```

3.26.8 Giac [F]

$$\int x^3(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\arctan(cx))^3\,dx=\int x^3\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

input
```
int(x^3*(a + b*atan(c*x))^3,x)
```

output
```
int(x^3*(a + b*atan(c*x))^3, x)
```

3.27 $\int x^2(a+b\arctan(cx))^3\,dx$

3.27.1 Optimal result 242
3.27.2 Mathematica [A] (verified) 243
3.27.3 Rubi [A] (verified) 243
3.27.4 Maple [C] (warning: unable to verify) 247
3.27.5 Fricas [F] 248
3.27.6 Sympy [F] 248
3.27.7 Maxima [F] 248
3.27.8 Giac [F] 249
3.27.9 Mupad [F(-1)] 249

3.27.1 Optimal result

Integrand size = 14, antiderivative size = 206

$$\begin{aligned}\int x^2(a+b\arctan(cx))^3\,dx = {} & \frac{ab^2x}{c^2}+\frac{b^3x\arctan(cx)}{c^2}-\frac{b(a+b\arctan(cx))^2}{2c^3} \\ & -\frac{bx^2(a+b\arctan(cx))^2}{2c}-\frac{i(a+b\arctan(cx))^3}{3c^3} \\ & +\frac{1}{3}x^3(a+b\arctan(cx))^3 \\ & -\frac{b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3}-\frac{b^3\log(1+c^2x^2)}{2c^3} \\ & -\frac{ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3} \\ & -\frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}\end{aligned}$$

output
```
a*b^2*x/c^2+b^3*x*arctan(c*x)/c^2-1/2*b*(a+b*arctan(c*x))^2/c^3-1/2*b*x^2*
(a+b*arctan(c*x))^2/c-1/3*I*(a+b*arctan(c*x))^3/c^3+1/3*x^3*(a+b*arctan(c*
x))^3-b*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^3-1/2*b^3*ln(c^2*x^2+1)/c^3-
I*b^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^3-1/2*b^3*polylog(3,1-2
/(1+I*c*x))/c^3
```

3.27.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.31

$$\int x^2(a+b\arctan(cx))^3\,dx$$
$$= \frac{-3a^2bc^2x^2+2a^3c^3x^3+6a^2bc^3x^3\arctan(cx)+3a^2b\log(1+c^2x^2)+6ab^2\left(cx+(i+c^3x^3)\arctan(cx)^2-\mathrm{a}\right.}{}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x])^3,x]
```

output
```
(-3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 + 6*a^2*b*c^3*x^3*ArcTan[c*x] + 3*a^2*b*
Log[1 + c^2*x^2] + 6*a*b^2*(c*x + (I + c^3*x^3)*ArcTan[c*x]^2 - ArcTan[c*x
]*(1 + c^2*x^2 + 2*Log[1 + E^((2*I)*ArcTan[c*x])]) + I*PolyLog[2, -E^((2*I
)*ArcTan[c*x])]) + b^3*(6*c*x*ArcTan[c*x] - 3*ArcTan[c*x]^2 - 3*c^2*x^2*Ar
cTan[c*x]^2 + (2*I)*ArcTan[c*x]^3 + 2*c^3*x^3*ArcTan[c*x]^3 - 6*ArcTan[c*x
]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 3*Log[1 + c^2*x^2] + (6*I)*ArcTan[c*x
]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - 3*PolyLog[3, -E^((2*I)*ArcTan[c*x])
]))/(6*c^3)
```

3.27.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5361, 5451, 5361, 5451, 2009, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arctan(cx))^3\,dx$$
$$\Big\downarrow \text{ 5361}$$
$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\int\frac{x^3(a+b\arctan(cx))^2}{c^2x^2+1}dx$$
$$\Big\downarrow \text{ 5451}$$
$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\left(\frac{\int x(a+b\arctan(cx))^2dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$
$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \; 5451$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{\int(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \; 5419$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \; 5455$$

$$\frac{1}{3}x^3(a+b\arctan(cx))^3-bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{c^2}-\frac{-\frac{\int\frac{(a+b\arctan(cx))^2}{i-cx}dx}{c}-\frac{i(a+b\arcta}{3bc^}}{c^2}\right.$$

$$\Big\downarrow \; 5379$$

3.27. $\int x^2(a+b\arctan(cx))^3\,dx$

$$\frac{1}{3}x^3(a+b\arctan(cx))^3 - bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2 - bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2} - \frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{c^2} - \frac{-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c} - 2b\int \frac{(a}{}}{c}}{}\right.$$

$\downarrow$ 5529

$$\frac{1}{3}x^3(a+b\arctan(cx))^3 - bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2 - bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2} - \frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{c^2} - \frac{-\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c} - 2b\left(\frac{1}{2}ib\right.}{}\right.$$

$\downarrow$ 7164

$$\frac{1}{3}x^3(a+b\arctan(cx))^3 - bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2 - bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2} - \frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{c^2} - \frac{-\frac{i(a+b\arctan(cx))^3}{3bc^2} - \frac{\log\left(\frac{2}{1+icx}\right)(a}{}}{}\right.$$

input
```
Int[x^2*(a + b*ArcTan[c*x])^3,x]
```

output
```
(x^3*(a + b*ArcTan[c*x])^3)/3 - b*c*(((x^2*(a + b*ArcTan[c*x])^2)/2 - b*c*
(-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) + (a*x + b*x*ArcTan[c*x] - (b*Log[1 +
c^2*x^2])/(2*c))/c^2))/c^2 - (((-1/3*I)*(a + b*ArcTan[c*x])^3)/(b*c^2) - (
((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c - 2*b*(((-1/2*I)*(a + b*ArcTa
n[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/(1 + I*c*x)
])/(4*c)))/c)/c^2)
```

3.27. $\int x^2(a+b\arctan(cx))^3\,dx$

3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.27.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.19 (sec) , antiderivative size = 1088, normalized size of antiderivative = 5.28

Expression too large to display

input
```
int(x^2*(a+b*arctan(c*x))^3,x)
```

output
```
1/c^3*(1/3*a^3*c^3*x^3+b^3*(1/3*c^3*x^3*arctan(c*x)^3-1/2*c^2*x^2*arctan(c
*x)^2+1/2*arctan(c*x)^2*ln(c^2*x^2+1)-arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+
1)^(1/2))+I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*polylog(3,
-(1+I*c*x)^2/(c^2*x^2+1))+1/12*I*arctan(c*x)*(-3*Pi*arctan(c*x)*csgn(I/(1+
(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/
(c^2*x^2+1))^2)^2+3*Pi*arctan(c*x)*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*c
sgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c
*x)^2/(c^2*x^2+1))+3*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I
*c*x)^2/(c^2*x^2+1))^2)^3-3*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/
(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))-3*Pi*arct
an(c*x)*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x
^2+1))^2)+6*Pi*arctan(c*x)*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(
1+I*c*x)^2/(c^2*x^2+1))^2)^2-3*Pi*arctan(c*x)*csgn(I*(1+(1+I*c*x)^2/(c^2*x
^2+1))^2)^3+3*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3-6*Pi*arctan
(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)+3*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))^2+4*arctan(c*x)^2+12*I*ln(2)*arctan(c*x)-12+6*I*arctan(c*x)-1
2*I*c*x)+ln(1+(1+I*c*x)^2/(c^2*x^2+1)))+3*a*b^2*(1/3*c^3*x^3*arctan(c*x)^2
-1/3*c^2*x^2*arctan(c*x)+1/3*arctan(c*x)*ln(c^2*x^2+1)+1/3*c*x-1/3*arctan(
c*x)+1/6*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I...
```

3.27.5 Fricas [F]

$$\int x^2(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arctan(c*x)^3 + 3*a*b^2*x^2*arctan(c*x)^2 + 3*a^2*b*x^2*a
rctan(c*x) + a^3*x^2, x)
```

3.27.6 Sympy [F]

$$\int x^2(a+b\arctan(cx))^3\,dx=\int x^2(a+b\,\mathrm{atan}\,(cx))^3\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))**3,x)
```

output
```
Integral(x**2*(a + b*atan(c*x))**3, x)
```

3.27.7 Maxima [F]

$$\int x^2(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
1/24*b^3*x^3*arctan(c*x)^3 - 1/32*b^3*x^3*arctan(c*x)*log(c^2*x^2 + 1)^2 +
 1/3*a^3*x^3 + 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4)
)*a^2*b + integrate(1/32*(4*b^3*c^2*x^4*arctan(c*x)*log(c^2*x^2 + 1) + 28*
(b^3*c^2*x^4 + b^3*x^2)*arctan(c*x)^3 + 4*(24*a*b^2*c^2*x^4 - b^3*c*x^3 +
24*a*b^2*x^2)*arctan(c*x)^2 + (b^3*c*x^3 + 3*(b^3*c^2*x^4 + b^3*x^2)*arcta
n(c*x))*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x)
```

3.27.8 Giac [F]

$$\int x^2(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arctan(cx))^3\,dx=\int x^2\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^3,x)
```

output
```
int(x^2*(a + b*atan(c*x))^3, x)
```

3.28 $\int x(a+b\arctan(cx))^3\,dx$

3.28.1 Optimal result . . . 250
3.28.2 Mathematica [A] (verified) . . . 250
3.28.3 Rubi [A] (verified) . . . 251
3.28.4 Maple [B] (verified) . . . 254
3.28.5 Fricas [F] . . . 255
3.28.6 Sympy [F] . . . 255
3.28.7 Maxima [F] . . . 255
3.28.8 Giac [F] . . . 256
3.28.9 Mupad [F(-1)] . . . 256

3.28.1 Optimal result

Integrand size = 12, antiderivative size = 131

$$\int x(a+b\arctan(cx))^3\,dx = -\frac{3ib(a+b\arctan(cx))^2}{2c^2}-\frac{3bx(a+b\arctan(cx))^2}{2c}+\frac{(a+b\arctan(cx))^3}{2c^2}+\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2}-\frac{3ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2}$$

output
```
-3/2*I*b*(a+b*arctan(c*x))^2/c^2-3/2*b*x*(a+b*arctan(c*x))^2/c+1/2*(a+b*ar
ctan(c*x))^3/c^2+1/2*x^2*(a+b*arctan(c*x))^3-3*b^2*(a+b*arctan(c*x))*ln(2/
(1+I*c*x))/c^2-3/2*I*b^3*polylog(2,1-2/(1+I*c*x))/c^2
```

3.28.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.16

$$\int x(a+b\arctan(cx))^3\,dx$$
$$=\frac{3b^2(a+ac^2x^2+b(i-cx))\arctan(cx)^2+b^3(1+c^2x^2)\arctan(cx)^3+3b\arctan(cx)\left(a(a-2bcx+ac^2x^2)\right. \cdots}{2c^2 \cdots}$$

input
```
Integrate[x*(a + b*ArcTan[c*x])^3,x]
```

output
```
(3*b^2*(a + a*c^2*x^2 + b*(I - c*x))*ArcTan[c*x]^2 + b^3*(1 + c^2*x^2)*Arc
Tan[c*x]^3 + 3*b*ArcTan[c*x]*(a*(a - 2*b*c*x + a*c^2*x^2) - 2*b^2*Log[1 +
E^((2*I)*ArcTan[c*x])]) + a*(a*c*x*(-3*b + a*c*x) + 3*b^2*Log[1 + c^2*x^2]
) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(2*c^2)
```

3.28.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx))^3\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\int \frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(\frac{\int (a+b\arctan(cx))^2dx}{c^2}-\frac{\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5345}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(\frac{x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(\frac{x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{(a+b\arctan(cx))^3}{3bc^3}\right)$$

$$\Big\downarrow \text{5455}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)}{c^2}\right)$$

$$\Big\downarrow \text{5379}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)}{c^2}\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)}{c^2}\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}}{c}\right)}{c^2}\right)$$

input
```
Int[x*(a + b*ArcTan[c*x])^3,x]
```

output
```
(x^2*(a + b*ArcTan[c*x])^3)/2 - (3*b*c*(-1/3*(a + b*ArcTan[c*x])^3/(b*c^3)
 + (x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c
^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1
- 2/(1 + I*c*x)])/c)/c))/c^2))/2
```

3.28. $\int x(a+b\arctan(cx))^3\,dx$

3.28.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.28.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(117) = 234.

Time = 2.89 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.11

method	result
derivativedivides	$\frac{\frac{c^2x^2a^3}{2}+b^3\left(\frac{c^2x^2\arctan(cx)^3}{2}+\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)^2cx}{2}+\frac{3\arctan(cx)\ln\left(c^2x^2+1\right)}{2}+\frac{3i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i)^2}{2}\right.}{}\right.}{}$
default	$\frac{\frac{c^2x^2a^3}{2}+b^3\left(\frac{c^2x^2\arctan(cx)^3}{2}+\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)^2cx}{2}+\frac{3\arctan(cx)\ln\left(c^2x^2+1\right)}{2}+\frac{3i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i)^2}{2}\right.}{}\right.}{}$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\frac{c^2x^2\arctan(cx)^3}{2}+\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)^2cx}{2}+\frac{3\arctan(cx)\ln\left(c^2x^2+1\right)}{2}+\frac{3i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i)^2}{2}\right.}{}\right.}{}$
risch	$-\frac{ib^3\ln(-icx+1)^3}{16c^2}+\frac{9ib^3\ln(-icx+1)^2}{32c^2}-\frac{3ib^3\ln(-icx+1)x^2}{32}-\frac{ib^3\ln(-icx+1)^3x^2}{16}+\frac{3ib^3\ln(-icx+1)^2x^2}{32}-$

input
```
int(x*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/2*c^2*x^2*a^3+b^3*(1/2*c^2*x^2*arctan(c*x)^3+1/2*arctan(c*x)^3-3/
2*arctan(c*x)^2*c*x+3/2*arctan(c*x)*ln(c^2*x^2+1)+3/4*I*(ln(c*x-I)*ln(c^2*
x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))
-3/4*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*
x+I)*ln(1/2*I*(c*x-I))))+3*a*b^2*(1/2*c^2*x^2*arctan(c*x)^2+1/2*arctan(c*x
)^2-c*x*arctan(c*x)+1/2*ln(c^2*x^2+1))+3*a^2*b*(1/2*c^2*x^2*arctan(c*x)-1/
2*c*x+1/2*arctan(c*x)))
```

3.28.5 Fricas [F]

$$\int x(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arctan(c*x)^3 + 3*a*b^2*x*arctan(c*x)^2 + 3*a^2*b*x*arctan(
c*x) + a^3*x, x)
```

3.28.6 Sympy [F]

$$\int x(a+b\arctan(cx))^3\,dx=\int x(a+b\operatorname{atan}(cx))^3\,dx$$

input
```
integrate(x*(a+b*atan(c*x))**3,x)
```

output
```
Integral(x*(a + b*atan(c*x))**3, x)
```

3.28.7 Maxima [F]

$$\int x(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
3/2*a*b^2*x^2*arctan(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arctan(c*x) - c*(x/c^
2 - arctan(c*x)/c^3))*a^2*b - 3/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*
x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*a*b^2 - 1/32*(12*c*x*arctan(c
*x)^2 - 8*(c^2*x^2 + 1)*arctan(c*x)^3 - 3*c*x*log(c^2*x^2 + 1)^2 - 4*(128*
c^3*integrate(1/32*x^3*arctan(c*x)^3/(c^3*x^2 + c), x) - 96*c^2*integrate(
1/32*x^2*arctan(c*x)^2/(c^3*x^2 + c), x) - 24*c^2*integrate(1/32*x^2*log(c
^2*x^2 + 1)^2/(c^3*x^2 + c), x) - 96*c^2*integrate(1/32*x^2*log(c^2*x^2 +
1)/(c^3*x^2 + c), x) + 128*c*integrate(1/32*x*arctan(c*x)^3/(c^3*x^2 + c),
 x) + 192*c*integrate(1/32*x*arctan(c*x)/(c^3*x^2 + c), x) - arctan(c*x)^3
/c^2 - 24*integrate(1/32*log(c^2*x^2 + 1)^2/(c^3*x^2 + c), x))*c^2)*b^3/c^
2
```

3.28.8 Giac [F]

$$\int x(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int x(a+b\arctan(cx))^3\,dx=\int x\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

input
```
int(x*(a + b*atan(c*x))^3,x)
```

output
```
int(x*(a + b*atan(c*x))^3, x)
```

3.29 $\int (a+b\arctan(cx))^3\,dx$

3.29.1 Optimal result . . . 257
3.29.2 Mathematica [A] (verified) . . . 257
3.29.3 Rubi [A] (verified) . . . 258
3.29.4 Maple [B] (verified) . . . 260
3.29.5 Fricas [F] . . . 261
3.29.6 Sympy [F] . . . 261
3.29.7 Maxima [F] . . . 261
3.29.8 Giac [F] . . . 262
3.29.9 Mupad [F(-1)] . . . 262

3.29.1 Optimal result

Integrand size = 10, antiderivative size = 119

$$\int (a+b\arctan(cx))^3\,dx = \frac{i(a+b\arctan(cx))^3}{c}+x(a+b\arctan(cx))^3 + \frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c} + \frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c} + \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c}$$

output
```
I*(a+b*arctan(c*x))^3/c+x*(a+b*arctan(c*x))^3+3*b*(a+b*arctan(c*x))^2*ln(2
/(1+I*c*x))/c+3*I*b^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c+3/2*b^3
*polylog(3,1-2/(1+I*c*x))/c
```

3.29.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.61

$$\int (a+b\arctan(cx))^3\,dx = a^3x+3a^2bx\arctan(cx)-\frac{3a^2b\log(1+c^2x^2)}{2c} + \frac{3ab^2\left(-i\arctan(cx)^2+cx\arctan(cx)^2+2\arctan(cx)\log\left(1+e^{2i\arctan(cx)}\right)-i\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx)}\right.\right.}{c} + \frac{b^3\left(-i\arctan(cx)^3+cx\arctan(cx)^3+3\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right)-3i\arctan(cx)\operatorname{PolyLog}\left(2,-\right.\right.}{c}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3,x]
```

output
```
a^3*x + 3*a^2*b*x*ArcTan[c*x] - (3*a^2*b*Log[1 + c^2*x^2])/(2*c) + (3*a*b^
2*((-I)*ArcTan[c*x]^2 + c*x*ArcTan[c*x]^2 + 2*ArcTan[c*x]*Log[1 + E^((2*I)
*ArcTan[c*x])] - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c + (b^3*((-I)*Arc
Tan[c*x]^3 + c*x*ArcTan[c*x]^3 + 3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c
*x])] - (3*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (3*PolyLog[
3, -E^((2*I)*ArcTan[c*x])])/2))/c
```

3.29.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5345, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arctan(cx))^3\,dx$$

$$\Big\downarrow \text{5345}$$

$$x(a+b\arctan(cx))^3-3bc\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5455}$$

$$x(a+b\arctan(cx))^3-3bc\left(-\frac{\int \frac{(a+b\arctan(cx))^2}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^3}{3bc^2}\right)$$

$$\Big\downarrow \text{5379}$$

$$x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^3}{3bc^2}\right)$$

$$\Big\downarrow \text{5529}$$

$$x(a+b\arctan(cx))^3-$$
$$3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right)}{c}-\frac{i(a+b\arc}{3b}\right.$$

$\downarrow$ 7164

$$x(a+b\arctan(cx))^3-$$
$$3bc\left(-\frac{i(a+b\arctan(cx))^3}{3bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{icx}\right.}{4c}\right.}{c}\right.$$

input
```
Int[(a + b*ArcTan[c*x])^3,x]
```

output
```
x*(a + b*ArcTan[c*x])^3 - 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x])^3)/(b*c^2)
- (((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c - 2*b*(((-1/2*I)*(a + b*Ar
cTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/(1 + I*c
*x)])/(4*c)))/c)
```

3.29.3.1 Defintions of rubi rules used

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.29.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(112) = 224.

Time = 5.71 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{c\,a^3x+b^3\left(\arctan(cx)^3(cx+i)-2i\arctan(cx)^3+3\arctan(cx)^2\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)-3i\arctan(cx)\operatorname{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)+\right.}{}$
default	$\frac{c\,a^3x+b^3\left(\arctan(cx)^3(cx+i)-2i\arctan(cx)^3+3\arctan(cx)^2\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)-3i\arctan(cx)\operatorname{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)+\right.}{}$
parts	$x\,a^3+\frac{b^3\left(\arctan(cx)^3(cx+i)-2i\arctan(cx)^3+3\arctan(cx)^2\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)-3i\arctan(cx)\operatorname{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)\right.}{c}$

input
```
int((a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*a^3*x+b^3*(arctan(c*x)^3*(c*x+I)-2*I*arctan(c*x)^3+3*arctan(c*x)^2*
ln(1+(1+I*c*x)^2/(c^2*x^2+1))-3*I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*
x^2+1))+3/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1)))+3*a^2*b*(c*x*arctan(c*x)-
1/2*ln(c^2*x^2+1))+3*b^2*a*(arctan(c*x)^2*(c*x+I)+2*arctan(c*x)*ln(1+(1+I*
c*x)^2/(c^2*x^2+1))-2*I*arctan(c*x)^2-I*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)
)))
```

3.29.5 Fricas [F]

$$\int (a + b\arctan(cx))^3 \, dx = \int (b\arctan(cx) + a)^3 \, dx$$

input
```
integrate((a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3, x)
```

3.29.6 Sympy [F]

$$\int (a + b\arctan(cx))^3 \, dx = \int (a + b\,\mathrm{atan}(cx))^3 \, dx$$

input
```
integrate((a+b*atan(c*x))**3,x)
```

output
```
Integral((a + b*atan(c*x))**3, x)
```

3.29.7 Maxima [F]

$$\int (a + b\arctan(cx))^3 \, dx = \int (b\arctan(cx) + a)^3 \, dx$$

input
```
integrate((a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
1/8*b^3*x*arctan(c*x)^3 - 3/32*b^3*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 7/32
*b^3*arctan(c*x)^4/c + 28*b^3*c^2*integrate(1/32*x^2*arctan(c*x)^3/(c^2*x^
2 + 1), x) + 3*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(
c^2*x^2 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^2*arctan(c*x)^2/(c^2*x^2
+ 1), x) + 12*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2
*x^2 + 1), x) + a*b^2*arctan(c*x)^3/c - 12*b^3*c*integrate(1/32*x*arctan(c
*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2
*x^2 + 1), x) + a^3*x + 3*b^3*integrate(1/32*arctan(c*x)*log(c^2*x^2 + 1)^
2/(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b/c
```

3.29.8 Giac [F]

$$\int (a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3\,dx$$

input
```
integrate((a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\arctan(cx))^3\,dx=\int (a+b\,\mathrm{atan}(c\,x))^3\,dx$$

input
```
int((a + b*atan(c*x))^3,x)
```

output
```
int((a + b*atan(c*x))^3, x)
```

3.30 $\int \frac{(a+b\arctan(cx))^3}{x}\,dx$

3.30.1 Optimal result 263
3.30.2 Mathematica [A] (verified) 264
3.30.3 Rubi [A] (verified) 265
3.30.4 Maple [C] (warning: unable to verify) 267
3.30.5 Fricas [F] 267
3.30.6 Sympy [F] 268
3.30.7 Maxima [F] 268
3.30.8 Giac [F(-1)] 268
3.30.9 Mupad [F(-1)] 269

3.30.1 Optimal result

Integrand size = 14, antiderivative size = 206

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x}\,dx &= 2(a+b\arctan(cx))^3\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\ &-\frac{3}{2}ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\ &+\frac{3}{2}ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\ &-\frac{3}{2}b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\ &+\frac{3}{2}b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\\ &+\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx}\right)\\ &-\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-2*(a+b*arctan(c*x))^3*arctanh(-1+2/(1+I*c*x))-3/2*I*b*(a+b*arctan(c*x))^2
*polylog(2,1-2/(1+I*c*x))+3/2*I*b*(a+b*arctan(c*x))^2*polylog(2,-1+2/(1+I*
c*x))-3/2*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x))+3/2*b^2*(a+b*arct
an(c*x))*polylog(3,-1+2/(1+I*c*x))+3/4*I*b^3*polylog(4,1-2/(1+I*c*x))-3/4*
I*b^3*polylog(4,-1+2/(1+I*c*x))
```

3.30.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.79

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = a^3\log(cx)+\frac{3}{2}ia^2b(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx)) + 3ab^2\left(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx)^3 + \arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - \arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + \frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\right) - \frac{1}{64}ib^3\big(\pi^4-32\arctan(cx)^4 + 64i\arctan(cx)^3\log\left(1-e^{-2i\arctan(cx)}\right) - 64i\arctan(cx)^3\log\left(1+e^{2i\arctan(cx)}\right) - 96\arctan(cx)^2\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) - 96\arctan(cx)^2\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + 96i\arctan(cx)\,\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - 96i\arctan(cx)\,\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right) + 48\,\text{PolyLog}\left(4,e^{-2i\arctan(cx)}\right)+48\,\text{PolyLog}\left(4,-e^{2i\arctan(cx)}\right)\big)$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/x,x]
```

output
```
a^3*Log[c*x] + ((3*I)/2)*a^2*b*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x])
+ 3*a*b^2*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1
- E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] +
 I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[
2, -E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLo
g[3, -E^((2*I)*ArcTan[c*x])]/2) - (I/64)*b^3*(Pi^4 - 32*ArcTan[c*x]^4 + (6
4*I)*ArcTan[c*x]^3*Log[1 - E^((-2*I)*ArcTan[c*x])] - (64*I)*ArcTan[c*x]^3*
Log[1 + E^((2*I)*ArcTan[c*x])] - 96*ArcTan[c*x]^2*PolyLog[2, E^((-2*I)*Arc
Tan[c*x])] - 96*ArcTan[c*x]^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (96*I)*
ArcTan[c*x]*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - (96*I)*ArcTan[c*x]*PolyLo
g[3, -E^((2*I)*ArcTan[c*x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[c*x])] + 48*
PolyLog[4, -E^((2*I)*ArcTan[c*x])])
```

3.30.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5357, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx$$

$$\Big\downarrow \text{5357}$$

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-6bc\int \frac{(a+b\arctan(cx))^2\text{arctanh}\left(1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx$$

$$\Big\downarrow \text{5523}$$

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-$$
$$6bc\left(\frac{1}{2}\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{1}{2}\int \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5529}$$

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-$$
$$6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}-ib\int \frac{(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+\frac{1}{2}\left(\right.\right.$$

$$\Big\downarrow \text{5533}$$

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-$$
$$6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}-ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\right.\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-$$
$$6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}-ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}+\frac{b\,\text{PolyI}}{}\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])^3/x,x]
```

output
```
2*(a + b*ArcTan[c*x])^3*ArcTanh[1 - 2/(1 + I*c*x)] - 6*b*c*((((I/2)*(a + b
*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - I*b*(((I/2)*(a + b*ArcT
an[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/c + (b*PolyLog[4, 1 - 2/(1 + I*c*x
)])/(4*c)))/2 + (((-1/2*I)*(a + b*ArcTan[c*x])^2*PolyLog[2, -1 + 2/(1 + I*
c*x)])/c + I*b*(((I/2)*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 + I*c*x)])
/c + (b*PolyLog[4, -1 + 2/(1 + I*c*x)])/(4*c)))/2)
```

3.30.3.1 Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.30.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 2026, normalized size of antiderivative = 9.83

Expression too large to display

input
```
int((a+b*arctan(c*x))^3/x,x)
```

output
```
a^3*ln(c*x)+b^3*(ln(c*x)*arctan(c*x)^3-arctan(c*x)^3*ln((1+I*c*x)^2/(c^2*x
^2+1)-1)+arctan(c*x)^3*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2
*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*arctan(c*x)*polylog(3,(1+I*c*x)/
(c^2*x^2+1)^(1/2))+6*I*polylog(4,(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^
3*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)
/(c^2*x^2+1)^(1/2))+6*arctan(c*x)*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+
6*I*polylog(4,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-
1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c
*x)^2/(c^2*x^2+1)))^2+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c
*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2
*x^2+1)))-csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*
csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I*(
(1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3-csgn(I*((1+I*c*x
)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2
+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+
(1+I*c*x)^2/(c^2*x^2+1)))^3+1)*arctan(c*x)^3+3/2*I*arctan(c*x)^2*polylog(2
,-(1+I*c*x)^2/(c^2*x^2+1))-3/2*arctan(c*x)*polylog(3,-(1+I*c*x)^2/(c^2*x^2
+1))-3/4*I*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1)))+3*a*b^2*(ln(c*x)*arctan...
```

3.30.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = \int \frac{(b\arctan{(cx)}+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x)
+ a^3)/x, x)
```

3.30.6 Sympy [F]

$$\int \frac{(a + b\arctan(cx))^3}{x} \, dx = \int \frac{(a + b\,\mathrm{atan}\,(cx))^3}{x} \, dx$$

input
```
integrate((a+b*atan(c*x))**3/x,x)
```

output
```
Integral((a + b*atan(c*x))**3/x, x)
```

3.30.7 Maxima [F]

$$\int \frac{(a + b\arctan(cx))^3}{x} \, dx = \int \frac{(b\arctan(cx) + a)^3}{x} \, dx$$

input
```
integrate((a+b*arctan(c*x))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x)^3 + 3*b^3*arctan(c*x)*log(
c^2*x^2 + 1)^2 + 96*a*b^2*arctan(c*x)^2 + 96*a^2*b*arctan(c*x))/x, x)
```

3.30.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx))^3}{x} \, dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))^3/x,x, algorithm="giac")
```

output
```
Timed out
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x}\,dx$$

input

```
int((a + b*atan(c*x))^3/x,x)
```

output

```
int((a + b*atan(c*x))^3/x, x)
```

3.31 $\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx$

3.31.1 Optimal result 270
3.31.2 Mathematica [A] (verified) 271
3.31.3 Rubi [A] (verified) 271
3.31.4 Maple [C] (warning: unable to verify) 273
3.31.5 Fricas [F] 274
3.31.6 Sympy [F] 275
3.31.7 Maxima [F] 275
3.31.8 Giac [F(-1)] 275
3.31.9 Mupad [F(-1)] 276

3.31.1 Optimal result

Integrand size = 14, antiderivative size = 116

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx = &-ic(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{x}\\ &+3bc(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right)\\ &-3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\ &+\frac{3}{2}b^3c\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-I*c*(a+b*arctan(c*x))^3-(a+b*arctan(c*x))^3/x+3*b*c*(a+b*arctan(c*x))^2*l
n(2-2/(1-I*c*x))-3*I*b^2*c*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))+3/2
*b^3*c*polylog(3,-1+2/(1-I*c*x))
```

3.31.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.84

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx = -\frac{a^3}{x}-\frac{3a^2b\arctan(cx)}{x}+3a^2bc\log(x)-\frac{3}{2}a^2bc\log\left(1+c^2x^2\right)+3ab^2c\left(-\frac{\arctan(cx)^2}{cx}+2\arctan(cx)\log\left(1-e^{2i\arctan(cx)}\right)-i\left(\arctan(cx)^2+\operatorname{PolyLog}\left(2,e^{2i\arctan(cx)}\right)\right)\right)+b^3c\left(-\frac{i\pi^3}{8}+i\arctan(cx)^3-\frac{\arctan(cx)^3}{cx}+3\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right)+3i\arctan(cx)\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx)}\right)+\frac{3}{2}\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/x^2,x]
```

output
```
-(a^3/x) - (3*a^2*b*ArcTan[c*x])/x + 3*a^2*b*c*Log[x] - (3*a^2*b*c*Log[1 +
 c^2*x^2])/2 + 3*a*b^2*c*(-(ArcTan[c*x]^2/(c*x)) + 2*ArcTan[c*x]*Log[1 - E
^((2*I)*ArcTan[c*x])] - I*(ArcTan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x]
)])) + b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x]^3 - ArcTan[c*x]^3/(c*x) + 3*Ar
cTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (3*I)*ArcTan[c*x]*PolyLog[2,
 E^((-2*I)*ArcTan[c*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*x])])/2)
```

3.31.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5361, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx$$

$$\Big\downarrow \text{5361}$$

$$3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{x}$$

$$\Big\downarrow \; 5459$$

$$-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\int \frac{(a+b\arctan(cx))^2}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^3}{3b}\right)$$

$$\Big\downarrow \; 5403$$

$$-\frac{(a+b\arctan(cx))^3}{x}+$$
$$3bc\left(i\left(2ibc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2\right)-\frac{i(a+b\arctan$$

$$\Big\downarrow \; 5527$$

$$-\frac{(a+b\arctan(cx))^3}{x}+$$
$$3bc\left(i\left(2ibc\left(\frac{i\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-i$$

$$\Big\downarrow \; 7164$$

$$-\frac{(a+b\arctan(cx))^3}{x}+$$
$$3bc\left(i\left(2ibc\left(\frac{i\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{b\operatorname{PolyLog}\left(3,\frac{2}{1-icx}-1\right)}{4c}\right)-i\log\left(2-\frac{2}{1-icx}\right)(a$$

input
```
Int[(a + b*ArcTan[c*x])^3/x^2,x]
```

output
```
-((a + b*ArcTan[c*x])^3/x) + 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x])^3)/b + I
*((-I)*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)] + (2*I)*b*c*(((I/2)*(a
 + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)])/c - (b*PolyLog[3, -1 + 2
/(1 - I*c*x)])/(4*c))))
```

3.31. $\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx$

3.31.3.1 Defintions of rubi rules used

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.31.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.91 (sec) , antiderivative size = 1862, normalized size of antiderivative = 16.05

Expression too large to display

input
```
int((a+b*arctan(c*x))^3/x^2,x)
```

output
```
c*(-a^3/c/x+b^3*(-1/c/x*arctan(c*x)^3+3*ln(c*x)*arctan(c*x)^2-3/2*arctan(c
*x)^2*ln(c^2*x^2+1)+3*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-3*arct
an(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-I*arctan(c*x)^3+3/4*(I*Pi*csgn(I*(
1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2
*x^2+1))^2)^2-2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*
x^2+1)))^2-I*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(
c^2*x^2+1))-2*I*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I*c*x)
^2/(c^2*x^2+1))^2)^2+I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+
I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2+2*I*Pi*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3-2*I*Pi*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x
^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+I*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^
2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)+2*I*Pi*csgn(((1+I*c*x)^2/(c
^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3+2*I*Pi*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/
(1+(1+I*c*x)^2/(c^2*x^2+1)))-I*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3-I*Pi*c
sgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3-2*I*Pi*csgn
(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)-1))+2*I*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*
(1+I*c*x)^2/(c^2*x^2+1))^2+2*I*Pi-I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1...
```

3.31.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx=\int \frac{(b\arctan(cx)+a)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x)
+ a^3)/x^2, x)
```

3.31.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx=\int \frac{(a+b\,\mathrm{atan}\,(cx))^3}{x^2}\,dx$$

input
```
integrate((a+b*atan(c*x))**3/x**2,x)
```

output
```
Integral((a + b*atan(c*x))**3/x**2, x)
```

3.31.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx=\int \frac{(b\arctan(cx)+a)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="maxima")
```

output
```
-3/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a^2*b - a^3/x - 1
/32*(4*b^3*arctan(c*x)^3 - 3*b^3*arctan(c*x)*log(c^2*x^2 + 1)^2 - (7*b^3*c
*arctan(c*x)^4 + 32*a*b^2*c*arctan(c*x)^3 + 96*b^3*c^2*integrate(1/32*x^2*
arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) - 384*b^3*c^2*integrate
(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + 384*b^3*c*int
egrate(1/32*x*arctan(c*x)^2/(c^2*x^4 + x^2), x) - 96*b^3*c*integrate(1/32*
x*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) + 896*b^3*integrate(1/32*arctan(c
*x)^3/(c^2*x^4 + x^2), x) + 96*b^3*integrate(1/32*arctan(c*x)*log(c^2*x^2
+ 1)^2/(c^2*x^4 + x^2), x) + 3072*a*b^2*integrate(1/32*arctan(c*x)^2/(c^2*
x^4 + x^2), x))*x)/x
```

3.31.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^2}\,dx$$

input
```
int((a + b*atan(c*x))^3/x^2,x)
```

output
```
int((a + b*atan(c*x))^3/x^2, x)
```

3.32 $\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx$

3.32.1 Optimal result 277
3.32.2 Mathematica [A] (verified) 277
3.32.3 Rubi [A] (verified) 278
3.32.4 Maple [B] (verified) 280
3.32.5 Fricas [F] 281
3.32.6 Sympy [F] 281
3.32.7 Maxima [F] 282
3.32.8 Giac [F(-1)] 282
3.32.9 Mupad [F(-1)] 282

3.32.1 Optimal result

Integrand size = 14, antiderivative size = 133

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = -\frac{3}{2}ibc^2(a+b\arctan(cx))^2 - \frac{3bc(a+b\arctan(cx))^2}{2x} - \frac{1}{2}c^2(a+b\arctan(cx))^3 - \frac{(a+b\arctan(cx))^3}{2x^2} + 3b^2c^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) - \frac{3}{2}ib^3c^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)$$

output
```
-3/2*I*b*c^2*(a+b*arctan(c*x))^2-3/2*b*c*(a+b*arctan(c*x))^2/x-1/2*c^2*(a+
b*arctan(c*x))^3-1/2*(a+b*arctan(c*x))^3/x^2+3*b^2*c^2*(a+b*arctan(c*x))*l
n(2-2/(1-I*c*x))-3/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x))
```

3.32.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx =$$

$$-\frac{3b^2(a+ac^2x^2+bcx(1+icx))\arctan(cx)^2+b^3(1+c^2x^2)\arctan(cx)^3+3b\arctan(cx)\left(a(a+2bcx+ac\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/x^3,x]
```

output
```
-1/2*(3*b^2*(a + a*c^2*x^2 + b*c*x*(1 + I*c*x))*ArcTan[c*x]^2 + b^3*(1 + c
^2*x^2)*ArcTan[c*x]^3 + 3*b*ArcTan[c*x]*(a*(a + 2*b*c*x + a*c^2*x^2) - 2*b
^2*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c*x])]) + a*(a*(a + 3*b*c*x) - 6*b^2*c^
2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + (3*I)*b^3*c^2*x^2*PolyLog[2, E^((2*I
)*ArcTan[c*x])])/x^2
```

3.32.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5361, 5453, 5361, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x^3}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{2}bc\int \frac{(a+b\arctan(cx))^2}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{2x^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{3}{2}bc\left(\int \frac{(a+b\arctan(cx))^2}{x^2}dx-c^2\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)-\frac{(a+b\arctan(cx))^3}{2x^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{2}bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)+2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}\right)-\frac{(a+b\arctan(cx))^3}{2x^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{3}{2}bc\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{c(a+b\arctan(cx))^3}{3b}-\frac{(a+b\arctan(cx))^2}{x}\right)-\frac{(a+b\arctan(cx))^3}{2x^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{(a+b\arctan(cx))^3}{2x^2}+$$
$$\frac{3}{2}bc\left(2bc\left(i\int\frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)-\frac{c(a+b\arctan(cx))^3}{3b}-\frac{(a+b\arctan(cx))^2}{x}\right)$$

$$\Big\downarrow \text{5403}$$

$$-\frac{(a+b\arctan(cx))^3}{2x^2}+$$
$$\frac{3}{2}bc\left(2bc\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)-\frac{c(a$$

$$\Big\downarrow \text{2897}$$

$$-\frac{(a+b\arctan(cx))^3}{2x^2}+$$
$$\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)-$$

input
```
Int[(a + b*ArcTan[c*x])^3/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c*x])^3/x^2 + (3*b*c*(-((a + b*ArcTan[c*x])^2/x) - (c*(
a + b*ArcTan[c*x])^3)/(3*b) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b +
I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2/
(1 - I*c*x)])/2))))/2
```

3.32.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.32.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(119) = 238$.

Time = 4.62 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.66

method	result
derivativedivides	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\arctan(cx)^3}{2c^2x^2}-\frac{3\arctan(cx)^2}{2cx}-\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)\ln(c^2x^2+1)}{2}+3\ln(cx)\,\mathrm{a}\right.\right.$
default	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\arctan(cx)^3}{2c^2x^2}-\frac{3\arctan(cx)^2}{2cx}-\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)\ln(c^2x^2+1)}{2}+3\ln(cx)\,\mathrm{a}\right.\right.$
parts	$-\frac{a^3}{2x^2}+b^3c^2\left(-\frac{\arctan(cx)^3}{2c^2x^2}-\frac{3\arctan(cx)^2}{2cx}-\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)\ln(c^2x^2+1)}{2}+3\ln(cx)\,\mathrm{arct}\right.$

input
```
int((a+b*arctan(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^3/c^2/x^2+b^3*(-1/2/c^2/x^2*arctan(c*x)^3-3/2*arctan(c*x)^2/c/
x-1/2*arctan(c*x)^3-3/2*arctan(c*x)*ln(c^2*x^2+1)+3*ln(c*x)*arctan(c*x)+3/
2*I*ln(c*x)*ln(1+I*c*x)-3/2*I*ln(c*x)*ln(1-I*c*x)+3/2*I*dilog(1+I*c*x)-3/2
*I*dilog(1-I*c*x)-3/4*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/
2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+3/4*I*(ln(c*x+I)*ln(c^2*x^2+1)-
1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))))+3*a*b^2
*(-1/2/c^2/x^2*arctan(c*x)^2-1/c/x*arctan(c*x)-1/2*arctan(c*x)^2-1/2*ln(c^
2*x^2+1)+ln(c*x))+3*a^2*b*(-1/2/c^2/x^2*arctan(c*x)-1/2*arctan(c*x)-1/2/c/
x))
```

3.32.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = \int \frac{(b\arctan(cx)+a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x)
+ a^3)/x^3, x)
```

3.32.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = \int \frac{(a+b\operatorname{atan}(cx))^3}{x^3}\,dx$$

input
```
integrate((a+b*atan(c*x))**3/x**3,x)
```

output
```
Integral((a + b*atan(c*x))**3/x**3, x)
```

3.32.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = \int \frac{(b\arctan(cx)+a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="maxima")
```

output
```
-3/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a^2*b + 3/2*((arctan(c*x)
^2 - log(c^2*x^2 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c
*x))*a*b^2 - 3/2*a*b^2*arctan(c*x)^2/x^2 - 1/32*(12*c*x*arctan(c*x)^2 + 8*
(c^2*x^2 + 1)*arctan(c*x)^3 - 3*c*x*log(c^2*x^2 + 1)^2 - 4*(c^2*arctan(c*x
)^3 + 24*c^3*integrate(1/32*x^3*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) - 9
6*c^3*integrate(1/32*x^3*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 128*c^2*in
tegrate(1/32*x^2*arctan(c*x)^3/(c^2*x^5 + x^3), x) + 192*c^2*integrate(1/3
2*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 96*c*integrate(1/32*x*arctan(c*x)^
2/(c^2*x^5 + x^3), x) + 24*c*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2*x^5
+ x^3), x) + 128*integrate(1/32*arctan(c*x)^3/(c^2*x^5 + x^3), x))*x^2)*b^
3/x^2 - 1/2*a^3/x^2
```

3.32.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^3}\,dx$$

input
```
int((a + b*atan(c*x))^3/x^3,x)
```

output
```
int((a + b*atan(c*x))^3/x^3, x)
```

3.33 $\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx$

3.33.1 Optimal result . . . 283
3.33.2 Mathematica [A] (verified) . . . 284
3.33.3 Rubi [A] (verified) . . . 284
3.33.4 Maple [C] (warning: unable to verify) . . . 288
3.33.5 Fricas [F] . . . 289
3.33.6 Sympy [F] . . . 290
3.33.7 Maxima [F] . . . 290
3.33.8 Giac [F(-1)] . . . 290
3.33.9 Mupad [F(-1)] . . . 291

3.33.1 Optimal result

Integrand size = 14, antiderivative size = 213

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx = &-\frac{b^2c^2(a+b\arctan(cx))}{x}-\frac{1}{2}bc^3(a+b\arctan(cx))^2\\&-\frac{bc(a+b\arctan(cx))^2}{2x^2}+\frac{1}{3}ic^3(a+b\arctan(cx))^3\\&-\frac{(a+b\arctan(cx))^3}{3x^3}+b^3c^3\log(x)-\frac{1}{2}b^3c^3\log\left(1+c^2x^2\right)\\&-bc^3(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right)\\&+ib^2c^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\&-\frac{1}{2}b^3c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-b^2*c^2*(a+b*arctan(c*x))/x-1/2*b*c^3*(a+b*arctan(c*x))^2-1/2*b*c*(a+b*ar
ctan(c*x))^2/x^2+1/3*I*c^3*(a+b*arctan(c*x))^3-1/3*(a+b*arctan(c*x))^3/x^3
+b^3*c^3*ln(x)-1/2*b^3*c^3*ln(c^2*x^2+1)-b*c^3*(a+b*arctan(c*x))^2*ln(2-2/
(1-I*c*x))+I*b^2*c^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))-1/2*b^3*c
^3*polylog(3,-1+2/(1-I*c*x))
```

3.33.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.43

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx$$
$$=\frac{1}{6}\left(-\frac{2a^3}{x^3}-\frac{3a^2bc}{x^2}-\frac{6a^2b\arctan(cx)}{x^3}-6a^2bc^3\log(x)+3a^2bc^3\log\left(1+c^2x^2\right)\right.$$
$$+\frac{6iab^2\left(ic^2x^2+\left(i+c^3x^3\right)\arctan(cx)^2+icx\arctan(cx)\left(1+c^2x^2+2c^2x^2\log\left(1-e^{2i\arctan(cx)}\right)\right)+c^3x^3\,\mathrm{P}\right.}{x^3}$$
$$+6b^3c^3\left(-i\arctan(cx)\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(cx)}\right)\right.$$
$$+\frac{1}{24}\left(i\pi^3-\frac{24\arctan(cx)}{cx}+\left(-8i-\frac{8}{c^3x^3}\right)\arctan(cx)^3+\arctan(cx)^2\left(-12-\frac{12}{c^2x^2}-24\log\left(1-e^{-2i\arctan(c}\right.\right.\right.$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/x^4,x]
```

output
```
((-2*a^3)/x^3 - (3*a^2*b*c)/x^2 - (6*a^2*b*ArcTan[c*x])/x^3 - 6*a^2*b*c^3*
Log[x] + 3*a^2*b*c^3*Log[1 + c^2*x^2] + ((6*I)*a*b^2*(I*c^2*x^2 + (I + c^3
*x^3)*ArcTan[c*x]^2 + I*c*x*ArcTan[c*x]*(1 + c^2*x^2 + 2*c^2*x^2*Log[1 - E
^((2*I)*ArcTan[c*x])]) + c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/x^3 +
 6*b^3*c^3*((-I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (I*Pi^3
- (24*ArcTan[c*x])/(c*x) + (-8*I - 8/(c^3*x^3))*ArcTan[c*x]^3 + ArcTan[c*x
]^2*(-12 - 12/(c^2*x^2) - 24*Log[1 - E^((-2*I)*ArcTan[c*x])]) + 24*Log[c*x
] - 12*Log[1 + c^2*x^2] - 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])])/24))/6
```

3.33.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5361, 5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx$$
$$\Big\downarrow \; 5361$$

$$bc\int \frac{(a+b\arctan(cx))^2}{x^3(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{3x^3}$$

$\downarrow$ 5453

$$bc\left(\int \frac{(a+b\arctan(cx))^2}{x^3}dx-c^2\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx\right)-\frac{(a+b\arctan(cx))^3}{3x^3}$$

$\downarrow$ 5361

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx\right)+bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}\right)-\frac{(a+b\arctan(cx))^3}{3x^3}$$

$\downarrow$ 5453

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx\right)+bc\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{(a+b\arctan(cx}{2x^2}\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$\downarrow$ 5361

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx\right)+bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(c}{x}\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$\downarrow$ 243

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx\right)+bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arcta}{x}\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$\downarrow$ 47

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx\right)+bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$\downarrow$ 14

3.33. $\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx$

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x\left(c^2x^2+1\right)}dx\right)+bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$$\Big\downarrow \; 16$$

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x\left(c^2x^2+1\right)}dx\right)+bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc(\log\left(x^2\right)-\log\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$$\Big\downarrow \; 5419$$

$$bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x\left(c^2x^2+1\right)}dx\right)+bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc(\log\left(x^2\right)-\log\left(c^2x^2\right.\right.\right.$$
$$\frac{(a+b\arctan(cx))^3}{3x^3}$$

$$\Big\downarrow \; 5459$$

$$-\frac{(a+b\arctan(cx))^3}{3x^3}+$$
$$bc\left(-\left(c^2\left(i\int \frac{(a+b\arctan(cx))^2}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^3}{3b}\right)\right)+bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}\right.\right.$$

$$\Big\downarrow \; 5403$$

$$-\frac{(a+b\arctan(cx))^3}{3x^3}+$$
$$bc\left(-\left(c^2\left(i\left(2ibc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2\right)-\frac{i(a+b\right.\right.\right.$$

$$\Big\downarrow \; 5527$$

$$-\frac{(a+b\arctan(cx))^3}{3x^3}+$$
$$bc\left(-\left(c^2\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2\right.\right.\right.\right.\right.$$

$$\Big\downarrow \; 7164$$

$$-\frac{(a+b\arctan(cx))^3}{3x^3}+$$
$$bc\left(-\left(c^2\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,\frac{2}{1-icx}-1\right)}{4c}\right)-i\log\left(2-\frac{2}{1-ic}\right.\right.\right.\right.\right.$$

3.33. $\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx$

input
```
Int[(a + b*ArcTan[c*x])^3/x^4,x]
```

output
```
-1/3*(a + b*ArcTan[c*x])^3/x^3 + b*c*(-1/2*(a + b*ArcTan[c*x])^2/x^2 + b*c
*(-((a + b*ArcTan[c*x])/x) - (c*(a + b*ArcTan[c*x])^2)/(2*b) + (b*c*(Log[x
^2] - Log[1 + c^2*x^2]))/2) - c^2*(((-1/3*I)*(a + b*ArcTan[c*x])^3)/b + I*
((-I)*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)] + (2*I)*b*c*(((I/2)*(a
+ b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)])/c - (b*PolyLog[3, -1 + 2/
(1 - I*c*x)])/(4*c)))))
```

3.33.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.33.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.80 (sec) , antiderivative size = 2097, normalized size of antiderivative = 9.85

Expression too large to display

input
```
int((a+b*arctan(c*x))^3/x^4,x)
```

output
```
c^3*(-1/3/c^3/x^3*a^3+b^3*(-1/3/c^3/x^3*arctan(c*x)^3-1/2/c^2/x^2*arctan(c
*x)^2-ln(c*x)*arctan(c*x)^2+1/2*arctan(c*x)^2*ln(c^2*x^2+1)-arctan(c*x)^2*
ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1
)+1/12*arctan(c*x)*(3*I*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I
*c*x)^2/(c^2*x^2+1))^2)*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1))*Pi*c*x-6*I*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*
c*x)^2/(c^2*x^2+1)))*Pi*c*x-3*I*arctan(c*x)*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2
+1))^2)^3*Pi*c*x-3*I*arctan(c*x)*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csg
n(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*Pi*c*x-3*I*arctan(c*x)*csgn(I*(1+I*c*x)
^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*csgn(I/(1+(1+I*c*x)^2/(c^2
*x^2+1))^2)*Pi*c*x+6*I*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(
1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*Pi*c*x+3*I*
arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^
3*Pi*c*x-6*I*arctan(c*x)*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(
c^2*x^2+1)))^3*Pi*c*x-6*I*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(
1+(1+I*c*x)^2/(c^2*x^2+1)))^3*Pi*c*x-12*I*c*x-6*I*arctan(c*x)*Pi*c*x+6*I*a
rctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))
^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*Pi*c*x+3*I*arctan(c*x)*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1))^3*Pi*c*x-6*I*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x...
```

3.33.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx = \int \frac{(b\arctan{(cx)}+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x)
+ a^3)/x^4, x)
```

3.33.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^3}{x^4}\,dx$$

input
```
integrate((a+b*atan(c*x))**3/x**4,x)
```

output
```
Integral((a + b*atan(c*x))**3/x**4, x)
```

3.33.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx = \int \frac{(b\arctan(cx)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="maxima")
```

output
```
1/2*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
a^2*b - 1/3*a^3/x^3 - 1/96*(4*b^3*arctan(c*x)^3 - 3*b^3*arctan(c*x)*log(c^
2*x^2 + 1)^2 - 96*x^3*integrate(-1/32*(4*b^3*c^2*x^2*arctan(c*x)*log(c^2*x
^2 + 1) - 28*(b^3*c^2*x^2 + b^3)*arctan(c*x)^3 - 4*(24*a*b^2*c^2*x^2 + b^3
*c*x + 24*a*b^2)*arctan(c*x)^2 + (b^3*c*x - 3*(b^3*c^2*x^2 + b^3)*arctan(c
*x))*log(c^2*x^2 + 1)^2)/(c^2*x^6 + x^4), x))/x^3
```

3.33.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="giac")
```

output
```
Timed out
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3}{x^4} \,d x$$

input
```
int((a + b*atan(c*x))^3/x^4,x)
```

output
```
int((a + b*atan(c*x))^3/x^4, x)
```

3.34 $\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx$

3.34.1 Optimal result 292
3.34.2 Mathematica [A] (verified) 293
3.34.3 Rubi [A] (verified) 293
3.34.4 Maple [B] (verified) 297
3.34.5 Fricas [F] 297
3.34.6 Sympy [F] 298
3.34.7 Maxima [F(-1)] 298
3.34.8 Giac [F(-1)] 298
3.34.9 Mupad [F(-1)] 299

3.34.1 Optimal result

Integrand size = 14, antiderivative size = 198

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = &-\frac{b^3c^3}{4x}-\frac{1}{4}b^3c^4\arctan(cx)-\frac{b^2c^2(a+b\arctan(cx))}{4x^2}\\ &+ibc^4(a+b\arctan(cx))^2-\frac{bc(a+b\arctan(cx))^2}{4x^3}\\ &+\frac{3bc^3(a+b\arctan(cx))^2}{4x}\\ &+\frac{1}{4}c^4(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{4x^4}\\ &-2b^2c^4(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\ &+ib^3c^4\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-1/4*b^3*c^3/x-1/4*b^3*c^4*arctan(c*x)-1/4*b^2*c^2*(a+b*arctan(c*x))/x^2+I
*b*c^4*(a+b*arctan(c*x))^2-1/4*b*c*(a+b*arctan(c*x))^2/x^3+3/4*b*c^3*(a+b*
arctan(c*x))^2/x+1/4*c^4*(a+b*arctan(c*x))^3-1/4*(a+b*arctan(c*x))^3/x^4-2
*b^2*c^4*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+I*b^3*c^4*polylog(2,-1+2/(1-I
*c*x))
```

3.34.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx =$$

$$-\frac{a^3+a^2bcx+ab^2c^2x^2-3a^2bc^3x^3+b^3c^3x^3+ab^2c^4x^4+b^2(bcx(1-3c^2x^2-4ic^3x^3)+a(3-3c^4x^4))\,\text{arct}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/x^5,x]
```

output
```
-1/4*(a^3 + a^2*b*c*x + a*b^2*c^2*x^2 - 3*a^2*b*c^3*x^3 + b^3*c^3*x^3 + a*
b^2*c^4*x^4 + b^2*(b*c*x*(1 - 3*c^2*x^2 - (4*I)*c^3*x^3) + a*(3 - 3*c^4*x^
4))*ArcTan[c*x]^2 - b^3*(-1 + c^4*x^4)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(b^2*
c^2*x^2*(1 + c^2*x^2) + a*b*(2*c*x - 6*c^3*x^3) + a^2*(3 - 3*c^4*x^4) + 8*
b^2*c^4*x^4*Log[1 - E^((2*I)*ArcTan[c*x])]) + 8*a*b^2*c^4*x^4*Log[(c*x)/Sq
rt[1 + c^2*x^2]] - (4*I)*b^3*c^4*x^4*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x^
4
```

3.34.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.40, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {5361, 5453, 5361, 5453, 5361, 264, 216, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{4}bc\int \frac{(a+b\arctan(cx))^2}{x^4\,(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{4x^4}$$

$$\Big\downarrow \text{5453}$$

$$\frac{3}{4}bc\left(\int \frac{(a+b\arctan(cx))^2}{x^4}dx-c^2\int \frac{(a+b\arctan(cx))^2}{x^2\,(c^2x^2+1)}dx\right)-\frac{(a+b\arctan(cx))^3}{4x^4}$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{4}bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{x^2(c^2x^2+1)}dx\right)+\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{3x^3}\right)-\frac{(a+b\arctan(cx))^3}{4x^4}$$

$\downarrow$ 5453

$$\frac{3}{4}bc\left(-\left(c^2\left(\int \frac{(a+b\arctan(cx))^2}{x^2}dx-c^2\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)\right)+\frac{2}{3}bc\left(\int \frac{a+b\arctan(cx)}{x^3}dx-c^2\int \frac{a+}{x}\right.\right.\frac{(a+b\arctan(cx))^3}{4x^4}$$

$\downarrow$ 5361

$$\frac{3}{4}bc\left(-\left(c^2\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)+2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}\right)\right)+\frac{2}{3}bc\left(c^2\left(-\right.\right.\right.\frac{(a+b\arctan(cx))^3}{4x^4}$$

$\downarrow$ 264

$$\frac{3}{4}bc\left(-\left(c^2\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)+2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}\right)\right)+\frac{2}{3}bc\left(c^2\left(-\right.\right.\right.\frac{(a+b\arctan(cx))^3}{4x^4}$$

$\downarrow$ 216

$$\frac{3}{4}bc\left(-\left(c^2\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)+2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}\right)\right)+\frac{2}{3}bc\left(c^2\left(-\right.\right.\right.\frac{(a+b\arctan(cx))^3}{4x^4}$$

$\downarrow$ 5419

$$\frac{3}{4}bc\left(-\left(c^2\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{c(a+b\arctan(cx))^3}{3b}-\frac{(a+b\arctan(cx))^2}{x}\right)\right)+\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\mathrm{arc}}{x(c^2x}\right.\right.\right.\frac{(a+b\arctan(cx))^3}{4x^4}$$

$\downarrow$ 5459

$$-\frac{(a+b\arctan(cx))^3}{4x^4}+\frac{3}{4}bc\left(\frac{2}{3}bc\left(-\left(c^2\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\right.\right.\right.$$

$\downarrow$ 5403

3.34. $\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx$

$$-\frac{(a+b\arctan(cx))^3}{4x^4}+$$
$$\frac{3}{4}bc\left(-\left(c^2\left(2bc\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right.\right.\right.\right.$$

$\downarrow$ 2897

$$-\frac{(a+b\arctan(cx))^3}{4x^4}+$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(c\right.\right.\right.\right.}{2b}$$

input
```
Int[(a + b*ArcTan[c*x])^3/x^5,x]
```

output
```
-1/4*(a + b*ArcTan[c*x])^3/x^4 + (3*b*c*(-1/3*(a + b*ArcTan[c*x])^2/x^3 -
c^2*(-((a + b*ArcTan[c*x])^2/x) - (c*(a + b*ArcTan[c*x])^3)/(3*b) + 2*b*c*
(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b + I*((-I)*(a + b*ArcTan[c*x])*Log[2 -
2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2/(1 - I*c*x)])/2))) + (2*b*c*(-1/2*(a
 + b*ArcTan[c*x])/x^2 + (b*c*(-x^(-1) - c*ArcTan[c*x]))/2 - c^2*(((-1/2*I)
*(a + b*ArcTan[c*x])^2)/b + I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c
*x)] - (b*PolyLog[2, -1 + 2/(1 - I*c*x)])/2))))/3))/4
```

3.34.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.34. $\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx$

3.34.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(180) = 360.

Time = 4.50 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.14

method	result
derivativedivides	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\arctan(cx)^3}{4c^4x^4}+\frac{\arctan(cx)^3}{4}-\frac{\arctan(cx)^2}{4c^3x^3}+\frac{3\arctan(cx)^2}{4cx}+\arctan(cx)\ln(c^2x\right.\right.$
default	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\arctan(cx)^3}{4c^4x^4}+\frac{\arctan(cx)^3}{4}-\frac{\arctan(cx)^2}{4c^3x^3}+\frac{3\arctan(cx)^2}{4cx}+\arctan(cx)\ln(c^2x\right.\right.$
parts	$-\frac{a^3}{4x^4}+b^3c^4\left(-\frac{\arctan(cx)^3}{4c^4x^4}+\frac{\arctan(cx)^3}{4}-\frac{\arctan(cx)^2}{4c^3x^3}+\frac{3\arctan(cx)^2}{4cx}+\arctan(cx)\ln(c^2x^2+\right.$

input
```
int((a+b*arctan(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
c^4*(-1/4*a^3/c^4/x^4+b^3*(-1/4/c^4/x^4*arctan(c*x)^3+1/4*arctan(c*x)^3-1/
4/c^3/x^3*arctan(c*x)^2+3/4*arctan(c*x)^2/c/x+arctan(c*x)*ln(c^2*x^2+1)-1/
4/c^2/x^2*arctan(c*x)-2*ln(c*x)*arctan(c*x)-1/4*arctan(c*x)-1/4/c/x-I*ln(c
*x)*ln(1+I*c*x)+I*ln(c*x)*ln(1-I*c*x)-I*dilog(1+I*c*x)+I*dilog(1-I*c*x)+1/
2*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-
I)*ln(-1/2*I*(c*x+I)))-1/2*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilo
g(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))))+3*a*b^2*(-1/4/c^4/x^4*arcta
n(c*x)^2-1/6/c^3/x^3*arctan(c*x)+1/2/c/x*arctan(c*x)+1/4*arctan(c*x)^2-1/1
2/c^2/x^2-2/3*ln(c*x)+1/3*ln(c^2*x^2+1))+3*a^2*b*(-1/4/c^4/x^4*arctan(c*x)
-1/12/c^3/x^3+1/4/c/x+1/4*arctan(c*x)))
```

3.34.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx=\int \frac{(b\arctan(cx)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x)
+ a^3)/x^5, x)
```

3.34.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx=\int \frac{(a+b\operatorname{atan}(cx))^3}{x^5}\,dx$$

input
```
integrate((a+b*atan(c*x))**3/x**5,x)
```

output
```
Integral((a + b*atan(c*x))**3/x**5, x)
```

3.34.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="maxima")
```

output
```
Timed out
```

3.34.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="giac")
```

output
```
Timed out
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^3}{x^5}\,dx$$

input
```
int((a + b*atan(c*x))^3/x^5,x)
```

output
```
int((a + b*atan(c*x))^3/x^5, x)
```

3.35 $\int \frac{x}{\arctan(ax)}\,dx$

3.35.1 Optimal result 300
3.35.2 Mathematica [N/A] 300
3.35.3 Rubi [N/A] 301
3.35.4 Maple [N/A] (verified) 301
3.35.5 Fricas [N/A] 302
3.35.6 Sympy [N/A] 302
3.35.7 Maxima [N/A] 302
3.35.8 Giac [N/A] 303
3.35.9 Mupad [N/A] 303

3.35.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{x}{\arctan(ax)}\,dx = \text{Int}\left(\frac{x}{\arctan(ax)},x\right)$$

output
```
Unintegrable(x/arctan(a*x),x)
```

3.35.2 Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan(ax)}\,dx$$

input
```
Integrate[x/ArcTan[a*x],x]
```

output
```
Integrate[x/ArcTan[a*x], x]
```

3.35.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)}\,dx$$

$$\Big\downarrow \text{5377}$$

$$\int \frac{x}{\arctan(ax)}dx$$

input
```
Int[x/ArcTan[a*x],x]
```

output
```
$Aborted
```

3.35.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.35.4 Maple [N/A] (verified)

Not integrable

Time = 10.58 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)}dx$$

input
```
int(x/arctan(a*x),x)
```

output
```
int(x/arctan(a*x),x)
```

3.35.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x/arctan(a*x), x)
```

3.35.6 Sympy [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(x/atan(a*x),x)
```

output
```
Integral(x/atan(a*x), x)
```

3.35.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/arctan(a*x), x)
```

3.35.8 Giac [N/A]

Not integrable

Time = 23.92 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.35.9 Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int(x/atan(a*x),x)
```

output
```
int(x/atan(a*x), x)
```

3.36 $\int \frac{1}{\arctan(ax)}\,dx$

3.36.1 Optimal result 304
3.36.2 Mathematica [N/A] 304
3.36.3 Rubi [N/A] 305
3.36.4 Maple [N/A] (verified) 305
3.36.5 Fricas [N/A] 306
3.36.6 Sympy [N/A] 306
3.36.7 Maxima [N/A] 306
3.36.8 Giac [N/A] 307
3.36.9 Mupad [N/A] 307

3.36.1 Optimal result

Integrand size = 6, antiderivative size = 6

$$\int \frac{1}{\arctan(ax)}\,dx = \text{Int}\left(\frac{1}{\arctan(ax)}, x\right)$$

output
```
Unintegrable(1/arctan(a*x),x)
```

3.36.2 Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\arctan(ax)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(-1),x]
```

output
```
Integrate[ArcTan[a*x]^(-1), x]
```

3.36.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)}\,dx$$
$$\Big\downarrow \; 5353$$
$$\int \frac{1}{\arctan(ax)}dx$$

input
```
Int[ArcTan[a*x]^(-1),x]
```

output
```
$Aborted
```

3.36.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

3.36.4 Maple [N/A] (verified)

Not integrable

Time = 4.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arctan\left(ax\right)}dx$$

input
```
int(1/arctan(a*x),x)
```

output
```
int(1/arctan(a*x),x)
```

3.36.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\arctan{(ax)}}\,dx$$

input
```
integrate(1/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/arctan(a*x), x)
```

3.36.6 Sympy [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(1/atan(a*x),x)
```

output
```
Integral(1/atan(a*x), x)
```

3.36.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\arctan{(ax)}}\,dx$$

input
```
integrate(1/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/arctan(a*x), x)
```

3.36.8 Giac [N/A]

Not integrable

Time = 21.74 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.36.9 Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int(1/atan(a*x),x)
```

output
```
int(1/atan(a*x), x)
```

3.37 $\int \frac{1}{x \arctan(ax)} \, dx$

3.37.1 Optimal result 308
3.37.2 Mathematica [N/A] 308
3.37.3 Rubi [N/A] 309
3.37.4 Maple [N/A] (verified) 309
3.37.5 Fricas [N/A] 310
3.37.6 Sympy [N/A] 310
3.37.7 Maxima [N/A] 310
3.37.8 Giac [N/A] 311
3.37.9 Mupad [N/A] 311

3.37.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arctan(ax)} \, dx = \text{Int}\left(\frac{1}{x \arctan(ax)}, x\right)$$

output
```
Unintegrable(1/x/arctan(a*x),x)
```

3.37.2 Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)} \, dx = \int \frac{1}{x \arctan(ax)} \, dx$$

input
```
Integrate[1/(x*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*ArcTan[a*x]), x]
```

3.37.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)}\,dx$$
$$\Big\downarrow \ 5377$$
$$\int \frac{1}{x\arctan(ax)}dx$$

input
```
Int[1/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

3.37.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.37.4 Maple [N/A] (verified)

Not integrable

Time = 4.59 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\arctan\left(ax\right)}dx$$

input
```
int(1/x/arctan(a*x),x)
```

output
```
int(1/x/arctan(a*x),x)
```

3.37.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)} \, dx = \int \frac{1}{x \arctan{(ax)}} \, dx$$

input
```
integrate(1/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/(x*arctan(a*x)), x)
```

3.37.6 Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x \arctan(ax)} \, dx = \int \frac{1}{x \operatorname{atan}{(ax)}} \, dx$$

input
```
integrate(1/x/atan(a*x),x)
```

output
```
Integral(1/(x*atan(a*x)), x)
```

3.37.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)} \, dx = \int \frac{1}{x \arctan{(ax)}} \, dx$$

input
```
integrate(1/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(x*arctan(a*x)), x)
```

3.37.8 Giac [N/A]

Not integrable

Time = 24.44 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int \frac{1}{x\arctan(ax)}\,dx = \int \frac{1}{x\arctan{(ax)}}\,dx$$

input
```
integrate(1/x/arctan(a*x),x, algorithm="giac")
```

output
```
sage0*x
```

3.37.9 Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arctan(ax)}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int(1/(x*atan(a*x)),x)
```

output
```
int(1/(x*atan(a*x)), x)
```

3.38 $\int \frac{x}{\arctan(ax)^2}\,dx$

3.38.1 Optimal result 312
3.38.2 Mathematica [N/A] 312
3.38.3 Rubi [N/A] 313
3.38.4 Maple [N/A] (verified) 313
3.38.5 Fricas [N/A] 314
3.38.6 Sympy [N/A] 314
3.38.7 Maxima [N/A] 314
3.38.8 Giac [N/A] 315
3.38.9 Mupad [N/A] 315

3.38.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{x}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{x}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable(x/arctan(a*x)^2,x)
```

3.38.2 Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)^2}\,dx = \int \frac{x}{\arctan(ax)^2}\,dx$$

input
```
Integrate[x/ArcTan[a*x]^2,x]
```

output
```
Integrate[x/ArcTan[a*x]^2, x]
```

3.38.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2} \, dx$$
$$\Big\downarrow \text{5377}$$
$$\int \frac{x}{\arctan(ax)^2} dx$$

input
```
Int[x/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

3.38.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.38.4 Maple [N/A] (verified)

Not integrable

Time = 10.51 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan\left(ax\right)^{2}}dx$$

input
```
int(x/arctan(a*x)^2,x)
```

output
```
int(x/arctan(a*x)^2,x)
```

3.38.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)^2}\,dx = \int \frac{x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x/arctan(a*x)^2, x)
```

3.38.6 Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)^2}\,dx = \int \frac{x}{\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(x/atan(a*x)**2,x)
```

output
```
Integral(x/atan(a*x)**2, x)
```

3.38.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 5.62

$$\int \frac{x}{\arctan(ax)^2}\,dx = \int \frac{x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^2*x^3 - arctan(a*x)*integrate((3*a^2*x^2 + 1)/arctan(a*x), x) + x)/(a*
arctan(a*x))
```

3.38.8 Giac [N/A]

Not integrable

Time = 49.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \frac{x}{\arctan(ax)^2}\,dx = \int \frac{x}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.38.9 Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)^2}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int(x/atan(a*x)^2,x)
```

output
```
int(x/atan(a*x)^2, x)
```

3.39 $\int \frac{1}{\arctan(ax)^2}\,dx$

3.39.1 Optimal result 316
3.39.2 Mathematica [N/A] 316
3.39.3 Rubi [N/A] 317
3.39.4 Maple [N/A] (verified) 317
3.39.5 Fricas [N/A] 318
3.39.6 Sympy [N/A] 318
3.39.7 Maxima [N/A] 318
3.39.8 Giac [N/A] 319
3.39.9 Mupad [N/A] 319

3.39.1 Optimal result

Integrand size = 6, antiderivative size = 6

$$\int \frac{1}{\arctan(ax)^2}\,dx = \mathrm{Int}\left(\frac{1}{\arctan(ax)^2},x\right)$$

output
```
Unintegrable(1/arctan(a*x)^2,x)
```

3.39.2 Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\arctan(ax)^2}\,dx$$

input
```
Integrate[ArcTan[a*x]^(-2),x]
```

output
```
Integrate[ArcTan[a*x]^(-2), x]
```

3.39.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2} \, dx$$
$$\Big\downarrow \ 5353$$
$$\int \frac{1}{\arctan(ax)^2}dx$$

input
```
Int[ArcTan[a*x]^(-2),x]
```

output
```
$Aborted
```

3.39.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

3.39.4 Maple [N/A] (verified)

Not integrable

Time = 5.54 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arctan\left(ax\right)^{2}}dx$$

input
```
int(1/arctan(a*x)^2,x)
```

output
```
int(1/arctan(a*x)^2,x)
```

3.39.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2} \, dx = \int \frac{1}{\arctan{(ax)}^2} \, dx$$

input
```
integrate(1/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^(-2), x)
```

3.39.6 Sympy [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2} \, dx = \int \frac{1}{\operatorname{atan}^2{(ax)}} \, dx$$

input
```
integrate(1/atan(a*x)**2,x)
```

output
```
Integral(atan(a*x)**(-2), x)
```

3.39.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 6.50

$$\int \frac{1}{\arctan(ax)^2} \, dx = \int \frac{1}{\arctan{(ax)}^2} \, dx$$

input
```
integrate(1/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^2*x^2 - 2*a^2*arctan(a*x)*integrate(x/arctan(a*x), x) + 1)/(a*arctan(a
*x))
```

3.39.8 Giac [N/A]

Not integrable

Time = 47.80 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.39.9 Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\mathrm{atan}{(a\,x)}^2}\,dx$$

input
```
int(1/atan(a*x)^2,x)
```

output
```
int(1/atan(a*x)^2, x)
```

3.40 $\int \frac{1}{x \arctan(ax)^2} dx$

3.40.1 Optimal result . . . 320
3.40.2 Mathematica [N/A] . . . 320
3.40.3 Rubi [N/A] . . . 321
3.40.4 Maple [N/A] (verified) . . . 321
3.40.5 Fricas [N/A] . . . 322
3.40.6 Sympy [N/A] . . . 322
3.40.7 Maxima [N/A] . . . 322
3.40.8 Giac [N/A] . . . 323
3.40.9 Mupad [N/A] . . . 323

3.40.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arctan(ax)^2} dx = \operatorname{Int}\left(\frac{1}{x \arctan(ax)^2}, x\right)$$

output
```
Unintegrable(1/x/arctan(a*x)^2,x)
```

3.40.2 Mathematica [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)^2} dx = \int \frac{1}{x \arctan(ax)^2} dx$$

input
```
Integrate[1/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*ArcTan[a*x]^2), x]
```

3.40.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^2}\,dx$$

$$\Big\downarrow \ 5377$$

$$\int \frac{1}{x\arctan(ax)^2}dx$$

input
```
Int[1/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

3.40.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.40.4 Maple [N/A] (verified)

Not integrable

Time = 4.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\arctan\left(ax\right)^{2}}dx$$

input
```
int(1/x/arctan(a*x)^2,x)
```

output
```
int(1/x/arctan(a*x)^2,x)
```

3.40.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)^2} dx = \int \frac{1}{x \arctan{(ax)}^2} dx$$

input
```
integrate(1/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/(x*arctan(a*x)^2), x)
```

3.40.6 Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arctan(ax)^2} dx = \int \frac{1}{x \operatorname{atan}^2{(ax)}} dx$$

input
```
integrate(1/x/atan(a*x)**2,x)
```

output
```
Integral(1/(x*atan(a*x)**2), x)
```

3.40.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 5.10

$$\int \frac{1}{x \arctan(ax)^2} dx = \int \frac{1}{x \arctan{(ax)}^2} dx$$

input
```
integrate(1/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^2*x^2 - x*arctan(a*x)*integrate((a^2*x^2 - 1)/(x^2*arctan(a*x)), x) +
1)/(a*x*arctan(a*x))
```

3.40.8 Giac [N/A]

Not integrable

Time = 49.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \int \frac{1}{x \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.40.9 Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \int \frac{1}{x \operatorname{atan}{(a\,x)}^2} \, dx$$

input
```
int(1/(x*atan(a*x)^2),x)
```

output
```
int(1/(x*atan(a*x)^2), x)
```

3.41 $\int x\sqrt{\arctan(ax)}\,dx$

3.41.1 Optimal result . . . 324
3.41.2 Mathematica [N/A] . . . 324
3.41.3 Rubi [N/A] . . . 325
3.41.4 Maple [N/A] (verified) . . . 325
3.41.5 Fricas [F(-2)] . . . 326
3.41.6 Sympy [N/A] . . . 326
3.41.7 Maxima [F(-2)] . . . 326
3.41.8 Giac [N/A] . . . 327
3.41.9 Mupad [N/A] . . . 327

3.41.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x\sqrt{\arctan(ax)}\,dx = \text{Int}\left(x\sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable(x*arctan(a*x)^(1/2),x)
```

3.41.2 Mathematica [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x\sqrt{\arctan(ax)}\,dx = \int x\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*Sqrt[ArcTan[a*x]], x]
```

3.41.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\,dx$$
$$\Big\downarrow \; 5377$$
$$\int x\sqrt{\arctan(ax)}dx$$

input
```
Int[x*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.41.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.41.4 Maple [N/A] (verified)

Not integrable

Time = 2.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sqrt{\arctan\left(ax\right)}dx$$

input
```
int(x*arctan(a*x)^(1/2),x)
```

output
```
int(x*arctan(a*x)^(1/2),x)
```

3.41.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.41.6 Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\sqrt{\arctan(ax)}\,dx = \int x\sqrt{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x*atan(a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(atan(a*x)), x)
```

3.41.7 Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.41.8 Giac [N/A]

Not integrable

Time = 53.54 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int x\sqrt{\arctan(ax)}\,dx = \int x\sqrt{\arctan\left(ax\right)}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.41.9 Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\sqrt{\arctan(ax)}\,dx = \int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int(x*atan(a*x)^(1/2),x)
```

output
```
int(x*atan(a*x)^(1/2), x)
```

3.42 $\int \sqrt{\arctan(ax)}\,dx$

3.42.1 Optimal result . . . 328
3.42.2 Mathematica [N/A] . . . 328
3.42.3 Rubi [N/A] . . . 329
3.42.4 Maple [N/A] (verified) . . . 329
3.42.5 Fricas [F(-2)] . . . 330
3.42.6 Sympy [N/A] . . . 330
3.42.7 Maxima [F(-2)] . . . 330
3.42.8 Giac [N/A] . . . 331
3.42.9 Mupad [N/A] . . . 331

3.42.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \sqrt{\arctan(ax)}\,dx = \text{Int}\left(\sqrt{\arctan(ax)},x\right)$$

output
```
Unintegrable(arctan(a*x)^(1/2),x)
```

3.42.2 Mathematica [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \sqrt{\arctan(ax)}\,dx = \int \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]], x]
```

3.42.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\,dx$$

$$\Big\downarrow \text{ 5353}$$

$$\int \sqrt{\arctan(ax)}dx$$

input
```
Int[Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.42.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

3.42.4 Maple [N/A] (verified)

Not integrable

Time = 1.64 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sqrt{\arctan\left(ax\right)}dx$$

input
```
int(arctan(a*x)^(1/2),x)
```

output
```
int(arctan(a*x)^(1/2),x)
```

3.42.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\arctan(ax)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.42.6 Sympy [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sqrt{\arctan(ax)}\, dx = \int \sqrt{\operatorname{atan}(ax)}\, dx$$

input
```
integrate(atan(a*x)**(1/2),x)
```

output
```
Integral(sqrt(atan(a*x)), x)
```

3.42.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\arctan(ax)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.42.8 Giac [N/A]

Not integrable

Time = 53.95 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \sqrt{\arctan(ax)}\,dx = \int \sqrt{\arctan{(ax)}}\,dx$$

input
```
integrate(arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.42.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sqrt{\arctan(ax)}\,dx = \int \sqrt{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int(atan(a*x)^(1/2),x)
```

output
```
int(atan(a*x)^(1/2), x)
```

3.43 $\int \frac{\sqrt{\arctan(ax)}}{x} \, dx$

3.43.1 Optimal result 332
3.43.2 Mathematica [N/A] 332
3.43.3 Rubi [N/A] 333
3.43.4 Maple [N/A] (verified) 333
3.43.5 Fricas [F(-2)] 334
3.43.6 Sympy [N/A] 334
3.43.7 Maxima [F(-2)] 334
3.43.8 Giac [N/A] 335
3.43.9 Mupad [N/A] 335

3.43.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sqrt{\arctan(ax)}}{x} \, dx = \text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x}, x\right)$$

output
```
Unintegrable(arctan(a*x)^(1/2)/x,x)
```

3.43.2 Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{\arctan(ax)}}{x} \, dx = \int \frac{\sqrt{\arctan(ax)}}{x} \, dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/x,x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/x, x]
```

3.43.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x} \, dx$$
$$\Big\downarrow \text{5377}$$
$$\int \frac{\sqrt{\arctan(ax)}}{x}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/x,x]
```

output
```
$Aborted
```

3.43.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.43.4 Maple [N/A] (verified)

Not integrable

Time = 1.95 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan\left(a x\right)}}{x}dx$$

input
```
int(arctan(a*x)^(1/2)/x,x)
```

output
```
int(arctan(a*x)^(1/2)/x,x)
```

3.43.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.43.6 Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(atan(a*x))/x, x)
```

3.43.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.43.8 Giac [N/A]

Not integrable

Time = 148.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\arctan{(ax)}}}{x}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.43.9 Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x}\,dx$$

input
```
int(atan(a*x)^(1/2)/x,x)
```

output
```
int(atan(a*x)^(1/2)/x, x)
```

3.44 $\int x \arctan(ax)^{3/2} \, dx$

3.44.1 Optimal result 336
3.44.2 Mathematica [N/A] 336
3.44.3 Rubi [N/A] 337
3.44.4 Maple [N/A] (verified) 337
3.44.5 Fricas [F(-2)] 338
3.44.6 Sympy [N/A] 338
3.44.7 Maxima [F(-2)] 338
3.44.8 Giac [N/A] 339
3.44.9 Mupad [N/A] 339

3.44.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x \arctan(ax)^{3/2} \, dx = \text{Int}\left(x \arctan(ax)^{3/2}, x\right)$$

output
```
Unintegrable(x*arctan(a*x)^(3/2),x)
```

3.44.2 Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x \arctan(ax)^{3/2} \, dx = \int x \arctan(ax)^{3/2} \, dx$$

input
```
Integrate[x*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*ArcTan[a*x]^(3/2), x]
```

3.44.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2} \, dx$$

$$\Big\downarrow \text{5377}$$

$$\int x \arctan(ax)^{3/2} dx$$

input
```
Int[x*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.44.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.44.4 Maple [N/A] (verified)

Not integrable

Time = 1.79 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x \arctan\left(ax\right)^{\frac{3}{2}} dx$$

input
```
int(x*arctan(a*x)^(3/2),x)
```

output
```
int(x*arctan(a*x)^(3/2),x)
```

3.44.5 Fricas [F(-2)]

Exception generated.

$$\int x\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.44.6 Sympy [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\arctan(ax)^{3/2}\,dx = \int x\,\text{atan}^{\frac{3}{2}}\,(ax)\,dx$$

input
```
integrate(x*atan(a*x)**(3/2),x)
```

output
```
Integral(x*atan(a*x)**(3/2), x)
```

3.44.7 Maxima [F(-2)]

Exception generated.

$$\int x\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.44.8 Giac [N/A]

Not integrable

Time = 81.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int x \arctan(ax)^{3/2} \, dx = \int x \arctan\left(ax\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.44.9 Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x \arctan(ax)^{3/2} \, dx = \int x \operatorname{atan}(a\,x)^{3/2} \, dx$$

input
```
int(x*atan(a*x)^(3/2),x)
```

output
```
int(x*atan(a*x)^(3/2), x)
```

3.45 $\int \arctan(ax)^{3/2}\,dx$

3.45.1 Optimal result . . . 340
3.45.2 Mathematica [N/A] . . . 340
3.45.3 Rubi [N/A] . . . 341
3.45.4 Maple [N/A] (verified) . . . 341
3.45.5 Fricas [F(-2)] . . . 342
3.45.6 Sympy [N/A] . . . 342
3.45.7 Maxima [F(-2)] . . . 342
3.45.8 Giac [N/A] . . . 343
3.45.9 Mupad [N/A] . . . 343

3.45.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \arctan(ax)^{3/2}\,dx = \text{Int}\left(\arctan(ax)^{3/2},x\right)$$

output
```
Unintegrable(arctan(a*x)^(3/2),x)
```

3.45.2 Mathematica [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \arctan(ax)^{3/2}\,dx = \int \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2), x]
```

3.45.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2} \, dx$$
$$\Big\downarrow \ 5353$$
$$\int \arctan(ax)^{3/2} dx$$

input
```
Int[ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.45.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

3.45.4 Maple [N/A] (verified)

Not integrable

Time = 1.53 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \arctan \left(a x \right)^{\frac{3}{2}} d x$$

input
```
int(arctan(a*x)^(3/2),x)
```

output
```
int(arctan(a*x)^(3/2),x)
```

3.45.5 Fricas [F(-2)]

Exception generated.

$$\int \arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.45.6 Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \arctan(ax)^{3/2}\,dx = \int \operatorname{atan}^{\frac{3}{2}}(ax)\,dx$$

input
```
integrate(atan(a*x)**(3/2),x)
```

output
```
Integral(atan(a*x)**(3/2), x)
```

3.45.7 Maxima [F(-2)]

Exception generated.

$$\int \arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.45.8 Giac [N/A]

Not integrable

Time = 81.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \arctan(ax)^{3/2}\,dx = \int \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.45.9 Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \arctan(ax)^{3/2}\,dx = \int \mathrm{atan}(a\,x)^{3/2}\,dx$$

input
```
int(atan(a*x)^(3/2),x)
```

output
```
int(atan(a*x)^(3/2), x)
```

3.46 $\int \frac{\arctan(ax)^{3/2}}{x} \, dx$

3.46.1 Optimal result 344
3.46.2 Mathematica [N/A] 344
3.46.3 Rubi [N/A] 345
3.46.4 Maple [N/A] (verified) 345
3.46.5 Fricas [F(-2)] 346
3.46.6 Sympy [N/A] 346
3.46.7 Maxima [F(-2)] 346
3.46.8 Giac [N/A] 347
3.46.9 Mupad [N/A] 347

3.46.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\arctan(ax)^{3/2}}{x} \, dx = \text{Int}\left(\frac{\arctan(ax)^{3/2}}{x}, x\right)$$

output
```
Unintegrable(arctan(a*x)^(3/2)/x,x)
```

3.46.2 Mathematica [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\arctan(ax)^{3/2}}{x} \, dx = \int \frac{\arctan(ax)^{3/2}}{x} \, dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/x,x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/x, x]
```

3.46.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{5377}$$

$$\int \frac{\arctan(ax)^{3/2}}{x}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/x,x]
```

output
```
$Aborted
```

3.46.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.46.4 Maple [N/A] (verified)

Not integrable

Time = 2.73 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arctan \left(a x \right)^{\frac{3}{2}}}{x}dx$$

input
```
int(arctan(a*x)^(3/2)/x,x)
```

output
```
int(arctan(a*x)^(3/2)/x,x)
```

3.46.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.46.6 Sympy [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x,x)
```

output
```
Integral(atan(a*x)**(3/2)/x, x)
```

3.46.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.46.8 Giac [N/A]

Not integrable

Time = 142.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.25

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\arctan{(ax)}^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.46.9 Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x}\,dx$$

input
```
int(atan(a*x)^(3/2)/x,x)
```

output
```
int(atan(a*x)^(3/2)/x, x)
```

3.47 $\int \frac{x}{\sqrt{\arctan(ax)}}\,dx$

3.47.1 Optimal result 348
3.47.2 Mathematica [N/A] 348
3.47.3 Rubi [N/A] 349
3.47.4 Maple [N/A] (verified) 349
3.47.5 Fricas [F(-2)] 350
3.47.6 Sympy [N/A] 350
3.47.7 Maxima [F(-2)] 350
3.47.8 Giac [N/A] 351
3.47.9 Mupad [N/A] 351

3.47.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(x/arctan(a*x)^(1/2),x)
```

3.47.2 Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x/Sqrt[ArcTan[a*x]], x]
```

3.47.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}} \, dx$$

$$\Big\downarrow \; 5377$$

$$\int \frac{x}{\sqrt{\arctan(ax)}} dx$$

input
```
Int[x/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.47.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.47.4 Maple [N/A] (verified)

Not integrable

Time = 1.75 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{\arctan (ax)}} dx$$

input
```
int(x/arctan(a*x)^(1/2),x)
```

output
```
int(x/arctan(a*x)^(1/2),x)
```

3.47.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.47.6 Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x/atan(a*x)**(1/2),x)
```

output
```
Integral(x/sqrt(atan(a*x)), x)
```

3.47.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.47.8 Giac [N/A]

Not integrable

Time = 70.97 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.47.9 Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int(x/atan(a*x)^(1/2),x)
```

output
```
int(x/atan(a*x)^(1/2), x)
```

3.48 $\int \frac{1}{\sqrt{\arctan(ax)}}\,dx$

3.48.1 Optimal result . . . 352
3.48.2 Mathematica [N/A] . . . 352
3.48.3 Rubi [N/A] . . . 353
3.48.4 Maple [N/A] (verified) . . . 353
3.48.5 Fricas [F(-2)] . . . 354
3.48.6 Sympy [N/A] . . . 354
3.48.7 Maxima [F(-2)] . . . 354
3.48.8 Giac [N/A] . . . 355
3.48.9 Mupad [N/A] . . . 355

3.48.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(1/arctan(a*x)^(1/2),x)
```

3.48.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[1/Sqrt[ArcTan[a*x]], x]
```

3.48.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}} \, dx$$
$$\Big\downarrow \text{5353}$$
$$\int \frac{1}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[1/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

3.48.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

3.48.4 Maple [N/A] (verified)

Not integrable

Time = 1.59 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{\arctan (ax)}}dx$$

input
```
int(1/arctan(a*x)^(1/2),x)
```

output
```
int(1/arctan(a*x)^(1/2),x)
```

3.48.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.48.6 Sympy [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(1/atan(a*x)**(1/2),x)
```

output
```
Integral(1/sqrt(atan(a*x)), x)
```

3.48.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.48.8 Giac [N/A]

Not integrable

Time = 64.90 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(1/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.48.9 Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int(1/atan(a*x)^(1/2),x)
```

output
```
int(1/atan(a*x)^(1/2), x)
```

3.49 $\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$

3.49.1 Optimal result 356
3.49.2 Mathematica [N/A] 356
3.49.3 Rubi [N/A] 357
3.49.4 Maple [N/A] (verified) 357
3.49.5 Fricas [F(-2)] 358
3.49.6 Sympy [N/A] 358
3.49.7 Maxima [F(-2)] 358
3.49.8 Giac [N/A] 359
3.49.9 Mupad [N/A] 359

3.49.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Unintegrable(1/x/arctan(a*x)^(1/2),x)
```

3.49.2 Mathematica [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[ArcTan[a*x]]), x]
```

3.49.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{5377}$$
$$\int \frac{1}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[1/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

3.49.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.49.4 Maple [N/A] (verified)

Not integrable

Time = 2.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(1/x/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/arctan(a*x)^(1/2),x)
```

3.49.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.49.6 Sympy [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(1/x/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(atan(a*x))), x)
```

3.49.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.49.8 Giac [N/A]

Not integrable

Time = 68.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.25

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(1/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.49.9 Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)), x)
```

3.50 $\int \frac{x}{\arctan(ax)^{3/2}}\,dx$

3.50.1 Optimal result 360
3.50.2 Mathematica [N/A] 360
3.50.3 Rubi [N/A] 361
3.50.4 Maple [N/A] (verified) 361
3.50.5 Fricas [F(-2)] 362
3.50.6 Sympy [N/A] 362
3.50.7 Maxima [F(-2)] 362
3.50.8 Giac [N/A] 363
3.50.9 Mupad [N/A] 363

3.50.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(x/arctan(a*x)^(3/2),x)
```

3.50.2 Mathematica [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x/ArcTan[a*x]^(3/2), x]
```

3.50.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}} \, dx$$
$$\Big\downarrow \; 5377$$
$$\int \frac{x}{\arctan(ax)^{3/2}}dx$$

input
```
Int[x/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

3.50.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.50.4 Maple [N/A] (verified)

Not integrable

Time = 1.94 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{\arctan\left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x/arctan(a*x)^(3/2),x)
```

output
```
int(x/arctan(a*x)^(3/2),x)
```

3.50.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.50.6 Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x/atan(a*x)**(3/2),x)
```

output
```
Integral(x/atan(a*x)**(3/2), x)
```

3.50.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.50.8 Giac [N/A]

Not integrable

Time = 249.37 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.50.9 Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(x/atan(a*x)^(3/2),x)
```

output
```
int(x/atan(a*x)^(3/2), x)
```

3.51 $\int \frac{1}{\arctan(ax)^{3/2}}\,dx$

3.51.1 Optimal result . . . 364
3.51.2 Mathematica [N/A] . . . 364
3.51.3 Rubi [N/A] . . . 365
3.51.4 Maple [N/A] (verified) . . . 365
3.51.5 Fricas [F(-2)] . . . 366
3.51.6 Sympy [N/A] . . . 366
3.51.7 Maxima [F(-2)] . . . 366
3.51.8 Giac [N/A] . . . 367
3.51.9 Mupad [N/A] . . . 367

3.51.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{1}{\arctan(ax)^{3/2}},x\right)$$

output
```
Unintegrable(1/arctan(a*x)^(3/2),x)
```

3.51.2 Mathematica [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(-3/2),x]
```

output
```
Integrate[ArcTan[a*x]^(-3/2), x]
```

3.51.3 Rubi [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}} \, dx$$
$$\Big\downarrow \; 5353$$
$$\int \frac{1}{\arctan(ax)^{3/2}}dx$$

input
```
Int[ArcTan[a*x]^(-3/2),x]
```

output
```
$Aborted
```

3.51.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

3.51.4 Maple [N/A] (verified)

Not integrable

Time = 1.52 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\arctan \left(a x \right)^{\frac{3}{2}}}dx$$

input
```
int(1/arctan(a*x)^(3/2),x)
```

output
```
int(1/arctan(a*x)^(3/2),x)
```

3.51.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.51.6 Sympy [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/atan(a*x)**(3/2),x)
```

output
```
Integral(atan(a*x)**(-3/2), x)
```

3.51.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.51.8 Giac [N/A]

Not integrable

Time = 239.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.51.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(1/atan(a*x)^(3/2),x)
```

output
```
int(1/atan(a*x)^(3/2), x)
```

3.52 $\int \frac{1}{x \arctan(ax)^{3/2}} \, dx$

3.52.1 Optimal result 368
3.52.2 Mathematica [N/A] 368
3.52.3 Rubi [N/A] 369
3.52.4 Maple [N/A] (verified) 369
3.52.5 Fricas [F(-2)] 370
3.52.6 Sympy [N/A] 370
3.52.7 Maxima [F(-2)] 370
3.52.8 Giac [N/A] 371
3.52.9 Mupad [N/A] 371

3.52.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x \arctan(ax)^{3/2}} \, dx = \text{Int}\left(\frac{1}{x \arctan(ax)^{3/2}}, x\right)$$

output
```
Unintegrable(1/x/arctan(a*x)^(3/2),x)
```

3.52.2 Mathematica [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x \arctan(ax)^{3/2}} \, dx$$

input
```
Integrate[1/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*ArcTan[a*x]^(3/2)), x]
```

3.52.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{3/2}} \, dx$$
$$\Big\downarrow \ 5377$$
$$\int \frac{1}{x \arctan(ax)^{3/2}} dx$$

input
```
Int[1/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

3.52.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.52.4 Maple [N/A] (verified)

Not integrable

Time = 2.64 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \arctan \left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/arctan(a*x)^(3/2),x)
```

3.52.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.52.6 Sympy [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x \operatorname{atan}^{\frac{3}{2}}(ax)} \, dx$$

input
```
integrate(1/x/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(x*atan(a*x)**(3/2)), x)
```

3.52.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.52.8 Giac [N/A]

Not integrable

Time = 184.34 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.25

$$\int \frac{1}{x\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.52.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)), x)
```

3.53 $\int \sqrt{x} \arctan(x) \, dx$

3.53.1 Optimal result 372
3.53.2 Mathematica [A] (verified) 372
3.53.3 Rubi [A] (verified) 373
3.53.4 Maple [A] (verified) 376
3.53.5 Fricas [C] (verification not implemented) 377
3.53.6 Sympy [A] (verification not implemented) 377
3.53.7 Maxima [A] (verification not implemented) 378
3.53.8 Giac [A] (verification not implemented) 378
3.53.9 Mupad [B] (verification not implemented) 379

3.53.1 Optimal result

Integrand size = 8, antiderivative size = 117

$$\int \sqrt{x} \arctan(x) \, dx = -\frac{4\sqrt{x}}{3} - \frac{1}{3}\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{x}\right) + \frac{1}{3}\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right) + \frac{2}{3}x^{3/2}\arctan(x) - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}}$$

output
```
2/3*x^(3/2)*arctan(x)-1/6*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/6*ln(1+x+2^(1/
2)*x^(1/2))*2^(1/2)+1/3*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/3*arctan(1+2^
(1/2)*x^(1/2))*2^(1/2)-4/3*x^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \sqrt{x} \arctan(x) \, dx = \frac{1}{6}\left(-8\sqrt{x} - 2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{x}\right) + 2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right) + 4x^{3/2}\arctan(x) - \sqrt{2}\log\left(1-\sqrt{2}\sqrt{x}+x\right) + \sqrt{2}\log\left(1+\sqrt{2}\sqrt{x}+x\right)\right)$$

input
```
Integrate[Sqrt[x]*ArcTan[x],x]
```

output
```
(-8*Sqrt[x] - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 2*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*Sqrt[x]] + 4*x^(3/2)*ArcTan[x] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x]
+ x] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/6
```

3.53.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {5361, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\arctan(x)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\int \frac{x^{3/2}}{x^2+1}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-\int \frac{1}{\sqrt{x}\,(x^2+1)}dx\right)$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\int \frac{1}{x^2+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{755}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right)$$

$$\Big\downarrow \; 1479$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow \; 25$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right.$$

$$\Big\downarrow \; 27$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow \; 1103$$

$$\frac{2}{3}x^{3/2}\arctan(x)-\frac{2}{3}\left(2\sqrt{x}-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right.\right.\right.$$

input
```
Int[Sqrt[x]*ArcTan[x],x]
```

output
```
(2*x^(3/2)*ArcTan[x])/3 - (2*(2*Sqrt[x] - 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]
]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2
]*Sqrt[x] + x]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)))/3
```

3.53. $\int \sqrt{x}\arctan(x)\,dx$

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.53.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{2x^{\frac{3}{2}}\arctan(x)}{3}-\frac{4\sqrt{x}}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{6}$
default	$\frac{2x^{\frac{3}{2}}\arctan(x)}{3}-\frac{4\sqrt{x}}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{6}$
meijerg	$-\frac{4\sqrt{x}}{3}+\frac{\sqrt{x}\left(-\frac{\sqrt{2}\ln\left(1-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}\right)}{\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\ln\left(1+\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\cdots}{2+\sqrt{\cdots}}\right)}{\left(x^{2}\right)^{\frac{1}{4}}}\cdots\right.}{3}$

input
```
int(arctan(x)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)*arctan(x)-4/3*x^(1/2)+1/6*2^(1/2)*(ln((1+x+2^(1/2)*x^(1/2))/(1
+x-2^(1/2)*x^(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/
2)))
```

3.53.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2}{3}\,(x\arctan{(x)} - 2)\sqrt{x} + \left(\frac{1}{6}i + \frac{1}{6}\right)\,\sqrt{2}\log\left((i+1)\,\sqrt{2} + 2\,\sqrt{x}\right) - \left(\frac{1}{6}i - \frac{1}{6}\right)\,\sqrt{2}\log\left(-(i-1)\,\sqrt{2} + 2\,\sqrt{x}\right) + \left(\frac{1}{6}i - \frac{1}{6}\right)\,\sqrt{2}\log\left((i-1)\,\sqrt{2} + 2\,\sqrt{x}\right) - \left(\frac{1}{6}i + \frac{1}{6}\right)\,\sqrt{2}\log\left(-(i+1)\,\sqrt{2} + 2\,\sqrt{x}\right)$$

input
```
integrate(arctan(x)*x^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x*arctan(x) - 2)*sqrt(x) + (1/6*I + 1/6)*sqrt(2)*log((I + 1)*sqrt(2)
+ 2*sqrt(x)) - (1/6*I - 1/6)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*sqrt(x)) + (
1/6*I - 1/6)*sqrt(2)*log((I - 1)*sqrt(2) + 2*sqrt(x)) - (1/6*I + 1/6)*sqrt
(2)*log(-(I + 1)*sqrt(2) + 2*sqrt(x))
```

3.53.6 Sympy [A] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2x^{\frac{3}{2}}\operatorname{atan}{(x)}}{3} - \frac{4\sqrt{x}}{3} - \frac{\sqrt{2}\log{\left(-4\sqrt{2}\sqrt{x} + 4x + 4\right)}}{6} + \frac{\sqrt{2}\log{\left(4\sqrt{2}\sqrt{x} + 4x + 4\right)}}{6} + \frac{\sqrt{2}\operatorname{atan}{\left(\sqrt{2}\sqrt{x} - 1\right)}}{3} + \frac{\sqrt{2}\operatorname{atan}{\left(\sqrt{2}\sqrt{x} + 1\right)}}{3}$$

input
```
integrate(atan(x)*x**(1/2),x)
```

output
```
2*x**(3/2)*atan(x)/3 - 4*sqrt(x)/3 - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x
+ 4)/6 + sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/6 + sqrt(2)*atan(sqrt(2)
*sqrt(x) - 1)/3 + sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/3
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(x\right) + \frac{1}{3}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2} + 2\,\sqrt{x}\right)\right) + \frac{1}{3}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2} - 2\,\sqrt{x}\right)\right) + \frac{1}{6}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x} + x + 1\right) - \frac{1}{6}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x} + x + 1\right) - \frac{4}{3}\,\sqrt{x}$$

input
```
integrate(arctan(x)*x^(1/2),x, algorithm="maxima")
```

output
```
2/3*x^(3/2)*arctan(x) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x
))) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/6*sqrt(2)
*log(sqrt(2)*sqrt(x) + x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
- 4/3*sqrt(x)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(x\right) + \frac{1}{3}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2} + 2\,\sqrt{x}\right)\right) + \frac{1}{3}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2} - 2\,\sqrt{x}\right)\right) + \frac{1}{6}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x} + x + 1\right) - \frac{1}{6}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x} + x + 1\right) - \frac{4}{3}\,\sqrt{x}$$

input
```
integrate(arctan(x)*x^(1/2),x, algorithm="giac")
```

output
```
2/3*x^(3/2)*arctan(x) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x
))) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/6*sqrt(2)
*log(sqrt(2)*sqrt(x) + x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
- 4/3*sqrt(x)
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2\,x^{3/2}\,\mathrm{atan}(x)}{3} - \frac{4\sqrt{x}}{3} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)$$

input
```
int(x^(1/2)*atan(x),x)
```

output
```
(2*x^(3/2)*atan(x))/3 + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/3 +
1i/3) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/3 - 1i/3) - (4*x^(1/
2))/3
```

3.54 $\int (dx)^m (a+b\arctan(cx))^3 \, dx$

3.54.1 Optimal result 380
3.54.2 Mathematica [N/A] 380
3.54.3 Rubi [N/A] 381
3.54.4 Maple [N/A] (verified) 381
3.54.5 Fricas [N/A] 382
3.54.6 Sympy [N/A] 382
3.54.7 Maxima [N/A] 382
3.54.8 Giac [N/A] 383
3.54.9 Mupad [N/A] 383

3.54.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m (a+b\arctan(cx))^3 \, dx = \text{Int}\left((dx)^m (a+b\arctan(cx))^3, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctan(c*x))^3,x)
```

3.54.2 Mathematica [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m (a+b\arctan(cx))^3 \, dx = \int (dx)^m (a+b\arctan(cx))^3 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x])^3, x]
```

3.54.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m(a+b\arctan(cx))^3\,dx$$

$$\Big\downarrow \text{ 5377}$$

$$\int (dx)^m(a+b\arctan(cx))^3dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x])^3,x]
```

output
```
$Aborted
```

3.54.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.54.4 Maple [N/A] (verified)

Not integrable

Time = 2.56 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m\,(a+b\arctan\,(cx))^3\,dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x))^3,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x))^3,x)
```

3.54.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x)
+ a^3)*(d*x)^m, x)
```

3.54.6 Sympy [N/A]

Not integrable

Time = 7.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (dx)^m\,(a+b\,\mathrm{atan}\,(cx))^3\,dx$$

input
```
integrate((d*x)**m*(a+b*atan(c*x))**3,x)
```

output
```
Integral((d*x)**m*(a + b*atan(c*x))**3, x)
```

3.54.7 Maxima [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 387, normalized size of antiderivative = 24.19

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x)^3 - 3*b^
3*d^m*x*x^m*arctan(c*x)*log(c^2*x^2 + 1)^2 + 32*(m + 1)*integrate(1/32*(12
*b^3*c^2*d^m*x^2*x^m*arctan(c*x)*log(c^2*x^2 + 1) + 28*(b^3*d^m*m + b^3*d^
m + (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*arctan(c*x)^3 - 12*(b^3*c*d^m*x
 - 8*a*b^2*d^m*m - 8*a*b^2*d^m - 8*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*
x^m*arctan(c*x)^2 + 96*(a^2*b*d^m*m + a^2*b*d^m + (a^2*b*c^2*d^m*m + a^2*b
*c^2*d^m)*x^2)*x^m*arctan(c*x) + 3*(b^3*c*d^m*x*x^m + (b^3*d^m*m + b^3*d^m
 + (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*arctan(c*x))*log(c^2*x^2 + 1)^2)
/((c^2*m + c^2)*x^2 + m + 1), x))/(m + 1)
```

3.54.8 Giac [N/A]

Not integrable

Time = 72.75 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.19

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (b\arctan{(cx)}+a)^3(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.54.9 Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (a+b\,\mathrm{atan}(c\,x))^3\,(d\,x)^m\,dx$$

input
```
int((a + b*atan(c*x))^3*(d*x)^m,x)
```

output
```
int((a + b*atan(c*x))^3*(d*x)^m, x)
```

3.55 $\int (dx)^m (a+b\arctan(cx))^2\,dx$

3.55.1 Optimal result . . . 384
3.55.2 Mathematica [N/A] . . . 384
3.55.3 Rubi [N/A] . . . 385
3.55.4 Maple [N/A] (verified) . . . 385
3.55.5 Fricas [N/A] . . . 386
3.55.6 Sympy [N/A] . . . 386
3.55.7 Maxima [N/A] . . . 386
3.55.8 Giac [N/A] . . . 387
3.55.9 Mupad [N/A] . . . 387

3.55.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m (a+b\arctan(cx))^2\,dx = \mathrm{Int}\left((dx)^m (a+b\arctan(cx))^2, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctan(c*x))^2,x)
```

3.55.2 Mathematica [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m (a+b\arctan(cx))^2\,dx = \int (dx)^m (a+b\arctan(cx))^2\,dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x])^2, x]
```

3.55.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \ 5377$$

$$\int (dx)^m(a+b\arctan(cx))^2dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x])^2,x]
```

output
```
$Aborted
```

3.55.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.55.4 Maple [N/A] (verified)

Not integrable

Time = 3.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m\,(a+b\arctan(cx))^2\,dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x))^2,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x))^2,x)
```

3.55.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m(a+b\arctan(cx))^2\,dx = \int (b\arctan(cx)+a)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)*(d*x)^m, x)
```

3.55.6 Sympy [N/A]

Not integrable

Time = 4.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m(a+b\arctan(cx))^2\,dx = \int (dx)^m\,(a+b\,\mathrm{atan}\,(cx))^2\,dx$$

input
```
integrate((d*x)**m*(a+b*atan(c*x))**2,x)
```

output
```
Integral((d*x)**m*(a + b*atan(c*x))**2, x)
```

3.55.7 Maxima [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 295, normalized size of antiderivative = 18.44

$$\int (dx)^m(a+b\arctan(cx))^2\,dx = \int (b\arctan(cx)+a)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x)^2 - b^2*
d^m*x*x^m*log(c^2*x^2 + 1)^2 + 16*(m + 1)*integrate(1/16*(4*b^2*c^2*d^m*x^
2*x^m*log(c^2*x^2 + 1) + 12*(b^2*d^m*m + b^2*d^m + (b^2*c^2*d^m*m + b^2*c^
2*d^m)*x^2)*x^m*arctan(c*x)^2 + (b^2*d^m*m + b^2*d^m + (b^2*c^2*d^m*m + b^
2*c^2*d^m)*x^2)*x^m*log(c^2*x^2 + 1)^2 - 8*(b^2*c*d^m*x - 4*a*b*d^m*m - 4*
a*b*d^m - 4*(a*b*c^2*d^m*m + a*b*c^2*d^m)*x^2)*x^m*arctan(c*x))/((c^2*m +
c^2)*x^2 + m + 1), x))/(m + 1)
```

3.55.8 Giac [N/A]

Not integrable

Time = 72.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.19

$$\int (dx)^m(a+b\arctan(cx))^2\,dx=\int (b\arctan{(cx)}+a)^2(dx)^m\;dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.55.9 Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^2\,dx=\int (a+b\,\mathrm{atan}(c\,x))^2\,(d\,x)^m\,dx$$

input
```
int((a + b*atan(c*x))^2*(d*x)^m,x)
```

output
```
int((a + b*atan(c*x))^2*(d*x)^m, x)
```

3.56 $\int (dx)^m(a+b\arctan(cx))\,dx$

3.56.1 Optimal result 388
3.56.2 Mathematica [A] (verified) 388
3.56.3 Rubi [A] (verified) 389
3.56.4 Maple [F] 390
3.56.5 Fricas [F] 390
3.56.6 Sympy [F] 390
3.56.7 Maxima [F] 391
3.56.8 Giac [F] 391
3.56.9 Mupad [F(-1)] 391

3.56.1 Optimal result

Integrand size = 14, antiderivative size = 73

$$\int (dx)^m(a+b\arctan(cx))\,dx=\frac{(dx)^{1+m}(a+b\arctan(cx))}{d(1+m)}-\frac{bc(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{d^2(1+m)(2+m)}$$

output
```
(d*x)^(1+m)*(a+b*arctan(c*x))/d/(1+m)-b*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*
m],[2+1/2*m],-c^2*x^2)/d^2/(1+m)/(2+m)
```

3.56.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int (dx)^m(a+b\arctan(cx))\,dx=-\frac{x(dx)^m\left(-((2+m)(a+b\arctan(cx)))+bcx\,\text{Hypergeometric2F1}\left(1,1+\frac{m}{2},2+\frac{m}{2},-c^2x^2\right)\right)}{(1+m)(2+m)}$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x]),x]
```

output
```
-((x*(d*x)^m*(-((2 + m)*(a + b*ArcTan[c*x])) + b*c*x*Hypergeometric2F1[1,
1 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)))
```

3.56.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5373, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \ 5373$$

$$\frac{(dx)^{m+1}(a+b\arctan(cx))}{d(m+1)}-\frac{bc\int \frac{(dx)^{m+1}}{c^2x^2+1}dx}{d(m+1)}$$

$$\Big\downarrow \ 278$$

$$\frac{(dx)^{m+1}(a+b\arctan(cx))}{d(m+1)}-\frac{bc(dx)^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{d^2(m+1)(m+2)}$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcTan[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d^2*(1 + m)*(2 + m))
```

3.56.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 5373
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n/(
d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.56.4 Maple [F]

$$\int (dx)^m (a + b\arctan (cx))\,dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x)),x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x)),x)
```

3.56.5 Fricas [F]

$$\int (dx)^m(a + b\arctan(cx))\,dx = \int (b\arctan (cx) + a)(dx)^m\;dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(d*x)^m, x)
```

3.56.6 Sympy [F]

$$\int (dx)^m(a + b\arctan(cx))\,dx = \int (dx)^m\,(a + b\,\mathrm{atan}\,(cx))\;dx$$

input
```
integrate((d*x)**m*(a+b*atan(c*x)),x)
```

output
```
Integral((d*x)**m*(a + b*atan(c*x)), x)
```

3.56.7 Maxima [F]

$$\int (dx)^m(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
(d^m*x*x^m*arctan(c*x) - (c*d^m*m + c*d^m)*integrate(x*x^m/((c^2*m + c^2)*
x^2 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

3.56.8 Giac [F]

$$\int (dx)^m(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m(a+b\arctan(cx))\,dx = \int (a+b\,\mathrm{atan}(c\,x))\,(d\,x)^m\,dx$$

input
```
int((a + b*atan(c*x))*(d*x)^m,x)
```

output
```
int((a + b*atan(c*x))*(d*x)^m, x)
```

3.57 $\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$

3.57.1 Optimal result 392
3.57.2 Mathematica [N/A] 392
3.57.3 Rubi [N/A] 393
3.57.4 Maple [N/A] (verified) 393
3.57.5 Fricas [N/A] 394
3.57.6 Sympy [N/A] 394
3.57.7 Maxima [N/A] 394
3.57.8 Giac [N/A] 395
3.57.9 Mupad [N/A] 395

3.57.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx = \text{Int}\left(\frac{(dx)^m}{a+b\arctan(cx)},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctan(c*x)),x)
```

3.57.2 Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx = \int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x]), x]
```

3.57.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$$

$$\Big\downarrow \ 5377$$

$$\int \frac{(dx)^m}{a+b\arctan(cx)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

3.57.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.57.4 Maple [N/A] (verified)

Not integrable

Time = 3.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\arctan(cx)}dx$$

input
```
int((d*x)^m/(a+b*arctan(c*x)),x)
```

output
```
int((d*x)^m/(a+b*arctan(c*x)),x)
```

3.57.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(dx)^m}{b\arctan(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arctan(c*x) + a), x)
```

3.57.6 Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(dx)^m}{a+b\,\mathrm{atan}(cx)}\,dx$$

input
```
integrate((d*x)**m/(a+b*atan(c*x)),x)
```

output
```
Integral((d*x)**m/(a + b*atan(c*x)), x)
```

3.57.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(dx)^m}{b\arctan(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arctan(c*x) + a), x)
```

3.57.8 Giac [N/A]

Not integrable

Time = 87.38 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.19

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(dx)^m}{b\arctan(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
sage0*x
```

3.57.9 Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(d\,x)^m}{a+b\,\mathrm{atan}\left(c\,x\right)}\,dx$$

input
```
int((d*x)^m/(a + b*atan(c*x)),x)
```

output
```
int((d*x)^m/(a + b*atan(c*x)), x)
```

3.58 $\int (a+b\arctan (cx))^p\,dx$

3.58.1 Optimal result 396
3.58.2 Mathematica [N/A] 396
3.58.3 Rubi [N/A] 397
3.58.4 Maple [N/A] (verified) 397
3.58.5 Fricas [N/A] 398
3.58.6 Sympy [N/A] 398
3.58.7 Maxima [N/A] 398
3.58.8 Giac [N/A] 399
3.58.9 Mupad [N/A] 399

3.58.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int (a+b\arctan (cx))^p\,dx=\text{Int}((a+b\arctan (cx))^p,x)$$

output
```
Unintegrable((a+b*arctan(c*x))^p,x)
```

3.58.2 Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\arctan (cx))^p\,dx=\int (a+b\arctan (cx))^p\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^p,x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^p, x]
```

3.58.3 Rubi [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5353}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\arctan(cx))^p\, dx$$

$$\Big\downarrow \text{5353}$$

$$\int (a + b\arctan(cx))^p dx$$

input
```
Int[(a + b*ArcTan[c*x])^p,x]
```

output
```
$Aborted
```

3.58.3.1 Defintions of rubi rules used

rule 5353
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrabl
e[(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

3.58.4 Maple [N/A] (verified)

Not integrable

Time = 1.69 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (a + b\arctan(cx))^p\, dx$$

input
```
int((a+b*arctan(c*x))^p,x)
```

output
```
int((a+b*arctan(c*x))^p,x)
```

3.58.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\arctan(cx))^p\,dx=\int (b\arctan{(cx)}+a)^p\,dx$$

input
```
integrate((a+b*arctan(c*x))^p,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)^p, x)
```

3.58.6 Sympy [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (a+b\arctan(cx))^p\,dx=\int (a+b\operatorname{atan}{(cx)})^p\;dx$$

input
```
integrate((a+b*atan(c*x))**p,x)
```

output
```
Integral((a + b*atan(c*x))**p, x)
```

3.58.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\arctan(cx))^p\,dx=\int (b\arctan{(cx)}+a)^p\,dx$$

input
```
integrate((a+b*arctan(c*x))^p,x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)^p, x)
```

3.58.8 Giac [N/A]

Not integrable

Time = 80.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int (a+b\arctan(cx))^p\,dx = \int (b\arctan(cx)+a)^p\,dx$$

input
```
integrate((a+b*arctan(c*x))^p,x, algorithm="giac")
```

output
```
sage0*x
```

3.58.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\arctan(cx))^p\,dx = \int (a+b\,\mathrm{atan}(c\,x))^p\,dx$$

input
```
int((a + b*atan(c*x))^p,x)
```

output
```
int((a + b*atan(c*x))^p, x)
```

3.59 $\int (dx)^m(a+b\arctan(cx))^p\,dx$

3.59.1 Optimal result 400
3.59.2 Mathematica [N/A] 400
3.59.3 Rubi [N/A] 401
3.59.4 Maple [N/A] (verified) 401
3.59.5 Fricas [N/A] 402
3.59.6 Sympy [N/A] 402
3.59.7 Maxima [N/A] 402
3.59.8 Giac [N/A] 403
3.59.9 Mupad [N/A] 403

3.59.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\text{Int}((dx)^m(a+b\arctan(cx))^p,x)$$

output
```
Unintegrable((d*x)^m*(a+b*arctan(c*x))^p,x)
```

3.59.2 Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (dx)^m(a+b\arctan(cx))^p\,dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x])^p,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x])^p, x]
```

3.59.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m (a + b \arctan(cx))^p \, dx$$
$$\Big\downarrow \text{5377}$$
$$\int (dx)^m (a + b \arctan(cx))^p dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x])^p,x]
```

output
```
$Aborted
```

3.59.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.59.4 Maple [N/A] (verified)

Not integrable

Time = 4.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a + b \arctan \left(cx\right)\right)^p dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x))^p,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x))^p,x)
```

3.59.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx = \int (dx)^m\,(b\arctan(cx)+a)^p\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="fricas")
```

output
```
integral((d*x)^m*(b*arctan(c*x) + a)^p, x)
```

3.59.6 Sympy [N/A]

Not integrable

Time = 155.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m(a+b\arctan(cx))^p\,dx = \int (dx)^m\,(a+b\,\mathrm{atan}\,(cx))^p\;dx$$

input
```
integrate((d*x)**m*(a+b*atan(c*x))**p,x)
```

output
```
Integral((d*x)**m*(a + b*atan(c*x))**p, x)
```

3.59.7 Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx = \int (dx)^m\,(b\arctan(cx)+a)^p\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="maxima")
```

output
```
integrate((d*x)^m*(b*arctan(c*x) + a)^p, x)
```

3.59.8 Giac [N/A]

Not integrable

Time = 79.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.19

$$\int (dx)^m(a+b\arctan(cx))^p\,dx = \int (dx)^m\left(b\arctan(cx)+a\right)^p\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="giac")
```

output
```
sage0*x
```

3.59.9 Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx = \int (a+b\,\mathrm{atan}(c\,x))^p\,(d\,x)^m\,dx$$

input
```
int((a + b*atan(c*x))^p*(d*x)^m,x)
```

output
```
int((a + b*atan(c*x))^p*(d*x)^m, x)
```

3.60 $\int x^7(a+b\arctan(cx^2))\,dx$

3.60.1 Optimal result . . . 404
3.60.2 Mathematica [A] (verified) . . . 404
3.60.3 Rubi [A] (verified) . . . 405
3.60.4 Maple [A] (verified) . . . 406
3.60.5 Fricas [A] (verification not implemented) . . . 407
3.60.6 Sympy [A] (verification not implemented) . . . 407
3.60.7 Maxima [A] (verification not implemented) . . . 407
3.60.8 Giac [A] (verification not implemented) . . . 408
3.60.9 Mupad [B] (verification not implemented) . . . 408

3.60.1 Optimal result

Integrand size = 14, antiderivative size = 54

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{bx^2}{8c^3}-\frac{bx^6}{24c}-\frac{b\arctan\left(cx^2\right)}{8c^4}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)$$

output
```
1/8*b*x^2/c^3-1/24*b*x^6/c-1/8*b*arctan(c*x^2)/c^4+1/8*x^8*(a+b*arctan(c*x
^2))
```

3.60.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{bx^2}{8c^3}-\frac{bx^6}{24c}+\frac{ax^8}{8}-\frac{b\arctan\left(cx^2\right)}{8c^4}+\frac{1}{8}bx^8\arctan\left(cx^2\right)$$

input
```
Integrate[x^7*(a + b*ArcTan[c*x^2]),x]
```

output
```
(b*x^2)/(8*c^3) - (b*x^6)/(24*c) + (a*x^8)/8 - (b*ArcTan[c*x^2])/(8*c^4) +
 (b*x^8*ArcTan[c*x^2])/8
```

3.60.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 807, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 (a + b\arctan(cx^2))\, dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{8}x^8(a + b\arctan(cx^2)) - \frac{1}{4}bc\int \frac{x^9}{c^2x^4+1}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{8}x^8(a + b\arctan(cx^2)) - \frac{1}{8}bc\int \frac{x^8}{c^2x^4+1}dx^2$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{8}x^8(a + b\arctan(cx^2)) - \frac{1}{8}bc\int \left(\frac{x^4}{c^2} + \frac{1}{c^4(c^2x^4+1)} - \frac{1}{c^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}x^8(a + b\arctan(cx^2)) - \frac{1}{8}bc\left(\frac{\arctan(cx^2)}{c^5} - \frac{x^2}{c^4} + \frac{x^6}{3c^2}\right)$$

input

```
Int[x^7*(a + b*ArcTan[c*x^2]),x]
```

output

```
(x^8*(a + b*ArcTan[c*x^2]))/8 - (b*c*(-(x^2/c^4) + x^6/(3*c^2) + ArcTan[c*
x^2]/c^5))/8
```

3.60.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.60.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{a\,x^{8}}{8}+\frac{b\,x^{8}\arctan\left(c\,x^{2}\right)}{8}-\frac{b\,x^{6}}{24c}+\frac{b\,x^{2}}{8c^{3}}-\frac{b\arctan\left(c\,x^{2}\right)}{8c^{4}}$	50
parts	$\frac{a\,x^{8}}{8}+\frac{b\,x^{8}\arctan\left(c\,x^{2}\right)}{8}-\frac{b\,x^{6}}{24c}+\frac{b\,x^{2}}{8c^{3}}-\frac{b\arctan\left(c\,x^{2}\right)}{8c^{4}}$	50
parallelrisch	$\frac{3b\arctan\left(c\,x^{2}\right)x^{8}c^{4}+3a\,c^{4}x^{8}-b\,c^{3}x^{6}+3bc\,x^{2}-3b\arctan\left(c\,x^{2}\right)}{24c^{4}}$	56
risch	$-\frac{i x^{8}b\ln\left(ic\,x^{2}+1\right)}{16}+\frac{i x^{8}b\ln\left(-ic\,x^{2}+1\right)}{16}+\frac{a\,x^{8}}{8}-\frac{b\,x^{6}}{24c}+\frac{b\,x^{2}}{8c^{3}}-\frac{b\arctan\left(c\,x^{2}\right)}{8c^{4}}$	72

input
```
int(x^7*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/8*a*x^8+1/8*b*x^8*arctan(c*x^2)-1/24*b*x^6/c+1/8*b*x^2/c^3-1/8*b*arctan(
c*x^2)/c^4
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int x^7 (a + b \arctan(cx^2)) \, dx = \frac{3\, ac^4x^8 - bc^3x^6 + 3\, bcx^2 + 3\,(bc^4x^8 - b) \arctan(cx^2)}{24\, c^4}$$

input
```
integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
1/24*(3*a*c^4*x^8 - b*c^3*x^6 + 3*b*c*x^2 + 3*(b*c^4*x^8 - b)*arctan(c*x^2
))/c^4
```

3.60.6 Sympy [A] (verification not implemented)

Time = 29.73 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int x^7 (a + b \arctan(cx^2)) \, dx = \begin{cases} \frac{ax^8}{8} + \frac{bx^8 \operatorname{atan}(cx^2)}{8} - \frac{bx^6}{24c} + \frac{bx^2}{8c^3} - \frac{b \operatorname{atan}(cx^2)}{8c^4} & \text{for } c \neq 0 \\ \frac{ax^8}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(a+b*atan(c*x**2)),x)
```

output
```
Piecewise((a*x**8/8 + b*x**8*atan(c*x**2)/8 - b*x**6/(24*c) + b*x**2/(8*c*
*3) - b*atan(c*x**2)/(8*c**4), Ne(c, 0)), (a*x**8/8, True))
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^7 (a + b \arctan(cx^2)) \, dx = \frac{1}{8}\, ax^8 + \frac{1}{24}\left(3\, x^8 \arctan(cx^2) - c\left(\frac{c^2x^6 - 3\, x^2}{c^4} + \frac{3\, \arctan(cx^2)}{c^5}\right)\right)b$$

input
```
integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
1/8*a*x^8 + 1/24*(3*x^8*arctan(c*x^2) - c*((c^2*x^6 - 3*x^2)/c^4 + 3*arcta
n(c*x^2)/c^5))*b
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{3\,acx^8+\left(3\,cx^8\arctan\left(cx^2\right)-\frac{3\,\arctan\left(cx^2\right)}{c^3}-\frac{c^9x^6-3\,c^7x^2}{c^9}\right)b}{24\,c}$$

input
```
integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
1/24*(3*a*c*x^8 + (3*c*x^8*arctan(c*x^2) - 3*arctan(c*x^2)/c^3 - (c^9*x^6
- 3*c^7*x^2)/c^9)*b)/c
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{a\,x^8}{8}+\frac{b\,x^2}{8\,c^3}-\frac{b\,x^6}{24\,c}-\frac{b\,\mathrm{atan}(c\,x^2)}{8\,c^4}+\frac{b\,x^8\,\mathrm{atan}(c\,x^2)}{8}$$

input
```
int(x^7*(a + b*atan(c*x^2)),x)
```

output
```
(a*x^8)/8 + (b*x^2)/(8*c^3) - (b*x^6)/(24*c) - (b*atan(c*x^2))/(8*c^4) + (
b*x^8*atan(c*x^2))/8
```

3.61 $\int x^5(a+b\arctan(cx^2))\,dx$

3.61.1 Optimal result 409
3.61.2 Mathematica [A] (verified) 409
3.61.3 Rubi [A] (verified) 410
3.61.4 Maple [A] (verified) 411
3.61.5 Fricas [A] (verification not implemented) 411
3.61.6 Sympy [B] (verification not implemented) 412
3.61.7 Maxima [A] (verification not implemented) 412
3.61.8 Giac [A] (verification not implemented) 413
3.61.9 Mupad [B] (verification not implemented) 413

3.61.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int x^5(a+b\arctan(cx^2))\,dx = -\frac{bx^4}{12c}+\frac{1}{6}x^6(a+b\arctan(cx^2))+\frac{b\log(1+c^2x^4)}{12c^3}$$

output
```
-1/12*b*x^4/c+1/6*x^6*(a+b*arctan(c*x^2))+1/12*b*ln(c^2*x^4+1)/c^3
```

3.61.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int x^5(a+b\arctan(cx^2))\,dx = -\frac{bx^4}{12c}+\frac{ax^6}{6}+\frac{1}{6}bx^6\arctan(cx^2)+\frac{b\log(1+c^2x^4)}{12c^3}$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x^2]),x]
```

output
```
-1/12*(b*x^4)/c + (a*x^6)/6 + (b*x^6*ArcTan[c*x^2])/6 + (b*Log[1 + c^2*x^4
])/(12*c^3)
```

3.61.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a + b \arctan(cx^2))\, dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{6}x^6 (a + b \arctan(cx^2)) - \frac{1}{3}bc \int \frac{x^7}{c^2x^4+1}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{6}x^6 (a + b \arctan(cx^2)) - \frac{1}{12}bc \int \frac{x^4}{c^2x^4+1}dx^4$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{6}x^6 (a + b \arctan(cx^2)) - \frac{1}{12}bc \int \left(\frac{1}{c^2} - \frac{1}{c^2(c^2x^4+1)}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}x^6 (a + b \arctan(cx^2)) - \frac{1}{12}bc \left(\frac{x^4}{c^2} - \frac{\log(c^2x^4+1)}{c^4}\right)$$

input
```
Int[x^5*(a + b*ArcTan[c*x^2]),x]
```

output
```
(x^6*(a + b*ArcTan[c*x^2]))/6 - (b*c*(x^4/c^2 - Log[1 + c^2*x^4]/c^4))/12
```

3.61.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.61.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{a\,x^{6}}{6}+\frac{b\,x^{6}\arctan\left(c\,x^{2}\right)}{6}-\frac{b\,x^{4}}{12c}+\frac{b\ln\left(c^{2}x^{4}+1\right)}{12c^{3}}$	45
parts	$\frac{a\,x^{6}}{6}+\frac{b\,x^{6}\arctan\left(c\,x^{2}\right)}{6}-\frac{b\,x^{4}}{12c}+\frac{b\ln\left(c^{2}x^{4}+1\right)}{12c^{3}}$	45
parallelrisch	$\frac{2x^{6}\arctan\left(c\,x^{2}\right)b\,c^{3}+2a\,c^{3}x^{6}-b\,c^{2}x^{4}+b\ln\left(c^{2}x^{4}+1\right)}{12c^{3}}$	52
risch	$-\frac{i x^{6} b\ln\left(i c\,x^{2}+1\right)}{12}+\frac{i x^{6} b\ln\left(-i c\,x^{2}+1\right)}{12}+\frac{a\,x^{6}}{6}-\frac{b\,x^{4}}{12c}+\frac{b\ln\left(-c^{2}x^{4}-1\right)}{12c^{3}}$	68

input
```
int(x^5*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x^6+1/6*b*x^6*arctan(c*x^2)-1/12*b*x^4/c+1/12*b*ln(c^2*x^4+1)/c^3
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{2\,bc^3x^6\arctan\left(cx^2\right)+2\,ac^3x^6-bc^2x^4+b\log\left(c^2x^4+1\right)}{12\,c^3}$$

input
```
integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
1/12*(2*b*c^3*x^6*arctan(c*x^2) + 2*a*c^3*x^6 - b*c^2*x^4 + b*log(c^2*x^4
+ 1))/c^3
```

3.61.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(39) = 78$.

Time = 22.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx = \begin{cases} \frac{ax^6}{6}+\frac{bx^6\operatorname{atan}\left(cx^2\right)}{6}-\frac{bx^4}{12c}+\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^2\right)}{6c^2}+\frac{b\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6c^3} & \text{for } c\neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(a+b*atan(c*x**2)),x)
```

output
```
Piecewise((a*x**6/6 + b*x**6*atan(c*x**2)/6 - b*x**4/(12*c) + b*sqrt(-1/c*
*2)*atan(c*x**2)/(6*c**2) + b*log(x**2 + sqrt(-1/c**2))/(6*c**3), Ne(c, 0)
), (a*x**6/6, True))
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx = \frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\arctan\left(cx^2\right)-\left(\frac{x^4}{c^2}-\frac{\log\left(c^2x^4+1\right)}{c^4}\right)c\right)b$$

input
```
integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/12*(2*x^6*arctan(c*x^2) - (x^4/c^2 - log(c^2*x^4 + 1)/c^4)*c
)*b
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int x^5 \left(a + b \arctan \left(cx^2\right)\right)\, dx = \frac{2\, acx^6 + \left(2\, cx^6 \arctan \left(cx^2\right) - x^4 + \frac{\log \left(c^2 x^4 + 1\right)}{c^2}\right) b}{12\, c}$$

input
```
integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
1/12*(2*a*c*x^6 + (2*c*x^6*arctan(c*x^2) - x^4 + log(c^2*x^4 + 1)/c^2)*b)/
c
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int x^5 \left(a + b \arctan \left(cx^2\right)\right)\, dx = \frac{a\, x^6}{6} + \frac{b\, \ln \left(c^2\, x^4 + 1\right)}{12\, c^3} - \frac{b\, x^4}{12\, c} + \frac{b\, x^6\, \mathrm{atan}(c\, x^2)}{6}$$

input
```
int(x^5*(a + b*atan(c*x^2)),x)
```

output
```
(a*x^6)/6 + (b*log(c^2*x^4 + 1))/(12*c^3) - (b*x^4)/(12*c) + (b*x^6*atan(c
*x^2))/6
```

3.62 $\int x^3(a+b\arctan(cx^2))\,dx$

3.62.1 Optimal result 414
3.62.2 Mathematica [A] (verified) 414
3.62.3 Rubi [A] (verified) 415
3.62.4 Maple [A] (verified) 416
3.62.5 Fricas [A] (verification not implemented) 417
3.62.6 Sympy [A] (verification not implemented) 417
3.62.7 Maxima [A] (verification not implemented) 417
3.62.8 Giac [A] (verification not implemented) 418
3.62.9 Mupad [B] (verification not implemented) 418

3.62.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int x^3(a+b\arctan(cx^2))\,dx=-\frac{bx^2}{4c}+\frac{b\arctan(cx^2)}{4c^2}+\frac{1}{4}x^4(a+b\arctan(cx^2))$$

output
```
-1/4*b*x^2/c+1/4*b*arctan(c*x^2)/c^2+1/4*x^4*(a+b*arctan(c*x^2))
```

3.62.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^3(a+b\arctan(cx^2))\,dx=-\frac{bx^2}{4c}+\frac{ax^4}{4}+\frac{b\arctan(cx^2)}{4c^2}+\frac{1}{4}bx^4\arctan(cx^2)$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x^2]),x]
```

output
```
-1/4*(b*x^2)/c + (a*x^4)/4 + (b*ArcTan[c*x^2])/(4*c^2) + (b*x^4*ArcTan[c*x
^2])/4
```

3.62.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 807, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\arctan(cx^2))\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4(a+b\arctan(cx^2))-\frac{1}{2}bc\int\frac{x^5}{c^2x^4+1}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{4}x^4(a+b\arctan(cx^2))-\frac{1}{4}bc\int\frac{x^4}{c^2x^4+1}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4}x^4(a+b\arctan(cx^2))-\frac{1}{4}bc\left(\frac{x^2}{c^2}-\frac{\int\frac{1}{c^2x^4+1}dx^2}{c^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}x^4(a+b\arctan(cx^2))-\frac{1}{4}bc\left(\frac{x^2}{c^2}-\frac{\arctan(cx^2)}{c^3}\right)$$

input
```
Int[x^3*(a + b*ArcTan[c*x^2]),x]
```

output
```
(x^4*(a + b*ArcTan[c*x^2]))/4 - (b*c*(x^2/c^2 - ArcTan[c*x^2]/c^3))/4
```

3.62.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.62.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{a\,x^{4}}{4}+\frac{b\,x^{4}\arctan\left(c\,x^{2}\right)}{4}-\frac{b\,x^{2}}{4c}+\frac{b\arctan\left(c\,x^{2}\right)}{4c^{2}}$	41
parts	$\frac{a\,x^{4}}{4}+\frac{b\,x^{4}\arctan\left(c\,x^{2}\right)}{4}-\frac{b\,x^{2}}{4c}+\frac{b\arctan\left(c\,x^{2}\right)}{4c^{2}}$	41
parallelrisch	$\frac{\arctan\left(c\,x^{2}\right)b\,c^{2}x^{4}+a\,c^{2}x^{4}-bc\,x^{2}+b\arctan\left(c\,x^{2}\right)}{4c^{2}}$	44
risch	$-\frac{i x^{4} b\ln\left(i c\,x^{2}+1\right)}{8}+\frac{i x^{4} b\ln\left(-i c\,x^{2}+1\right)}{8}+\frac{a\,x^{4}}{4}-\frac{b\,x^{2}}{4c}+\frac{b\arctan\left(c\,x^{2}\right)}{4c^{2}}+\frac{b^{2}}{16a\,c^{2}}$	74

input
```
int(x^3*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+1/4*b*x^4*arctan(c*x^2)-1/4*b*x^2/c+1/4*b*arctan(c*x^2)/c^2
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int x^3(a+b\arctan(cx^2))\,dx = \frac{ac^2x^4 - bcx^2 + (bc^2x^4+b)\arctan(cx^2)}{4\,c^2}$$

input
```
integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
1/4*(a*c^2*x^4 - b*c*x^2 + (b*c^2*x^4 + b)*arctan(c*x^2))/c^2
```

3.62.6 Sympy [A] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^3(a+b\arctan(cx^2))\,dx = \begin{cases} \frac{ax^4}{4} + \frac{bx^4\operatorname{atan}(cx^2)}{4} - \frac{bx^2}{4c} + \frac{b\operatorname{atan}(cx^2)}{4c^2} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(a+b*atan(c*x**2)),x)
```

output
```
Piecewise((a*x**4/4 + b*x**4*atan(c*x**2)/4 - b*x**2/(4*c) + b*atan(c*x**2
)/(4*c**2), Ne(c, 0)), (a*x**4/4, True))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^3(a+b\arctan(cx^2))\,dx = \frac{1}{4}ax^4 + \frac{1}{4}\left(x^4\arctan(cx^2) - c\left(\frac{x^2}{c^2} - \frac{\arctan(cx^2)}{c^3}\right)\right)b$$

input
```
integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/4*(x^4*arctan(c*x^2) - c*(x^2/c^2 - arctan(c*x^2)/c^3))*b
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^3(a+b\arctan(cx^2))\,dx=\frac{acx^4+\frac{(c^2x^4\arctan(cx^2)-cx^2+\arctan(cx^2))b}{c}}{4\,c}$$

input
```
integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
1/4*(a*c*x^4 + (c^2*x^4*arctan(c*x^2) - c*x^2 + arctan(c*x^2))*b/c)/c
```

3.62.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^3(a+b\arctan(cx^2))\,dx=\frac{a\,x^4}{4}-\frac{b\,x^2}{4\,c}+\frac{b\,\mathrm{atan}(c\,x^2)}{4\,c^2}+\frac{b\,x^4\,\mathrm{atan}(c\,x^2)}{4}$$

input
```
int(x^3*(a + b*atan(c*x^2)),x)
```

output
```
(a*x^4)/4 - (b*x^2)/(4*c) + (b*atan(c*x^2))/(4*c^2) + (b*x^4*atan(c*x^2))/
4
```

3.63 $\int x(a + b \arctan(cx^2)) \, dx$

3.63.1 Optimal result 419
3.63.2 Mathematica [A] (verified) 419
3.63.3 Rubi [A] (verified) 420
3.63.4 Maple [A] (verified) 421
3.63.5 Fricas [A] (verification not implemented) 421
3.63.6 Sympy [B] (verification not implemented) 422
3.63.7 Maxima [A] (verification not implemented) 422
3.63.8 Giac [A] (verification not implemented) 422
3.63.9 Mupad [B] (verification not implemented) 423

3.63.1 Optimal result

Integrand size = 12, antiderivative size = 36

$$\int x(a + b \arctan(cx^2)) \, dx = \frac{1}{2}x^2(a + b \arctan(cx^2)) - \frac{b \log(1 + c^2x^4)}{4c}$$

output
```
1/2*x^2*(a+b*arctan(c*x^2))-1/4*b*ln(c^2*x^4+1)/c
```

3.63.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int x(a + b \arctan(cx^2)) \, dx = \frac{ax^2}{2} + \frac{1}{2}bx^2 \arctan(cx^2) - \frac{b \log(1 + c^2x^4)}{4c}$$

input
```
Integrate[x*(a + b*ArcTan[c*x^2]),x]
```

output
```
(a*x^2)/2 + (b*x^2*ArcTan[c*x^2])/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

3.63.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5361, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)-bc\int \frac{x^3}{c^2x^4+1}dx$$

$$\Big\downarrow \text{ 792}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)-\frac{b\log\left(c^2x^4+1\right)}{4c}$$

input
```
Int[x*(a + b*ArcTan[c*x^2]),x]
```

output
```
(x^2*(a + b*ArcTan[c*x^2]))/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

3.63.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.63.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{b \arctan\left(c\,x^{2}\right) x^{2}}{2}-\frac{b \ln\left(c^{2} x^{4}+1\right)}{4c}$	36
derivativedivides	$\frac{c\,x^{2} a+b\left(c\,x^{2} \arctan\left(c\,x^{2}\right)-\frac{\ln\left(c^{2} x^{4}+1\right)}{2}\right)}{2c}$	39
default	$\frac{c\,x^{2} a+b\left(c\,x^{2} \arctan\left(c\,x^{2}\right)-\frac{\ln\left(c^{2} x^{4}+1\right)}{2}\right)}{2c}$	39
parallelrisch	$-\frac{-2x^{2} \arctan\left(c\,x^{2}\right) bc-2c\,x^{2} a+b \ln\left(c^{2} x^{4}+1\right)}{4c}$	39
risch	$-\frac{ib\,x^{2} \ln\left(ic\,x^{2}+1\right)}{4}+\frac{ib\,x^{2} \ln\left(-ic\,x^{2}+1\right)}{4}+\frac{a\,x^{2}}{2}-\frac{b \ln\left(-c^{2} x^{4}-1\right)}{4c}$	59

input
```
int(x*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/2*b*arctan(c*x^2)*x^2-1/4*b*ln(c^2*x^4+1)/c
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{2\,bcx^2\arctan\left(cx^2\right)+2\,acx^2-b\log\left(c^2x^4+1\right)}{4\,c}$$

input
```
integrate(x*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
1/4*(2*b*c*x^2*arctan(c*x^2) + 2*a*c*x^2 - b*log(c^2*x^4 + 1))/c
```

3.63.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(29) = 58.

Time = 5.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx = \begin{cases} \frac{ax^2}{2}+\frac{bx^2\operatorname{atan}\left(cx^2\right)}{2}-\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^2\right)}{2}-\frac{b\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2c} & \text{for } c\neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*atan(c*x**2)),x)
```

output
```
Piecewise((a*x**2/2 + b*x**2*atan(c*x**2)/2 - b*sqrt(-1/c**2)*atan(c*x**2)
/2 - b*log(x**2 + sqrt(-1/c**2))/(2*c), Ne(c, 0)), (a*x**2/2, True))
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx = \frac{1}{2}\,ax^2+\frac{\left(2\,cx^2\arctan\left(cx^2\right)-\log\left(c^2x^4+1\right)\right)b}{4\,c}$$

input
```
integrate(x*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/4*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b/c
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx = \frac{2\,acx^2+\left(2\,cx^2\arctan\left(cx^2\right)-\log\left(c^2x^4+1\right)\right)b}{4\,c}$$

input
```
integrate(x*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
1/4*(2*a*c*x^2 + (2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b)/c
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,\ln\left(c^2\,x^4+1\right)}{4\,c}+\frac{b\,x^2\,\mathrm{atan}(c\,x^2)}{2}$$

input
```
int(x*(a + b*atan(c*x^2)),x)
```

output
```
(a*x^2)/2 - (b*log(c^2*x^4 + 1))/(4*c) + (b*x^2*atan(c*x^2))/2
```

3.64 $\int \frac{a+b\arctan(cx^2)}{x}\,dx$

3.64.1 Optimal result 424
3.64.2 Mathematica [A] (verified) 424
3.64.3 Rubi [A] (verified) 425
3.64.4 Maple [C] (verified) 426
3.64.5 Fricas [F] 426
3.64.6 Sympy [F] 427
3.64.7 Maxima [F] 427
3.64.8 Giac [F] 427
3.64.9 Mupad [B] (verification not implemented) 428

3.64.1 Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx = a\log(x) + \frac{1}{4}ib\,\text{PolyLog}\left(2,-icx^2\right) - \frac{1}{4}ib\,\text{PolyLog}\left(2,icx^2\right)$$

output
```
a*ln(x)+1/4*I*b*polylog(2,-I*c*x^2)-1/4*I*b*polylog(2,I*c*x^2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx = a\log(x) + \frac{1}{4}ib\,\text{PolyLog}\left(2,-icx^2\right) - \frac{1}{4}ib\,\text{PolyLog}\left(2,icx^2\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x,x]
```

output
```
a*Log[x] + (I/4)*b*PolyLog[2, (-I)*c*x^2] - (I/4)*b*PolyLog[2, I*c*x^2]
```

3.64.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x}dx$$

$$\Big\downarrow \ 5359$$

$$\frac{1}{2}\int \frac{a+b\arctan\left(cx^2\right)}{x^2}dx^2$$

$$\Big\downarrow \ 5355$$

$$\frac{1}{2}\left(\frac{1}{2}ib\int \frac{\log\left(1-icx^2\right)}{x^2}dx^2-\frac{1}{2}ib\int \frac{\log\left(icx^2+1\right)}{x^2}dx^2+a\log\left(x^2\right)\right)$$

$$\Big\downarrow \ 2838$$

$$\frac{1}{2}\left(a\log\left(x^2\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,-icx^2\right)-\frac{1}{2}ib\,\text{PolyLog}\left(2,icx^2\right)\right)$$

input
```
Int[(a + b*ArcTan[c*x^2])/x,x]
```

output
```
(a*Log[x^2] + (I/2)*b*PolyLog[2, (-I)*c*x^2] - (I/2)*b*PolyLog[2, I*c*x^2]
)/2
```

3.64.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

3.64.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

method	result
default	$a\ln(x)+b\ln(x)\arctan(c\,x^2)-\frac{b\left(\sum_{\_R1=\mathrm{RootOf}(c^2\_Z^4+1)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^2}\right)}{2c}$
parts	$a\ln(x)+b\ln(x)\arctan(c\,x^2)-\frac{b\left(\sum_{\_R1=\mathrm{RootOf}(c^2\_Z^4+1)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^2}\right)}{2c}$
risch	$\frac{i\ln(-ic\,x^2+1)\ln(x)b}{2}-\frac{i\ln(x)\ln(1-ix\sqrt{-ic})b}{2}-\frac{i\ln(x)\ln(1+ix\sqrt{-ic})b}{2}-\frac{i\,\mathrm{dilog}(1-ix\sqrt{-ic})b}{2}-\frac{i\,\mathrm{dilog}(1+ix\sqrt{-ic})b}{2}+a$

input
```
int((a+b*arctan(c*x^2))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*ln(x)*arctan(c*x^2)-1/2*b/c*sum(1/_R1^2*(ln(x)*ln((_R1-x)/_R1)+d
ilog((_R1-x)/_R1)),_R1=RootOf(_Z^4*c^2+1))
```

3.64.5 Fricas [F]

$$\int\frac{a+b\arctan(cx^2)}{x}\,dx=\int\frac{b\arctan(cx^2)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))/x,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x^2) + a)/x, x)
```

3.64.6 Sympy [F]

$$\int \frac{a + b\arctan(cx^2)}{x} \, dx = \int \frac{a + b\,\mathrm{atan}(cx^2)}{x} \, dx$$

input
```
integrate((a+b*atan(c*x**2))/x,x)
```

output
```
Integral((a + b*atan(c*x**2))/x, x)
```

3.64.7 Maxima [F]

$$\int \frac{a + b\arctan(cx^2)}{x} \, dx = \int \frac{b\arctan(cx^2) + a}{x} \, dx$$

input
```
integrate((a+b*arctan(c*x^2))/x,x, algorithm="maxima")
```

output
```
b*integrate(arctan(c*x^2)/x, x) + a*log(x)
```

3.64.8 Giac [F]

$$\int \frac{a + b\arctan(cx^2)}{x} \, dx = \int \frac{b\arctan(cx^2) + a}{x} \, dx$$

input
```
integrate((a+b*arctan(c*x^2))/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)/x, x)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx = a\,\ln(x) - \frac{b\,(\mathrm{Li}_2(1-c\,x^2\,1\mathrm{i})-\mathrm{Li}_2(1\mathrm{i}\,c\,x^2+1))\;1\mathrm{i}}{4}$$

input
```
int((a + b*atan(c*x^2))/x,x)
```

output
```
a*log(x) - (b*(dilog(1 - c*x^2*1i) - dilog(c*x^2*1i + 1))*1i)/4
```

3.65 $\int \frac{a+b\arctan(cx^2)}{x^3}\,dx$

3.65.1 Optimal result 429
3.65.2 Mathematica [A] (verified) 429
3.65.3 Rubi [A] (verified) 430
3.65.4 Maple [A] (verified) 431
3.65.5 Fricas [A] (verification not implemented) 432
3.65.6 Sympy [A] (verification not implemented) 432
3.65.7 Maxima [A] (verification not implemented) 433
3.65.8 Giac [A] (verification not implemented) 433
3.65.9 Mupad [B] (verification not implemented) 433

3.65.1 Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx = -\frac{a+b\arctan(cx^2)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log\left(1+c^2x^4\right)$$

output
```
1/2*(-a-b*arctan(c*x^2))/x^2+b*c*ln(x)-1/4*b*c*ln(c^2*x^4+1)
```

3.65.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx = -\frac{a}{2x^2}-\frac{b\arctan(cx^2)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log\left(1+c^2x^4\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*ArcTan[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 + c^2*x^4
])/4
```

3.65.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5361, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^3}dx$$

$$\Big\downarrow \text{5361}$$

$$bc\int \frac{1}{x\left(c^2x^4+1\right)}dx-\frac{a+b\arctan\left(cx^2\right)}{2x^2}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}bc\int \frac{1}{x^4\left(c^2x^4+1\right)}dx^4-\frac{a+b\arctan\left(cx^2\right)}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{4}bc\left(\int \frac{1}{x^4}dx^4-c^2\int \frac{1}{c^2x^4+1}dx^4\right)-\frac{a+b\arctan\left(cx^2\right)}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{4}bc\left(\log\left(x^4\right)-c^2\int \frac{1}{c^2x^4+1}dx^4\right)-\frac{a+b\arctan\left(cx^2\right)}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{4}bc(\log\left(x^4\right)-\log\left(c^2x^4+1\right))-\frac{a+b\arctan\left(cx^2\right)}{2x^2}$$

input
```
Int[(a + b*ArcTan[c*x^2])/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c*x^2])/x^2 + (b*c*(Log[x^4] - Log[1 + c^2*x^4]))/4
```

3.65.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.65.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{a}{2x^2}+b\left(-\frac{\arctan(c\,x^2)}{2x^2}+c\left(\ln(x)-\frac{\ln(c^2x^4+1)}{4}\right)\right)$	39
parts	$-\frac{a}{2x^2}+b\left(-\frac{\arctan(c\,x^2)}{2x^2}+c\left(\ln(x)-\frac{\ln(c^2x^4+1)}{4}\right)\right)$	39
parallelrisch	$\frac{4bc\ln(x)x^2-bc\ln(c^2x^4+1)x^2-2b\arctan(c\,x^2)-2a}{4x^2}$	45
risch	$\frac{ib\ln(ic\,x^2+1)}{4x^2}-\frac{-4bc\ln(x)x^2+bc\ln(-c^2x^4-1)x^2+ib\ln(-ic\,x^2+1)+2a}{4x^2}$	68

input
```
int((a+b*arctan(c*x^2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*(-1/2/x^2*arctan(c*x^2)+c*(ln(x)-1/4*ln(c^2*x^4+1)))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx = -\frac{bcx^2\log(c^2x^4+1)-4\,bcx^2\log(x)+2\,b\arctan(cx^2)+2\,a}{4\,x^2}$$

input
```
integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="fricas")
```

output
```
-1/4*(b*c*x^2*log(c^2*x^4 + 1) - 4*b*c*x^2*log(x) + 2*b*arctan(c*x^2) + 2*
a)/x^2
```

3.65.6 Sympy [A] (verification not implemented)

Time = 12.93 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx = \begin{cases} -\frac{a}{2x^2}+bc\log(x)-\frac{bc\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2}+\frac{b\,\mathrm{atan}(cx^2)}{2\sqrt{-\frac{1}{c^2}}}-\frac{b\,\mathrm{atan}(cx^2)}{2x^2} & \text{for } c\neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atan(c*x**2))/x**3,x)
```

output
```
Piecewise((-a/(2*x**2) + b*c*log(x) - b*c*log(x**2 + sqrt(-1/c**2))/2 + b*
atan(c*x**2)/(2*sqrt(-1/c**2)) - b*atan(c*x**2)/(2*x**2), Ne(c, 0)), (-a/(
2*x**2), True))
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx = -\frac{1}{4}\left(c\left(\log\left(c^2x^4+1\right)-\log\left(x^4\right)\right)+\frac{2\arctan(cx^2)}{x^2}\right)b-\frac{a}{2x^2}$$

input
```
integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="maxima")
```

output
```
-1/4*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*b - 1/2*a/x^2
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx = -\frac{bc^3x^2\log\left(c^2x^4+1\right)-2\,bc^3x^2\log\left(cx^2\right)+2\,bc^2\arctan\left(cx^2\right)+2\,ac^2}{4\,c^2x^2}$$

input
```
integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="giac")
```

output
```
-1/4*(b*c^3*x^2*log(c^2*x^4 + 1) - 2*b*c^3*x^2*log(c*x^2) + 2*b*c^2*arctan
(c*x^2) + 2*a*c^2)/(c^2*x^2)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx = b\,c\,\ln(x)-\frac{a}{2\,x^2}-\frac{b\,\mathrm{atan}(c\,x^2)}{2\,x^2}-\frac{b\,c\,\ln\left(c^2\,x^4+1\right)}{4}$$

input
```
int((a + b*atan(c*x^2))/x^3,x)
```

output
```
b*c*log(x) - a/(2*x^2) - (b*atan(c*x^2))/(2*x^2) - (b*c*log(c^2*x^4 + 1))/
4
```

3.66 $\int \frac{a+b\arctan(cx^2)}{x^5}\,dx$

3.66.1 Optimal result 434
3.66.2 Mathematica [C] (verified) 434
3.66.3 Rubi [A] (verified) 435
3.66.4 Maple [A] (verified) 436
3.66.5 Fricas [A] (verification not implemented) 437
3.66.6 Sympy [A] (verification not implemented) 437
3.66.7 Maxima [A] (verification not implemented) 437
3.66.8 Giac [C] (verification not implemented) 438
3.66.9 Mupad [B] (verification not implemented) 438

3.66.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{a+b\arctan(cx^2)}{x^5}\,dx = -\frac{bc}{4x^2}-\frac{1}{4}bc^2\arctan(cx^2)-\frac{a+b\arctan(cx^2)}{4x^4}$$

output
```
-1/4*b*c/x^2-1/4*b*c^2*arctan(c*x^2)+1/4*(-a-b*arctan(c*x^2))/x^4
```

3.66.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{a+b\arctan(cx^2)}{x^5}\,dx = -\frac{a}{4x^4}-\frac{b\arctan(cx^2)}{4x^4}-\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^4\right)}{4x^2}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x^5,x]
```

output
```
-1/4*a/x^4 - (b*ArcTan[c*x^2])/(4*x^4) - (b*c*Hypergeometric2F1[-1/2, 1, 1
/2, -(c^2*x^4)])/(4*x^2)
```

3.66.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 807, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^5}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}bc\int \frac{1}{x^3\left(c^2x^4+1\right)}dx-\frac{a+b\arctan\left(cx^2\right)}{4x^4}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{4}bc\int \frac{1}{x^4\left(c^2x^4+1\right)}dx^2-\frac{a+b\arctan\left(cx^2\right)}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{4}bc\left(c^2\left(-\int \frac{1}{c^2x^4+1}dx^2\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^2\right)}{4x^4}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}bc\left(-c\arctan\left(cx^2\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^2\right)}{4x^4}$$

input
```
Int[(a + b*ArcTan[c*x^2])/x^5,x]
```

output
```
-1/4*(a + b*ArcTan[c*x^2])/x^4 + (b*c*(-x^(-2) - c*ArcTan[c*x^2]))/4
```

3.66.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.66.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a}{4x^4}-\frac{b\arctan\left(c\,x^2\right)}{4x^4}-\frac{bc}{4x^2}-\frac{b\,c^2\arctan\left(c\,x^2\right)}{4}$	39
parts	$-\frac{a}{4x^4}-\frac{b\arctan\left(c\,x^2\right)}{4x^4}-\frac{bc}{4x^2}-\frac{b\,c^2\arctan\left(c\,x^2\right)}{4}$	39
parallelrisch	$-\frac{\arctan\left(c\,x^2\right)b\,c^2x^4-a\,c^2x^4+bc\,x^2+b\arctan\left(c\,x^2\right)+a}{4x^4}$	45
risch	$\frac{ib\ln\left(ic\,x^2+1\right)}{8x^4}-\frac{-ib\,c^2\ln\left(c\,x^2-i\right)x^4+ib\,c^2\ln\left(c\,x^2+i\right)x^4+2bc\,x^2+ib\ln\left(-ic\,x^2+1\right)+2a}{8x^4}$	87

input
```
int((a+b*arctan(c*x^2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/x^4-1/4*b/x^4*arctan(c*x^2)-1/4*b*c/x^2-1/4*b*c^2*arctan(c*x^2)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{a + b \arctan(cx^2)}{x^5} \, dx = -\frac{bcx^2 + (bc^2x^4 + b) \arctan(cx^2) + a}{4 x^4}$$

input
```
integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="fricas")
```

output
```
-1/4*(b*c*x^2 + (b*c^2*x^4 + b)*arctan(c*x^2) + a)/x^4
```

3.66.6 Sympy [A] (verification not implemented)

Time = 12.69 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{a + b \arctan(cx^2)}{x^5} \, dx = -\frac{a}{4x^4} - \frac{bc^2 \operatorname{atan}(cx^2)}{4} - \frac{bc}{4x^2} - \frac{b \operatorname{atan}(cx^2)}{4x^4}$$

input
```
integrate((a+b*atan(c*x**2))/x**5,x)
```

output
```
-a/(4*x**4) - b*c**2*atan(c*x**2)/4 - b*c/(4*x**2) - b*atan(c*x**2)/(4*x**
4)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{a + b \arctan(cx^2)}{x^5} \, dx = -\frac{1}{4} \left(\left(c \arctan(cx^2) + \frac{1}{x^2} \right) c + \frac{\arctan(cx^2)}{x^4} \right) b - \frac{a}{4 x^4}$$

input
```
integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="maxima")
```

output
```
-1/4*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*b - 1/4*a/x^4
```

3.66.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\int \frac{a + b\arctan(cx^2)}{x^5}\,dx = \frac{i\,bc^5x^4\log(i\,cx^2+1) - i\,bc^5x^4\log(-i\,cx^2+1) - 2\,bc^4x^2 - 2\,bc^3\arctan(cx^2) - 2\,ac^3}{8\,c^3x^4}$$

input
```
integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="giac")
```

output
```
1/8*(I*b*c^5*x^4*log(I*c*x^2 + 1) - I*b*c^5*x^4*log(-I*c*x^2 + 1) - 2*b*c^
4*x^2 - 2*b*c^3*arctan(c*x^2) - 2*a*c^3)/(c^3*x^4)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{a + b\arctan(cx^2)}{x^5}\,dx = -\frac{\frac{b\,c\,x^2}{2} + \frac{a}{2}}{2\,x^4} - \frac{b\,c^2\,\mathrm{atan}(c\,x^2)}{4} - \frac{b\,\mathrm{atan}(c\,x^2)}{4\,x^4}$$

input
```
int((a + b*atan(c*x^2))/x^5,x)
```

output
```
- (a/2 + (b*c*x^2)/2)/(2*x^4) - (b*c^2*atan(c*x^2))/4 - (b*atan(c*x^2))/(4
*x^4)
```

3.67 $\int \frac{a+b\arctan(cx^2)}{x^7}\,dx$

3.67.1 Optimal result 439
3.67.2 Mathematica [A] (verified) 439
3.67.3 Rubi [A] (verified) 440
3.67.4 Maple [A] (verified) 441
3.67.5 Fricas [A] (verification not implemented) 442
3.67.6 Sympy [A] (verification not implemented) 442
3.67.7 Maxima [A] (verification not implemented) 443
3.67.8 Giac [A] (verification not implemented) 443
3.67.9 Mupad [B] (verification not implemented) 443

3.67.1 Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx=-\frac{bc}{12x^4}-\frac{a+b\arctan(cx^2)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log\left(1+c^2x^4\right)$$

output
```
-1/12*b*c/x^4+1/6*(-a-b*arctan(c*x^2))/x^6-1/3*b*c^3*ln(x)+1/12*b*c^3*ln(c
^2*x^4+1)
```

3.67.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx=-\frac{a}{6x^6}-\frac{bc}{12x^4}-\frac{b\arctan(cx^2)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log\left(1+c^2x^4\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x^7,x]
```

output
```
-1/6*a/x^6 - (b*c)/(12*x^4) - (b*ArcTan[c*x^2])/(6*x^6) - (b*c^3*Log[x])/3
 + (b*c^3*Log[1 + c^2*x^4])/12
```

3.67.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^7}dx$$

$$\Big\downarrow \ 5361$$

$$\frac{1}{3}bc\int \frac{1}{x^5\left(c^2x^4+1\right)}dx-\frac{a+b\arctan\left(cx^2\right)}{6x^6}$$

$$\Big\downarrow \ 798$$

$$\frac{1}{12}bc\int \frac{1}{x^8\left(c^2x^4+1\right)}dx^4-\frac{a+b\arctan\left(cx^2\right)}{6x^6}$$

$$\Big\downarrow \ 54$$

$$\frac{1}{12}bc\int \left(\frac{c^4}{c^2x^4+1}-\frac{c^2}{x^4}+\frac{1}{x^8}\right)dx^4-\frac{a+b\arctan\left(cx^2\right)}{6x^6}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{12}bc\left(c^2\left(-\log\left(x^4\right)\right)+c^2\log\left(c^2x^4+1\right)-\frac{1}{x^4}\right)-\frac{a+b\arctan\left(cx^2\right)}{6x^6}$$

input
```
Int[(a + b*ArcTan[c*x^2])/x^7,x]
```

output
```
-1/6*(a + b*ArcTan[c*x^2])/x^6 + (b*c*(-x^(-4) - c^2*Log[x^4] + c^2*Log[1
+ c^2*x^4]))/12
```

3.67.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.67.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a}{6x^{6}}+b\left(-\frac{\arctan\left(c\,x^{2}\right)}{6x^{6}}+\frac{c\left(-\frac{1}{4x^{4}}-c^{2}\ln\left(x\right)+\frac{c^{2}\ln\left(c^{2}x^{4}+1\right)}{4}\right)}{3}\right)$	53
parts	$-\frac{a}{6x^{6}}+b\left(-\frac{\arctan\left(c\,x^{2}\right)}{6x^{6}}+\frac{c\left(-\frac{1}{4x^{4}}-c^{2}\ln\left(x\right)+\frac{c^{2}\ln\left(c^{2}x^{4}+1\right)}{4}\right)}{3}\right)$	53
parallelrisch	$-\frac{4b\,c^{3}\ln\left(x\right)x^{6}-b\,c^{3}\ln\left(c^{2}x^{4}+1\right)x^{6}-b\,c^{3}x^{6}+bc\,x^{2}+2b\arctan\left(c\,x^{2}\right)+2a}{12x^{6}}$	64
risch	$\frac{ib\ln\left(ic\,x^{2}+1\right)}{12x^{6}}-\frac{4b\,c^{3}\ln\left(x\right)x^{6}-b\,c^{3}\ln\left(c^{2}x^{4}+1\right)x^{6}+bc\,x^{2}+ib\ln\left(-ic\,x^{2}+1\right)+2a}{12x^{6}}$	78

input
```
int((a+b*arctan(c*x^2))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6+b*(-1/6/x^6*arctan(c*x^2)+1/3*c*(-1/4/x^4-c^2*ln(x)+1/4*c^2*ln(
c^2*x^4+1)))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = \frac{bc^3x^6\log(c^2x^4+1) - 4\,bc^3x^6\log(x) - bcx^2 - 2\,b\arctan(cx^2) - 2\,a}{12\,x^6}$$

input
```
integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="fricas")
```

output
```
1/12*(b*c^3*x^6*log(c^2*x^4 + 1) - 4*b*c^3*x^6*log(x) - b*c*x^2 - 2*b*arct
an(c*x^2) - 2*a)/x^6
```

3.67.6 Sympy [A] (verification not implemented)

Time = 38.96 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.67

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = \begin{cases} -\frac{a}{6x^6} - \frac{bc^3\log(x)}{3} + \frac{bc^3\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6} - \frac{bc^2\,\mathrm{atan}(cx^2)}{6\sqrt{-\frac{1}{c^2}}} - \frac{bc}{12x^4} - \frac{b\,\mathrm{atan}(cx^2)}{6x^6} & \text{for } c \neq 0 \\ -\frac{a}{6x^6} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atan(c*x**2))/x**7,x)
```

output
```
Piecewise((-a/(6*x**6) - b*c**3*log(x)/3 + b*c**3*log(x**2 + sqrt(-1/c**2)
)/6 - b*c**2*atan(c*x**2)/(6*sqrt(-1/c**2)) - b*c/(12*x**4) - b*atan(c*x**
2)/(6*x**6), Ne(c, 0)), (-a/(6*x**6), True))
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = \frac{1}{12}\left(\left(c^2\log\left(c^2x^4+1\right)-c^2\log\left(x^4\right)-\frac{1}{x^4}\right)c-\frac{2\arctan(cx^2)}{x^6}\right)b - \frac{a}{6x^6}$$

input
```
integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="maxima")
```

output
```
1/12*((c^2*log(c^2*x^4 + 1) - c^2*log(x^4) - 1/x^4)*c - 2*arctan(c*x^2)/x^
6)*b - 1/6*a/x^6
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = \frac{bc^7x^6\log\left(c^2x^4+1\right)-2\,bc^7x^6\log\left(cx^2\right)-bc^5x^2-2\,bc^4\arctan\left(cx^2\right)-2\,ac^4}{12\,c^4x^6}$$

input
```
integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="giac")
```

output
```
1/12*(b*c^7*x^6*log(c^2*x^4 + 1) - 2*b*c^7*x^6*log(c*x^2) - b*c^5*x^2 - 2*
b*c^4*arctan(c*x^2) - 2*a*c^4)/(c^4*x^6)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = \frac{b\,c^3\,\ln\left(c^2\,x^4+1\right)}{12}-\frac{a}{6\,x^6}-\frac{b\,c^3\,\ln(x)}{3}-\frac{b\,\mathrm{atan}(c\,x^2)}{6\,x^6}-\frac{b\,c}{12\,x^4}$$

input
```
int((a + b*atan(c*x^2))/x^7,x)
```

output
```
(b*c^3*log(c^2*x^4 + 1))/12 - a/(6*x^6) - (b*c^3*log(x))/3 - (b*atan(c*x^2
))/(6*x^6) - (b*c)/(12*x^4)
```

3.68 $\int x^4(a + b\arctan(cx^2))\,dx$

3.68.1 Optimal result . . . 444
3.68.2 Mathematica [A] (verified) . . . 444
3.68.3 Rubi [A] (verified) . . . 445
3.68.4 Maple [A] (verified) . . . 448
3.68.5 Fricas [C] (verification not implemented) . . . 449
3.68.6 Sympy [A] (verification not implemented) . . . 450
3.68.7 Maxima [A] (verification not implemented) . . . 451
3.68.8 Giac [A] (verification not implemented) . . . 451
3.68.9 Mupad [B] (verification not implemented) . . . 452

3.68.1 Optimal result

Integrand size = 14, antiderivative size = 161

$$\int x^4(a+b\arctan(cx^2))\,dx = -\frac{2bx^3}{15c}+\frac{1}{5}x^5(a+b\arctan(cx^2)) - \frac{b\arctan(1-\sqrt{2}\sqrt{c}x)}{5\sqrt{2}c^{5/2}}+\frac{b\arctan(1+\sqrt{2}\sqrt{c}x)}{5\sqrt{2}c^{5/2}} + \frac{b\log(1-\sqrt{2}\sqrt{c}x+cx^2)}{10\sqrt{2}c^{5/2}}-\frac{b\log(1+\sqrt{2}\sqrt{c}x+cx^2)}{10\sqrt{2}c^{5/2}}$$

output
```
-2/15*b*x^3/c+1/5*x^5*(a+b*arctan(c*x^2))+1/10*b*arctan(-1+x*2^(1/2)*c^(1/
2))/c^(5/2)*2^(1/2)+1/10*b*arctan(1+x*2^(1/2)*c^(1/2))/c^(5/2)*2^(1/2)+1/2
0*b*ln(1+c*x^2-x*2^(1/2)*c^(1/2))/c^(5/2)*2^(1/2)-1/20*b*ln(1+c*x^2+x*2^(1
/2)*c^(1/2))/c^(5/2)*2^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.11

$$\int x^4(a+b\arctan(cx^2))\,dx = -\frac{2bx^3}{15c}+\frac{ax^5}{5}+\frac{1}{5}bx^5\arctan(cx^2) + \frac{b\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{5\sqrt{2}c^{5/2}}+\frac{b\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{5\sqrt{2}c^{5/2}} + \frac{b\log(1-\sqrt{2}\sqrt{c}x+cx^2)}{10\sqrt{2}c^{5/2}}-\frac{b\log(1+\sqrt{2}\sqrt{c}x+cx^2)}{10\sqrt{2}c^{5/2}}$$

input
```
Integrate[x^4*(a + b*ArcTan[c*x^2]),x]
```

output
```
(-2*b*x^3)/(15*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x^2])/5 + (b*ArcTan[(-Sqrt
[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]*c^(5/2)) + (b*ArcTan[(Sqrt[2] + 2*
Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]*c^(5/2)) + (b*Log[1 - Sqrt[2]*Sqrt[c]*x +
c*x^2])/(10*Sqrt[2]*c^(5/2)) - (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*
Sqrt[2]*c^(5/2))
```

3.68.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5361, 843, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b \arctan \left(c x^2\right)\right) \, dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{5} x^5 \left(a+b \arctan \left(c x^2\right)\right)-\frac{2}{5} b c \int \frac{x^6}{c^2 x^4+1}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{1}{5} x^5 \left(a+b \arctan \left(c x^2\right)\right)-\frac{2}{5} b c \left(\frac{x^3}{3 c^2}-\frac{\int \frac{x^2}{c^2 x^4+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{ 826}$$

$$\frac{1}{5} x^5 \left(a+b \arctan \left(c x^2\right)\right)-\frac{2}{5} b c \left(\frac{x^3}{3 c^2}-\frac{\frac{\int \frac{c x^2+1}{c^2 x^4+1}dx}{2 c}-\frac{\int \frac{1-c x^2}{c^2 x^4+1}dx}{2 c}}{c^2}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{1}{5} x^5 \left(a+b \arctan \left(c x^2\right)\right)-\frac{2}{5} b c \left(\frac{x^3}{3 c^2}-\frac{\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2} x}{\sqrt{c}}+\frac{1}{c}}dx}{2 c}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2} x}{\sqrt{c}}+\frac{1}{c}}dx}{2 c}}{2 c}-\frac{\int \frac{1-c x^2}{c^2 x^4+1}dx}{2 c}}{c^2}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\int\frac{1}{-\left(1-\sqrt{2}\sqrt{c}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{\int\frac{1}{-\left(\sqrt{2}\sqrt{c}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\int\frac{1-cx^2}{c^2x^4+1}dx}{2c}}{c^2}\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\int\frac{1-cx^2}{c^2x^4+1}dx}{2c}}{c^2}\right)$$

$$\Big\downarrow \; 1479$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{-\frac{\int-\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}}{2c}}{c^2}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\frac{\int\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}}{2c}}{c^2}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\frac{\int\frac{\sqrt{2}-2\sqrt{c}x}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2\sqrt{2}c}+\frac{\int\frac{\sqrt{2}\sqrt{c}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}}{2c}}{c^2}\right)$$

$$\Big\downarrow \; 1103$$

3.68. $\int x^4\left(a+b\arctan\left(cx^2\right)\right)\,dx$

$$\frac{1}{5}x^5(a+b\arctan(cx^2))-\frac{2}{5}bc\left(\frac{x^3}{3c^2}-\frac{\frac{\frac{\arctan(\sqrt{2}\sqrt{c}x+1)}{\sqrt{2}\sqrt{c}}-\frac{\arctan(1-\sqrt{2}\sqrt{c}x)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\frac{\log(cx^2+\sqrt{2}\sqrt{c}x+1)}{2\sqrt{2}\sqrt{c}}-\frac{\log(cx^2-\sqrt{2}\sqrt{c}x+1)}{2\sqrt{2}\sqrt{c}}}{2c}}{c^2}\right)$$

input
```
Int[x^4*(a + b*ArcTan[c*x^2]),x]
```

output
```
(x^5*(a + b*ArcTan[c*x^2]))/5 - (2*b*c*(x^3/(3*c^2) - ((-(ArcTan[1 - Sqrt[
2]*Sqrt[c]*x]/(Sqrt[2]*Sqrt[c])) + ArcTan[1 + Sqrt[2]*Sqrt[c]*x]/(Sqrt[2]*
Sqrt[c]))/(2*c) - (-1/2*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]/(Sqrt[2]*Sqrt[c
]) + Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]/(2*Sqrt[2]*Sqrt[c]))/(2*c))/c^2))/
5
```

3.68.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.68.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

method	result	s
default	$\frac{a\,x^{5}}{5}+b\left(\frac{x^{5}\arctan\left(c\,x^{2}\right)}{5}-\frac{2c\left(\frac{x^{3}}{3c^{2}}-\frac{\sqrt{2}\,\left(\ln\left(\frac{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^{4}\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}\right)}{5}\right)$	1
parts	$\frac{a\,x^{5}}{5}+b\left(\frac{x^{5}\arctan\left(c\,x^{2}\right)}{5}-\frac{2c\left(\frac{x^{3}}{3c^{2}}-\frac{\sqrt{2}\,\left(\ln\left(\frac{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^{4}\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}\right)}{5}\right)$	1

input
```
int(x^4*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+b*(1/5*x^5*arctan(c*x^2)-2/5*c*(1/3/c^2*x^3-1/8/c^4/(1/c^2)^(1/4
)*2^(1/2)*(ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/
4)*x*2^(1/2)+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(
2^(1/2)/(1/c^2)^(1/4)*x-1))))
```

3.68.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06

$$\int x^{4}\left(a+b\arctan\left(cx^{2}\right)\right)\,dx$$

$$=\frac{6\,bcx^{5}\arctan\left(cx^{2}\right)+6\,acx^{5}-4\,bx^{3}+3\,c\left(-\frac{b^{4}}{c^{10}}\right)^{\frac{1}{4}}\log\left(c^{7}\left(-\frac{b^{4}}{c^{10}}\right)^{\frac{3}{4}}+b^{3}x\right)-3i\,c\left(-\frac{b^{4}}{c^{10}}\right)^{\frac{1}{4}}\log\left(i\,c^{7}\left(-\frac{b^{4}}{c^{10}}\right.\right.}{30\,c}$$

input
```
integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
1/30*(6*b*c*x^5*arctan(c*x^2) + 6*a*c*x^5 - 4*b*x^3 + 3*c*(-b^4/c^10)^(1/4
)*log(c^7*(-b^4/c^10)^(3/4) + b^3*x) - 3*I*c*(-b^4/c^10)^(1/4)*log(I*c^7*(
-b^4/c^10)^(3/4) + b^3*x) + 3*I*c*(-b^4/c^10)^(1/4)*log(-I*c^7*(-b^4/c^10)
^(3/4) + b^3*x) - 3*c*(-b^4/c^10)^(1/4)*log(-c^7*(-b^4/c^10)^(3/4) + b^3*x
))/c
```

3.68.6 Sympy [A] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.95

$$\int x^4\left(a+b\arctan\left(cx^2\right)\right)\,dx$$
$$=\begin{cases}\frac{ax^5}{5}+\frac{bx^5\operatorname{atan}\left(cx^2\right)}{5}-\frac{2bx^3}{15c}+\frac{b\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{5c^3\sqrt[4]{-\frac{1}{c^2}}}-\frac{b\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{10c^3\sqrt[4]{-\frac{1}{c^2}}}+\frac{b\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{5c^3\sqrt[4]{-\frac{1}{c^2}}}-\frac{b\operatorname{atan}\left(cx^2\right)}{5c^6\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}} & \text{for } c\neq 0\\ \frac{ax^5}{5} & \text{otherwise}\end{cases}$$

input
```
integrate(x**4*(a+b*atan(c*x**2)),x)
```

output
```
Piecewise((a*x**5/5 + b*x**5*atan(c*x**2)/5 - 2*b*x**3/(15*c) + b*log(x -
(-1/c**2)**(1/4))/(5*c**3*(-1/c**2)**(1/4)) - b*log(x**2 + sqrt(-1/c**2))/
(10*c**3*(-1/c**2)**(1/4)) + b*atan(x/(-1/c**2)**(1/4))/(5*c**3*(-1/c**2)*
*(1/4)) - b*atan(c*x**2)/(5*c**6*(-1/c**2)**(7/4)), Ne(c, 0)), (a*x**5/5,
True))
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int x^4 \left(a + b \arctan\left(cx^2\right)\right) \, dx = \frac{1}{5} a x^5 + \frac{1}{60} \left(12 x^5 \arctan\left(cx^2\right) - c \left(\frac{8 x^3}{c^2} - \frac{3 \left(\frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2 cx + \sqrt{2}\sqrt{c}\right)}{2 \sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2 \sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2 cx - \sqrt{2}\sqrt{c}\right)}{2 \sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{2} \log\left(cx^2 + \cdots\right.}{c^{\frac{3}{2}}} \cdots\right.}{c^2} \cdots\right.\right.$$

input
```
integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/60*(12*x^5*arctan(c*x^2) - c*(8*x^3/c^2 - 3*(2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c^(3/2) + 2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c^(3/2) - sqrt(2)*log(c*x
^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*
x + 1)/c^(3/2))/c^2))*b
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.05

$$\int x^4 \left(a + b \arctan\left(cx^2\right)\right) \, dx$$
$$= \frac{1}{20} b c^9 \left(\frac{2 \sqrt{2}\sqrt{|c|} \arctan\left(\frac{1}{2}\sqrt{2}\left(2x + \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{12}} + \frac{2 \sqrt{2}\sqrt{|c|} \arctan\left(\frac{1}{2}\sqrt{2}\left(2x - \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{12}} - \frac{\sqrt{2} \log \cdots}{\cdots}\right.$$
$$+ \frac{3 bcx^5 \arctan\left(cx^2\right) + 3 acx^5 - 2 bx^3}{15 c}$$

input
```
integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
1/20*b*c^9*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(
abs(c)))*sqrt(abs(c)))/c^12 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^12 - sqrt(2)*log(x^2 + sqrt(2)*
x/sqrt(abs(c)) + 1/abs(c))/(c^10*abs(c)^(3/2)) + sqrt(2)*sqrt(abs(c))*log(
x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^12) + 1/15*(3*b*c*x^5*arctan(c*
x^2) + 3*a*c*x^5 - 2*b*x^3)/c
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.40

$$\int x^4\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{a\,x^5}{5}-\frac{2\,b\,x^3}{15\,c}+\frac{b\,x^5\,\mathrm{atan}(c\,x^2)}{5}+\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)}{5\,c^{5/2}}+\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{5\,c^{5/2}}$$

input
```
int(x^4*(a + b*atan(c*x^2)),x)
```

output
```
(a*x^5)/5 - (2*b*x^3)/(15*c) + (b*x^5*atan(c*x^2))/5 + ((-1)^(1/4)*b*atan(
(-1)^(1/4)*c^(1/2)*x))/(5*c^(5/2)) + ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)
*x*1i)*1i)/(5*c^(5/2))
```

3.69 $\int x^2(a+b\arctan(cx^2))\,dx$

3.69.1 Optimal result 453
3.69.2 Mathematica [A] (verified) 453
3.69.3 Rubi [A] (verified) 454
3.69.4 Maple [A] (verified) 458
3.69.5 Fricas [C] (verification not implemented) 459
3.69.6 Sympy [A] (verification not implemented) 459
3.69.7 Maxima [A] (verification not implemented) 460
3.69.8 Giac [A] (verification not implemented) 460
3.69.9 Mupad [B] (verification not implemented) 461

3.69.1 Optimal result

Integrand size = 14, antiderivative size = 159

$$\int x^2(a+b\arctan(cx^2))\,dx = -\frac{2bx}{3c}+\frac{1}{3}x^3(a+b\arctan(cx^2)) - \frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}+\frac{b\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}} - \frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}+\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}$$

output
```
-2/3*b*x/c+1/3*x^3*(a+b*arctan(c*x^2))+1/6*b*arctan(-1+x*2^(1/2)*c^(1/2))/
c^(3/2)*2^(1/2)+1/6*b*arctan(1+x*2^(1/2)*c^(1/2))/c^(3/2)*2^(1/2)-1/12*b*l
n(1+c*x^2-x*2^(1/2)*c^(1/2))/c^(3/2)*2^(1/2)+1/12*b*ln(1+c*x^2+x*2^(1/2)*c
^(1/2))/c^(3/2)*2^(1/2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11

$$\int x^2(a+b\arctan(cx^2))\,dx = -\frac{2bx}{3c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\arctan(cx^2) + \frac{b\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{3\sqrt{2}c^{3/2}}+\frac{b\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{3\sqrt{2}c^{3/2}} - \frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}+\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x^2]),x]
```

output
```
(-2*b*x)/(3*c) + (a*x^3)/3 + (b*x^3*ArcTan[c*x^2])/3 + (b*ArcTan[(-Sqrt[2]
 + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]*c^(3/2)) + (b*ArcTan[(Sqrt[2] + 2*Sqr
t[c]*x)/Sqrt[2]])/(3*Sqrt[2]*c^(3/2)) - (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x
^2])/(6*Sqrt[2]*c^(3/2)) + (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[
2]*c^(3/2))
```

3.69.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5361, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \arctan \left(c x^2\right)\right) \, dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3} x^3 \left(a+b \arctan \left(c x^2\right)\right)-\frac{2}{3} b c \int \frac{x^4}{c^2 x^4+1}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{1}{3} x^3 \left(a+b \arctan \left(c x^2\right)\right)-\frac{2}{3} b c \left(\frac{x}{c^2}-\frac{\int \frac{1}{c^2 x^4+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{ 755}$$

$$\frac{1}{3} x^3 \left(a+b \arctan \left(c x^2\right)\right)-\frac{2}{3} b c \left(\frac{x}{c^2}-\frac{\frac{1}{2} \int \frac{1-c x^2}{c^2 x^4+1}dx+\frac{1}{2} \int \frac{c x^2+1}{c^2 x^4+1}dx}{c^2}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{1}{3} x^3 \left(a+b \arctan \left(c x^2\right)\right)-\frac{2}{3} b c \left(\frac{x}{c^2}-\frac{\frac{1}{2} \int \frac{1-c x^2}{c^2 x^4+1}dx+\frac{1}{2} \left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2} x}{\sqrt{c}}+\frac{1}{c}}dx}{2 c}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2} x}{\sqrt{c}}+\frac{1}{c}}dx}{2 c}\right)}{c^2}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\int\frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\int\frac{1}{-\left(1-\sqrt{2}\sqrt{c}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{\int\frac{1}{-\left(\sqrt{2}\sqrt{c}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}\right)}{c^2}\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\int\frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)}{c^2}\right)$$

$$\Big\downarrow \; 1479$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)}{c^2}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)}{c^2}\right)$$

$$\Big\downarrow \; 27$$

3.69. $\int x^2(a+b\arctan(cx^2))\ dx$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{c}x}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2\sqrt{2}c}+\frac{\int\frac{\sqrt{2}\sqrt{c}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)}{c^2}\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{2}{3}bc\left(\frac{x}{c^2}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}-\frac{\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}\right)}{c^2}\right)$$

input
```
Int[x^2*(a + b*ArcTan[c*x^2]),x]
```

output
```
(x^3*(a + b*ArcTan[c*x^2]))/3 - (2*b*c*(x/c^2 - ((-(ArcTan[1 - Sqrt[2]*Sqr
t[c]*x]/(Sqrt[2]*Sqrt[c])) + ArcTan[1 + Sqrt[2]*Sqrt[c]*x]/(Sqrt[2]*Sqrt[c
]))/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]/(Sqrt[2]*Sqrt[c]) + Log[1
 + Sqrt[2]*Sqrt[c]*x + c*x^2]/(2*Sqrt[2]*Sqrt[c]))/2)/c^2))/3
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.69. $\int x^2(a + b\arctan(cx^2))\,dx$

3.69.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

method	result
default	$\frac{x^3 a}{3}+b\left(\frac{x^3\arctan(c x^2)}{3}-\frac{2c\left(\frac{x}{c^2}-\frac{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^2}\right)}{3}\right)$
parts	$\frac{x^3 a}{3}+b\left(\frac{x^3\arctan(c x^2)}{3}-\frac{2c\left(\frac{x}{c^2}-\frac{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^2}\right)}{3}\right)$

input
```
int(x^2*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*a+b*(1/3*x^3*arctan(c*x^2)-2/3*c*(1/c^2*x-1/8/c^2*(1/c^2)^(1/4)*2^
(1/2)*(ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x
*2^(1/2)+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1
/2)/(1/c^2)^(1/4)*x-1))))
```

3.69.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

$$\int x^2(a+b\arctan(cx^2))\,dx$$

$$= \frac{2\,bcx^3\arctan(cx^2)+2\,acx^3+c\left(-\frac{b^4}{c^6}\right)^{\frac14}\log\left(bx+c\left(-\frac{b^4}{c^6}\right)^{\frac14}\right)+i\,c\left(-\frac{b^4}{c^6}\right)^{\frac14}\log\left(bx+i\,c\left(-\frac{b^4}{c^6}\right)^{\frac14}\right)-i\,c\Big(-}{6\,c}$$

input
```
integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
1/6*(2*b*c*x^3*arctan(c*x^2) + 2*a*c*x^3 + c*(-b^4/c^6)^(1/4)*log(b*x + c*
(-b^4/c^6)^(1/4)) + I*c*(-b^4/c^6)^(1/4)*log(b*x + I*c*(-b^4/c^6)^(1/4)) -
 I*c*(-b^4/c^6)^(1/4)*log(b*x - I*c*(-b^4/c^6)^(1/4)) - c*(-b^4/c^6)^(1/4)
*log(b*x - c*(-b^4/c^6)^(1/4)) - 4*b*x)/c
```

3.69.6 Sympy [A] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x^2(a+b\arctan(cx^2))\,dx$$

$$= \begin{cases} \frac{ax^3}{3}+\frac{bx^3\operatorname{atan}(cx^2)}{3}+\frac{b\left(-\frac{1}{c^2}\right)^{\frac34}\operatorname{atan}(cx^2)}{3}-\frac{2bx}{3c}-\frac{b\sqrt[4]{-\frac{1}{c^2}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{3c}+\frac{b\sqrt[4]{-\frac{1}{c^2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6c}+\frac{b\sqrt[4]{-\frac{1}{c^2}}\operatorname{at}}{} \\ \frac{ax^3}{3} \end{cases}$$

input
```
integrate(x**2*(a+b*atan(c*x**2)),x)
```

output
```
Piecewise((a*x**3/3 + b*x**3*atan(c*x**2)/3 + b*(-1/c**2)**(3/4)*atan(c*x*
*2)/3 - 2*b*x/(3*c) - b*(-1/c**2)**(1/4)*log(x - (-1/c**2)**(1/4))/(3*c) +
 b*(-1/c**2)**(1/4)*log(x**2 + sqrt(-1/c**2))/(6*c) + b*(-1/c**2)**(1/4)*a
tan(x/(-1/c**2)**(1/4))/(3*c), Ne(c, 0)), (a*x**3/3, True))
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int x^2(a+b\arctan(cx^2))\,dx = \frac{1}{3}ax^3 + \frac{1}{12}\left(4x^3\arctan(cx^2) - c\left(\frac{8x}{c^2} - \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(2cx+\sqrt{2}\sqrt{c})}{2\sqrt{c}}\right)}{\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(2cx-\sqrt{2}\sqrt{c})}{2\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{2}\log(cx^2+\sqrt{2}\sqrt{c}x+}{\sqrt{c}}}{c^2}\right.\right.$$

input
```
integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/12*(4*x^3*arctan(c*x^2) - c*(8*x/c^2 - (2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + 2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + sqrt(2)*log(c*x^2 +
sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1
)/sqrt(c))/c^2))*b
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.04

$$\int x^2(a+b\arctan(cx^2))\,dx$$

$$= \frac{1}{12}bc^5\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^6\sqrt{|c|}} + \frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^6\sqrt{|c|}} + \frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{}}{\sqrt{}}\right.}{c^6\sqrt{|c|}}\right.$$
$$+ \frac{bcx^3\arctan(cx^2) + acx^3 - 2bx}{3c}$$

input
```
integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
1/12*b*c^5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt
(abs(c)))/(c^6*sqrt(abs(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
/sqrt(abs(c)))*sqrt(abs(c)))/(c^6*sqrt(abs(c))) + sqrt(2)*log(x^2 + sqrt(2
)*x/sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c))) - sqrt(2)*log(x^2 - sqrt(2
)*x/sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c)))) + 1/3*(b*c*x^3*arctan(c*x
^2) + a*c*x^3 - 2*b*x)/c
```

3.69.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

$$\int x^2(a+b\arctan(cx^2))\,dx = \frac{a\,x^3}{3}+\frac{b\,x^3\,\mathrm{atan}(c\,x^2)}{3}-\frac{2\,b\,x}{3\,c} - \frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)\,1\mathrm{i}}{3\,c^{3/2}} - \frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)}{3\,c^{3/2}}$$

input
```
int(x^2*(a + b*atan(c*x^2)),x)
```

output
```
(a*x^3)/3 + (b*x^3*atan(c*x^2))/3 - (2*b*x)/(3*c) - ((-1)^(1/4)*b*atan((-1
)^(1/4)*c^(1/2)*x)*1i)/(3*c^(3/2)) - ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)
*x*1i))/(3*c^(3/2))
```

3.70 $\int (a+b \arctan (cx^2)) \, dx$

3.70.1 Optimal result 462
3.70.2 Mathematica [A] (verified) 462
3.70.3 Rubi [A] (verified) 463
3.70.4 Maple [A] (verified) 464
3.70.5 Fricas [C] (verification not implemented) 465
3.70.6 Sympy [A] (verification not implemented) 465
3.70.7 Maxima [A] (verification not implemented) 466
3.70.8 Giac [A] (verification not implemented) 467
3.70.9 Mupad [B] (verification not implemented) 467

3.70.1 Optimal result

Integrand size = 10, antiderivative size = 140

$$\int (a+b\arctan(cx^2))\,dx = ax+bx\arctan(cx^2)+\frac{b\arctan(1-\sqrt{2}\sqrt{c}x)}{\sqrt{2}\sqrt{c}} - \frac{b\arctan(1+\sqrt{2}\sqrt{c}x)}{\sqrt{2}\sqrt{c}} - \frac{b\log(1-\sqrt{2}\sqrt{c}x+cx^2)}{2\sqrt{2}\sqrt{c}} + \frac{b\log(1+\sqrt{2}\sqrt{c}x+cx^2)}{2\sqrt{2}\sqrt{c}}$$

output
```
a*x+b*x*arctan(c*x^2)-1/2*b*arctan(-1+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2)-1
/2*b*arctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2)-1/4*b*ln(1+c*x^2-x*2^(1/2
)*c^(1/2))*2^(1/2)/c^(1/2)+1/4*b*ln(1+c*x^2+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(
1/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int (a+b\arctan(cx^2))\,dx = ax+bx\arctan(cx^2) - \frac{b(-2\arctan(1-\sqrt{2}\sqrt{c}x)+2\arctan(1+\sqrt{2}\sqrt{c}x)+\log(1-\sqrt{2}\sqrt{c}x+cx^2)-\log(1+\sqrt{2}\sqrt{c}x+cx}{2\sqrt{2}\sqrt{c}}$$

input
```
Integrate[a + b*ArcTan[c*x^2],x]
```

output
```
a*x + b*x*ArcTan[c*x^2] - (b*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c]*x] + 2*ArcTan[
1 + Sqrt[2]*Sqrt[c]*x] + Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2] - Log[1 + Sqrt
[2]*Sqrt[c]*x + c*x^2]))/(2*Sqrt[2]*Sqrt[c])
```

3.70.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arctan\left(cx^2\right)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+bx\arctan\left(cx^2\right)+\frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{b\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}+\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}$$

input
```
Int[a + b*ArcTan[c*x^2],x]
```

output
```
a*x + b*x*ArcTan[c*x^2] + (b*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[
c]) - (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*Log[1 - Sqr
t[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]*Sqrt[c]) + (b*Log[1 + Sqrt[2]*Sqrt[c]*
x + c*x^2])/(2*Sqrt[2]*Sqrt[c])
```

3.70.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.70.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.74

method	result	siz
default	$ax+b\left(x\arctan\left(c\,x^{2}\right)-\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}\right)$	103
parts	$ax+b\left(x\arctan\left(c\,x^{2}\right)-\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}\right)$	103

input
```
int(a+b*arctan(c*x^2),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*(x*arctan(c*x^2)-1/4/c/(1/c^2)^(1/4)*2^(1/2)*(ln((x^2-(1/c^2)^(1/4)*
x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+2*ar
ctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)))
```

3.70.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int (a+b\arctan(cx^2))\,dx = bx\arctan(cx^2)+ax-\frac{1}{2}\left(-\frac{b^4}{c^2}\right)^{\frac{1}{4}}\log\left(b^3x+\left(-\frac{b^4}{c^2}\right)^{\frac{3}{4}}c\right)+\frac{1}{2}i\left(-\frac{b^4}{c^2}\right)^{\frac{1}{4}}\log\left(b^3x+i\left(-\frac{b^4}{c^2}\right)^{\frac{3}{4}}c\right)-\frac{1}{2}i\left(-\frac{b^4}{c^2}\right)^{\frac{1}{4}}\log\left(b^3x-i\left(-\frac{b^4}{c^2}\right)^{\frac{3}{4}}c\right)+\frac{1}{2}\left(-\frac{b^4}{c^2}\right)^{\frac{1}{4}}\log\left(b^3x-\left(-\frac{b^4}{c^2}\right)^{\frac{3}{4}}c\right)$$

input
```
integrate(a+b*arctan(c*x^2),x, algorithm="fricas")
```

output
```
b*x*arctan(c*x^2) + a*x - 1/2*(-b^4/c^2)^(1/4)*log(b^3*x + (-b^4/c^2)^(3/4
)*c) + 1/2*I*(-b^4/c^2)^(1/4)*log(b^3*x + I*(-b^4/c^2)^(3/4)*c) - 1/2*I*(-
b^4/c^2)^(1/4)*log(b^3*x - I*(-b^4/c^2)^(3/4)*c) + 1/2*(-b^4/c^2)^(1/4)*lo
g(b^3*x - (-b^4/c^2)^(3/4)*c)
```

3.70.6 Sympy [A] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 617, normalized size of antiderivative = 4.41

$$\int (a+b\arctan(cx^2))\,dx = ax + b\left(\begin{cases}0\\ -\infty ix\\ \infty ix\\ \frac{2c^5x^5\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}\operatorname{atan}(cx^2)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}} - \frac{2c^4x^4\left(-\frac{1}{c^2}\right)^{\frac{3}{2}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}} + \frac{c^4x^4\left(-\frac{1}{c^2}\right)^{\frac{3}{2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}} - \frac{2c^4x^4\left(-\frac{1}{c^2}\right)^{\frac{3}{2}}\operatorname{atan}\left(\right.}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(\right.}\end{cases}\right.$$

input
```
integrate(a+b*atan(c*x**2),x)
```

output
```
a*x + b*Piecewise((0, Eq(c, 0)), (-oo*I*x, Eq(c, -I/x**2)), (oo*I*x, Eq(c,
 I/x**2)), (2*c**5*x**5*(-1/c**2)**(7/4)*atan(c*x**2)/(2*c**5*x**4*(-1/c**
2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) - 2*c**4*x**4*(-1/c**2)**(3/2)*log(x
- (-1/c**2)**(1/4))/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4
)) + c**4*x**4*(-1/c**2)**(3/2)*log(x**2 + sqrt(-1/c**2))/(2*c**5*x**4*(-1
/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) - 2*c**4*x**4*(-1/c**2)**(3/2)*at
an(x/(-1/c**2)**(1/4))/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(
7/4)) + 2*c**3*x*(-1/c**2)**(7/4)*atan(c*x**2)/(2*c**5*x**4*(-1/c**2)**(7/
4) + 2*c**3*(-1/c**2)**(7/4)) - 2*c**2*(-1/c**2)**(3/2)*log(x - (-1/c**2)*
*(1/4))/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) + c**2*(-
1/c**2)**(3/2)*log(x**2 + sqrt(-1/c**2))/(2*c**5*x**4*(-1/c**2)**(7/4) + 2
*c**3*(-1/c**2)**(7/4)) - 2*c**2*(-1/c**2)**(3/2)*atan(x/(-1/c**2)**(1/4))
/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) + 2*c*x**4*atan(
c*x**2)/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) + 2*atan(
c*x**2)/(2*c**6*x**4*(-1/c**2)**(7/4) + 2*c**4*(-1/c**2)**(7/4)), True))
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \left(a+b\arctan\left(cx^2\right)\right)\,dx =$$

$$-\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{c^{\frac{3}{2}}}+\right.\right.$$

$$+ax$$

input
```
integrate(a+b*arctan(c*x^2),x, algorithm="maxima")
```

output
```
-1/4*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c
^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c
^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(
c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) - 4*x*arctan(c*x^2))*b + a*x
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06

$$\int \left(a+b\arctan\left(cx^2\right)\right)\,dx =$$

$$-\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}+\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}-\frac{\sqrt{2}\sqrt{}}{}\right.\right.$$

$$+\,ax$$

input
```
integrate(a+b*arctan(c*x^2),x, algorithm="giac")
```

output
```
-1/4*(c*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs
(c)))*sqrt(abs(c)))/c^2 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^2 - sqrt(2)*sqrt(abs(c))*log(x^2 +
sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 + sqrt(2)*sqrt(abs(c))*log(x^2 - sq
rt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2) - 4*x*arctan(c*x^2))*b + a*x
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.35

$$\int \left(a+b\arctan\left(cx^2\right)\right)\,dx = a\,x+b\,x\,\mathrm{atan}\left(c\,x^2\right)-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{\sqrt{c}}$$

input
```
int(a + b*atan(c*x^2),x)
```

output
```
a*x + b*x*atan(c*x^2) - ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x))/c^(1/2)
- ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x*1i)*1i)/c^(1/2)
```

3.71 $\int \frac{a+b\arctan(cx^2)}{x^2}\,dx$

3.71.1 Optimal result 468
3.71.2 Mathematica [A] (verified) 468
3.71.3 Rubi [A] (verified) 469
3.71.4 Maple [A] (verified) 472
3.71.5 Fricas [C] (verification not implemented) 473
3.71.6 Sympy [A] (verification not implemented) 473
3.71.7 Maxima [A] (verification not implemented) 474
3.71.8 Giac [A] (verification not implemented) 474
3.71.9 Mupad [B] (verification not implemented) 475

3.71.1 Optimal result

Integrand size = 14, antiderivative size = 143

$$\int \frac{a+b\arctan(cx^2)}{x^2}\,dx = -\frac{a+b\arctan(cx^2)}{x} - \frac{b\sqrt{c}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}} + \frac{b\sqrt{c}\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}} - \frac{b\sqrt{c}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}} + \frac{b\sqrt{c}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}$$

output
```
(-a-b*arctan(c*x^2))/x+1/2*b*arctan(-1+x*2^(1/2)*c^(1/2))*c^(1/2)*2^(1/2)+
1/2*b*arctan(1+x*2^(1/2)*c^(1/2))*c^(1/2)*2^(1/2)-1/4*b*ln(1+c*x^2-x*2^(1/
2)*c^(1/2))*c^(1/2)*2^(1/2)+1/4*b*ln(1+c*x^2+x*2^(1/2)*c^(1/2))*c^(1/2)*2^
(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \frac{a+b\arctan(cx^2)}{x^2}\,dx = -\frac{a}{x} - \frac{b\arctan(cx^2)}{x} + \frac{b\sqrt{c}\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{b\sqrt{c}\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{b\sqrt{c}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}} + \frac{b\sqrt{c}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x^2,x]
```

output
```
-(a/x) - (b*ArcTan[c*x^2])/x + (b*Sqrt[c]*ArcTan[(-Sqrt[2] + 2*Sqrt[c]*x)/
Sqrt[2]])/Sqrt[2] + (b*Sqrt[c]*ArcTan[(Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/Sq
rt[2] - (b*Sqrt[c]*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]) + (b*Sq
rt[c]*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2])
```

3.71.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5361, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^2}dx$$

$$\Big\downarrow \text{5361}$$

$$2bc\int \frac{1}{c^2x^4+1}dx-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$$\Big\downarrow \text{755}$$

$$2bc\left(\frac{1}{2}\int \frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\int \frac{cx^2+1}{c^2x^4+1}dx\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$$\Big\downarrow \text{1476}$$

$$2bc\left(\frac{1}{2}\int \frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$$\Big\downarrow \text{1082}$$

$$2bc\left(\frac{1}{2}\int \frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{c}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{c}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$$\Big\downarrow \text{217}$$

$$2bc\left(\frac{1}{2}\int \frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$\downarrow$ 1479

$$2bc\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right) - \frac{a+b\arctan\left(cx^2\right)}{x}$$

$\downarrow$ 25

$$2bc\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$\downarrow$ 27

$$2bc\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{c}x}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2\sqrt{2}c}+\frac{\int \frac{\sqrt{2}\sqrt{c}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{x}$$

$\downarrow$ 1103

$$2bc\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}-\frac{\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}\right)\right) - \frac{a+b\arctan\left(cx^2\right)}{x}$$

input
```
Int[(a + b*ArcTan[c*x^2])/x^2,x]
```

3.71. $\int \frac{a+b\arctan\left(cx^2\right)}{x^2}\,dx$

output
```
-((a + b*ArcTan[c*x^2])/x) + 2*b*c*((-(ArcTan[1 - Sqrt[2]*Sqrt[c]*x]/(Sqrt
[2]*Sqrt[c])) + ArcTan[1 + Sqrt[2]*Sqrt[c]*x]/(Sqrt[2]*Sqrt[c]))/2 + (-1/2
*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]/(Sqrt[2]*Sqrt[c]) + Log[1 + Sqrt[2]*Sq
rt[c]*x + c*x^2]/(2*Sqrt[2]*Sqrt[c]))/2)
```

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.71.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

method	result
default	$-\frac{a}{x}+b\left(-\frac{\arctan\left(c\,x^{2}\right)}{x}+\frac{c\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4}\right)$
parts	$-\frac{a}{x}+b\left(-\frac{\arctan\left(c\,x^{2}\right)}{x}+\frac{c\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4}\right)$

input
```
int((a+b*arctan(c*x^2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*(-1/x*arctan(c*x^2)+1/4*c*(1/c^2)^(1/4)*2^(1/2)*(ln((x^2+(1/c^2)^(1
/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+
2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)))
```

3.71.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97

$$\int \frac{a + b\arctan(cx^2)}{x^2}\,dx$$
$$= \frac{(-b^4c^2)^{\frac{1}{4}}\,x\log\left(bcx + (-b^4c^2)^{\frac{1}{4}}\right) + i\,(-b^4c^2)^{\frac{1}{4}}\,x\log\left(bcx + i\,(-b^4c^2)^{\frac{1}{4}}\right) - i\,(-b^4c^2)^{\frac{1}{4}}\,x\log\left(bcx - i\,(-b^4c^2\right.}{2\,x}$$

input
```
integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="fricas")
```

output
```
1/2*((-b^4*c^2)^(1/4)*x*log(b*c*x + (-b^4*c^2)^(1/4)) + I*(-b^4*c^2)^(1/4)
*x*log(b*c*x + I*(-b^4*c^2)^(1/4)) - I*(-b^4*c^2)^(1/4)*x*log(b*c*x - I*(-
b^4*c^2)^(1/4)) - (-b^4*c^2)^(1/4)*x*log(b*c*x - (-b^4*c^2)^(1/4)) - 2*b*a
rctan(c*x^2) - 2*a)/x
```

3.71.6 Sympy [A] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int \frac{a + b\arctan(cx^2)}{x^2}\,dx$$
$$= \begin{cases} -\frac{a}{x} - bc\sqrt[4]{-\frac{1}{c^2}}\log\left(x - \sqrt[4]{-\frac{1}{c^2}}\right) + \frac{bc\sqrt[4]{-\frac{1}{c^2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2} + bc\sqrt[4]{-\frac{1}{c^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right) - \frac{b\operatorname{atan}(cx^2)}{\sqrt[4]{-\frac{1}{c^2}}} - \frac{b\operatorname{ata}}{} \\ -\frac{a}{x} \end{cases}$$

input
```
integrate((a+b*atan(c*x**2))/x**2,x)
```

output
```
Piecewise((-a/x - b*c*(-1/c**2)**(1/4)*log(x - (-1/c**2)**(1/4)) + b*c*(-1
/c**2)**(1/4)*log(x**2 + sqrt(-1/c**2))/2 + b*c*(-1/c**2)**(1/4)*atan(x/(-
1/c**2)**(1/4)) - b*atan(c*x**2)/(-1/c**2)**(1/4) - b*atan(c*x**2)/x, Ne(c
, 0)), (-a/x, True))
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

$$\int \frac{a+b\arctan(cx^2)}{x^2}\,dx$$
$$=\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}}-\sqrt{\ }\right.\right.$$
$$-\frac{a}{x}$$

input
```
integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="maxima")
```

output
```
1/4*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sq
rt(c) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sq
rt(c) + sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c
*x^2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c)) - 4*arctan(c*x^2)/x)*b - a/x
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{a+b\arctan(cx^2)}{x^2}\,dx$$
$$=\frac{1}{4}bc\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}\right.}{\sqrt{|c|}}\right.$$
$$-\frac{b\arctan(cx^2)+a}{x}$$

input
```
integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="giac")
```

output
```
1/4*b*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(ab
s(c)))/sqrt(abs(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs
(c))*sqrt(abs(c)))/sqrt(abs(c)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(c)
) + 1/abs(c))/sqrt(abs(c)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/
abs(c))/sqrt(abs(c))) - (b*arctan(c*x^2) + a)/x
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.38

$$\int \frac{a + b\arctan(cx^2)}{x^2}\,dx = -\frac{a}{x} - \frac{b\,\mathrm{atan}(c\,x^2)}{x} - (-1)^{1/4}\,b\,\sqrt{c}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)\,1\mathrm{i} - (-1)^{1/4}\,b\,\sqrt{c}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)$$

input
```
int((a + b*atan(c*x^2))/x^2,x)
```

output
```
- a/x - (b*atan(c*x^2))/x - (-1)^(1/4)*b*c^(1/2)*atan((-1)^(1/4)*c^(1/2)*x
)*1i - (-1)^(1/4)*b*c^(1/2)*atan((-1)^(1/4)*c^(1/2)*x*1i)
```

3.72 $\int \frac{a+b\arctan(cx^2)}{x^4}\,dx$

3.72.1 Optimal result . . . 476
3.72.2 Mathematica [A] (verified) . . . 476
3.72.3 Rubi [A] (verified) . . . 477
3.72.4 Maple [A] (verified) . . . 480
3.72.5 Fricas [C] (verification not implemented) . . . 481
3.72.6 Sympy [C] (verification not implemented) . . . 482
3.72.7 Maxima [A] (verification not implemented) . . . 483
3.72.8 Giac [A] (verification not implemented) . . . 483
3.72.9 Mupad [B] (verification not implemented) . . . 484

3.72.1 Optimal result

Integrand size = 14, antiderivative size = 159

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = -\frac{2bc}{3x}-\frac{a+b\arctan(cx^2)}{3x^3}+\frac{bc^{3/2}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}$$

output
```
-2/3*b*c/x+1/3*(-a-b*arctan(c*x^2))/x^3-1/6*b*c^(3/2)*arctan(-1+x*2^(1/2)*
c^(1/2))*2^(1/2)-1/6*b*c^(3/2)*arctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)-1/12*b*
c^(3/2)*ln(1+c*x^2-x*2^(1/2)*c^(1/2))*2^(1/2)+1/12*b*c^(3/2)*ln(1+c*x^2+x*
2^(1/2)*c^(1/2))*2^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{2bc}{3x}-\frac{b\arctan(cx^2)}{3x^3}-\frac{bc^{3/2}\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{bc^{3/2}\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{bc^{3/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x^4,x]
```

output
```
-1/3*a/x^3 - (2*b*c)/(3*x) - (b*ArcTan[c*x^2])/(3*x^3) - (b*c^(3/2)*ArcTan
[(-Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]) - (b*c^(3/2)*ArcTan[(Sqrt[
2] + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]) - (b*c^(3/2)*Log[1 - Sqrt[2]*Sqrt[
c]*x + c*x^2])/(6*Sqrt[2]) + (b*c^(3/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]
)/(6*Sqrt[2])
```

3.72.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5361, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^4}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{3}bc\int \frac{1}{x^2\left(c^2x^4+1\right)}dx-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{847}$$

$$\frac{2}{3}bc\left(c^2\left(-\int \frac{x^2}{c^2x^4+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{826}$$

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\int \frac{cx^2+1}{c^2x^4+1}dx}{2c}-\frac{\int \frac{1-cx^2}{c^2x^4+1}dx}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}}{2c}-\frac{\int \frac{1-cx^2}{c^2x^4+1}dx}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\int \frac{1}{-\left(1-\sqrt{2}\sqrt{c}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{\int \frac{1}{-\left(\sqrt{2}\sqrt{c}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\int \frac{1-cx^2}{c^2x^4+1}dx}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$\downarrow$ 217

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\int \frac{1-cx^2}{c^2x^4+1}dx}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$\downarrow$ 1479

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{-\frac{\int -\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$\downarrow$ 25

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$\downarrow$ 27

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\frac{\int \frac{\sqrt{2}-2\sqrt{c}x}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2\sqrt{2}c}+\frac{\int \frac{\sqrt{2}\sqrt{c}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

$\downarrow$ 1103

3.72. $\int \frac{a+b\arctan\left(cx^2\right)}{x^4}\,dx$

$$\frac{2}{3}bc\left(-\left(c^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}}{2c}-\frac{\frac{\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}-\frac{\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}}{2c}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^2\right)}{3x^3}$$

input
```
Int[(a + b*ArcTan[c*x^2])/x^4,x]
```

output
```
-1/3*(a + b*ArcTan[c*x^2])/x^3 + (2*b*c*(-x^(-1) - c^2*((-(ArcTan[1 - Sqrt
[2]*Sqrt[c]*x]/(Sqrt[2]*Sqrt[c])) + ArcTan[1 + Sqrt[2]*Sqrt[c]*x]/(Sqrt[2]
*Sqrt[c]))/(2*c) - (-1/2*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]/(Sqrt[2]*Sqrt[
c]) + Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]/(2*Sqrt[2]*Sqrt[c]))/(2*c))))/3
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.72.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.72

method	result
default	$-\frac{a}{3x^3}+b\left(-\frac{\arctan(c\,x^2)}{3x^3}+\frac{2c\left(-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\frac{1}{x}\right)}{3}\right)$
parts	$-\frac{a}{3x^3}+b\left(-\frac{\arctan(c\,x^2)}{3x^3}+\frac{2c\left(-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\frac{1}{x}\right)}{3}\right)$

input
```
int((a+b*arctan(c*x^2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*(-1/3/x^3*arctan(c*x^2)+2/3*c*(-1/8/(1/c^2)^(1/4)*2^(1/2)*(ln
((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+
(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c^
2)^(1/4)*x-1))-1/x))
```

3.72.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx =$$

$$-\frac{(-b^4c^6)^{\frac{1}{4}}\,x^3\log\left(b^3c^5x+(-b^4c^6)^{\frac{3}{4}}\right)-i\,(-b^4c^6)^{\frac{1}{4}}\,x^3\log\left(b^3c^5x+i\,(-b^4c^6)^{\frac{3}{4}}\right)+i\,(-b^4c^6)^{\frac{1}{4}}\,x^3\log\left(b^3c^5x\right.}{6\,x^3}$$

input
```
integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="fricas")
```

output
```
-1/6*((-b^4*c^6)^(1/4)*x^3*log(b^3*c^5*x + (-b^4*c^6)^(3/4)) - I*(-b^4*c^6
)^(1/4)*x^3*log(b^3*c^5*x + I*(-b^4*c^6)^(3/4)) + I*(-b^4*c^6)^(1/4)*x^3*l
og(b^3*c^5*x - I*(-b^4*c^6)^(3/4)) - (-b^4*c^6)^(1/4)*x^3*log(b^3*c^5*x -
(-b^4*c^6)^(3/4)) + 4*b*c*x^2 + 2*b*arctan(c*x^2) + 2*a)/x^3
```

3.72.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.04 (sec) , antiderivative size = 529, normalized size of antiderivative = 3.33

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx$$
$$= \begin{cases} -\frac{a}{3x^3} \\ -\frac{a-\infty ib}{3x^3} \\ -\frac{a+\infty ib}{3x^3} \\ -\frac{2ax^4}{6x^7+\frac{6x^3}{c^2}} - \frac{2a}{6c^2x^7+6x^3} + \frac{2bc^3x^7\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{6x^7+\frac{6x^3}{c^2}} - \frac{bc^3x^7\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6x^7+\frac{6x^3}{c^2}} + \frac{2bc^3x^7\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{6x^7+\frac{6x^3}{c^2}} \end{cases}$$

input
```
integrate((a+b*atan(c*x**2))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3), Eq(c, 0)), (-(a - oo*I*b)/(3*x**3), Eq(c, -I/x**2)
), (-(a + oo*I*b)/(3*x**3), Eq(c, I/x**2)), (-2*a*x**4/(6*x**7 + 6*x**3/c*
*2) - 2*a/(6*c**2*x**7 + 6*x**3) + 2*b*c**3*x**7*(-1/c**2)**(3/4)*log(x -
(-1/c**2)**(1/4))/(6*x**7 + 6*x**3/c**2) - b*c**3*x**7*(-1/c**2)**(3/4)*lo
g(x**2 + sqrt(-1/c**2))/(6*x**7 + 6*x**3/c**2) + 2*b*c**3*x**7*(-1/c**2)**
(3/4)*atan(x/(-1/c**2)**(1/4))/(6*x**7 + 6*x**3/c**2) + 2*b*c**2*x**7*(-1/
c**2)**(1/4)*atan(c*x**2)/(6*x**7 + 6*x**3/c**2) - 4*b*c*x**6/(6*x**7 + 6*
x**3/c**2) + 2*b*c*x**3*(-1/c**2)**(3/4)*log(x - (-1/c**2)**(1/4))/(6*x**7
 + 6*x**3/c**2) - b*c*x**3*(-1/c**2)**(3/4)*log(x**2 + sqrt(-1/c**2))/(6*x
**7 + 6*x**3/c**2) + 2*b*c*x**3*(-1/c**2)**(3/4)*atan(x/(-1/c**2)**(1/4))/
(6*x**7 + 6*x**3/c**2) - 2*b*x**4*atan(c*x**2)/(6*x**7 + 6*x**3/c**2) + 2*
b*x**3*(-1/c**2)**(1/4)*atan(c*x**2)/(6*x**7 + 6*x**3/c**2) - 4*b*x**2/(6*
c*x**7 + 6*x**3/c) - 2*b*atan(c*x**2)/(6*c**2*x**7 + 6*x**3), True))
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx =$$

$$-\frac{1}{12}\left(\left(\left(c^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right.}{c^{\frac{3}{2}}}\right.\right.\right.\right.$$

$$-\frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="maxima")
```

output
```
-1/12*((c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c
))/c^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c
))/c^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*
log(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) + 8/x)*c + 4*arctan(c*x^2)/x^3
)*b - 1/3*a/x^3
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx =$$

$$-\frac{1}{12}\,bc^3\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}+\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}-\frac{\sqrt{2}}{\ }\right.$$

$$-\frac{2\,bcx^2+b\arctan(cx^2)+a}{3\,x^3}$$

input
```
integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="giac")
```

output
```
-1/12*b*c^3*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt
(abs(c)))*sqrt(abs(c)))/c^2 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^2 - sqrt(2)*sqrt(abs(c))*log(x^
2 + sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 + sqrt(2)*sqrt(abs(c))*log(x^2
- sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2) - 1/3*(2*b*c*x^2 + b*arctan(c*x^
2) + a)/x^3
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.40

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = -\frac{2\,b\,c\,x^2+a}{3\,x^3}-\frac{b\,\mathrm{atan}(c\,x^2)}{3\,x^3}-\frac{(-1)^{1/4}\,b\,c^{3/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)}{3} - \frac{(-1)^{1/4}\,b\,c^{3/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{3}$$

input

```
int((a + b*atan(c*x^2))/x^4,x)
```

output

```
- (a + 2*b*c*x^2)/(3*x^3) - (b*atan(c*x^2))/(3*x^3) - ((-1)^(1/4)*b*c^(3/2
)*atan((-1)^(1/4)*c^(1/2)*x))/3 - ((-1)^(1/4)*b*c^(3/2)*atan((-1)^(1/4)*c^
(1/2)*x*1i)*1i)/3
```

3.73 $\int \frac{a+b\arctan(cx^2)}{x^6}\,dx$

3.73.1 Optimal result . . . 485
3.73.2 Mathematica [A] (verified) . . . 485
3.73.3 Rubi [A] (verified) . . . 486
3.73.4 Maple [A] (verified) . . . 489
3.73.5 Fricas [C] (verification not implemented) . . . 490
3.73.6 Sympy [A] (verification not implemented) . . . 491
3.73.7 Maxima [A] (verification not implemented) . . . 491
3.73.8 Giac [A] (verification not implemented) . . . 492
3.73.9 Mupad [B] (verification not implemented) . . . 492

3.73.1 Optimal result

Integrand size = 14, antiderivative size = 159

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{2bc}{15x^3} - \frac{a+b\arctan(cx^2)}{5x^5} + \frac{bc^{5/2}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}} - \frac{bc^{5/2}\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}} + \frac{bc^{5/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}} - \frac{bc^{5/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}}$$

output
```
-2/15*b*c/x^3+1/5*(-a-b*arctan(c*x^2))/x^5-1/10*b*c^(5/2)*arctan(-1+x*2^(1
/2)*c^(1/2))*2^(1/2)-1/10*b*c^(5/2)*arctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)+1/
20*b*c^(5/2)*ln(1+c*x^2-x*2^(1/2)*c^(1/2))*2^(1/2)-1/20*b*c^(5/2)*ln(1+c*x
^2+x*2^(1/2)*c^(1/2))*2^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{a}{5x^5} - \frac{2bc}{15x^3} - \frac{b\arctan(cx^2)}{5x^5} - \frac{bc^{5/2}\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{5\sqrt{2}} - \frac{bc^{5/2}\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{5\sqrt{2}} + \frac{bc^{5/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}} - \frac{bc^{5/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])/x^6,x]
```

output
```
-1/5*a/x^5 - (2*b*c)/(15*x^3) - (b*ArcTan[c*x^2])/(5*x^5) - (b*c^(5/2)*Arc
Tan[(-Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]) - (b*c^(5/2)*ArcTan[(Sq
rt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]) + (b*c^(5/2)*Log[1 - Sqrt[2]*Sq
rt[c]*x + c*x^2])/(10*Sqrt[2]) - (b*c^(5/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*
x^2])/(10*Sqrt[2])
```

3.73.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5361, 847, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^6}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{5}bc\int \frac{1}{x^4\left(c^2x^4+1\right)}dx-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \text{847}$$

$$\frac{2}{5}bc\left(c^2\left(-\int \frac{1}{c^2x^4+1}dx\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \text{755}$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\int \frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\int \frac{cx^2+1}{c^2x^4+1}dx\right)\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\int \frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}\right)\right)\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\int\frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\int\frac{1}{-\left(1-\sqrt{2}\sqrt{c}x\right)^2-1}d\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}-\frac{\int\frac{1}{-\left(\sqrt{2}\sqrt{c}x+1\right)^2-1}d\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}\right)\right)\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \; 217$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\int\frac{1-cx^2}{c^2x^4+1}dx+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)\right)-\frac{1}{3x^3}\right)-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \; 1479$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \; 25$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{c}x}{\sqrt{c}\left(x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}\left(x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}\right)}dx}{2\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{c}x}{x^2-\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2\sqrt{2}c}+\frac{\int\frac{\sqrt{2}\sqrt{c}x+1}{x^2+\frac{\sqrt{2}x}{\sqrt{c}}+\frac{1}{c}}dx}{2c}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)\right)\right)\right)-\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

$$\Big\downarrow \; 1103$$

3.73. $\int \frac{a+b\arctan\left(cx^2\right)}{x^6}\,dx$

$$\frac{2}{5}bc\left(-\left(c^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\right)+\frac{1}{2}\left(\frac{\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}-\frac{\log\left(cx^2-\sqrt{2}\sqrt{c}\right.}{2\sqrt{2}\sqrt{c}}\right.\right.\right.\right.$$
$$\frac{a+b\arctan\left(cx^2\right)}{5x^5}$$

input
```
Int[(a + b*ArcTan[c*x^2])/x^6,x]
```

output
```
-1/5*(a + b*ArcTan[c*x^2])/x^5 + (2*b*c*(-1/3*1/x^3 - c^2*((-(ArcTan[1 - S
qrt[2]*Sqrt[c]*x]/(Sqrt[2]*Sqrt[c])) + ArcTan[1 + Sqrt[2]*Sqrt[c]*x]/(Sqrt
[2]*Sqrt[c]))/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]/(Sqrt[2]*Sqrt[c
]) + Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]/(2*Sqrt[2]*Sqrt[c]))/2)))/5
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.73.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

method	result
default	$-\frac{a}{5x^5}+b\left(-\frac{\arctan(c\,x^2)}{5x^5}+\frac{2c\left(-\frac{c^2\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8}-\right.}{5}\right.$
parts	$-\frac{a}{5x^5}+b\left(-\frac{\arctan(c\,x^2)}{5x^5}+\frac{2c\left(-\frac{c^2\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8}-\right.}{5}\right.$

input
```
int((a+b*arctan(c*x^2))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5+b*(-1/5/x^5*arctan(c*x^2)+2/5*c*(-1/8*c^2*(1/c^2)^(1/4)*2^(1/2)
*(ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1
/2)+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(
1/c^2)^(1/4)*x-1))-1/3/x^3))
```

3.73.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx =$$

$$-\frac{3\left(-b^4c^{10}\right)^{\frac{1}{4}}x^5\log\left(bc^3x+\left(-b^4c^{10}\right)^{\frac{1}{4}}\right)+3i\left(-b^4c^{10}\right)^{\frac{1}{4}}x^5\log\left(bc^3x+i\left(-b^4c^{10}\right)^{\frac{1}{4}}\right)-3i\left(-b^4c^{10}\right)^{\frac{1}{4}}x^5\log}{30\,x^5}$$

input
```
integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="fricas")
```

output
```
-1/30*(3*(-b^4*c^10)^(1/4)*x^5*log(b*c^3*x + (-b^4*c^10)^(1/4)) + 3*I*(-b^
4*c^10)^(1/4)*x^5*log(b*c^3*x + I*(-b^4*c^10)^(1/4)) - 3*I*(-b^4*c^10)^(1/
4)*x^5*log(b*c^3*x - I*(-b^4*c^10)^(1/4)) - 3*(-b^4*c^10)^(1/4)*x^5*log(b*
c^3*x - (-b^4*c^10)^(1/4)) + 4*b*c*x^2 + 6*b*arctan(c*x^2) + 6*a)/x^5
```

3.73.6 Sympy [A] (verification not implemented)

Time = 29.70 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97

$$\int \frac{a + b\arctan(cx^2)}{x^6}\,dx$$
$$= \begin{cases} -\frac{a}{5x^5} + \frac{bc^3\sqrt[4]{-\frac{1}{c^2}}\log\left(x - \sqrt[4]{-\frac{1}{c^2}}\right)}{5} - \frac{bc^3\sqrt[4]{-\frac{1}{c^2}}\log\left(x^2 + \sqrt{-\frac{1}{c^2}}\right)}{10} - \frac{bc^3\sqrt[4]{-\frac{1}{c^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{5} + \frac{bc^2\operatorname{atan}(cx^2)}{5\sqrt[4]{-\frac{1}{c^2}}} - \frac{2bc}{15x^3} \\ -\frac{a}{5x^5} \end{cases}$$

input
```
integrate((a+b*atan(c*x**2))/x**6,x)
```

output
```
Piecewise((-a/(5*x**5) + b*c**3*(-1/c**2)**(1/4)*log(x - (-1/c**2)**(1/4))
/5 - b*c**3*(-1/c**2)**(1/4)*log(x**2 + sqrt(-1/c**2))/10 - b*c**3*(-1/c**
2)**(1/4)*atan(x/(-1/c**2)**(1/4))/5 + b*c**2*atan(c*x**2)/(5*(-1/c**2)**(
1/4)) - 2*b*c/(15*x**3) - b*atan(c*x**2)/(5*x**5), Ne(c, 0)), (-a/(5*x**5)
, True))
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.87

$$\int \frac{a + b\arctan(cx^2)}{x^6}\,dx =$$
$$-\frac{1}{60}\left(\left(6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}(2cx + \sqrt{2}\sqrt{c})}{2\sqrt{c}}\right) + 6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}(2cx - \sqrt{2}\sqrt{c})}{2\sqrt{c}}\right) + 3\sqrt{2}c^{\frac{3}{2}}\log\left(cx^2\right.\right.\right.$$
$$-\frac{a}{5x^5}$$

input
```
integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="maxima")
```

output
```
-1/60*((6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqr
t(c)) + 6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqr
t(c)) + 3*sqrt(2)*c^(3/2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1) - 3*sqrt(2)*c
^(3/2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1) + 8/x^3)*c + 12*arctan(c*x^2)/x^
5)*b - 1/5*a/x^5
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{1}{20}\,bc^3\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\sqrt{2}\log\left(x^2+\right.}{\sqrt{|}}\right. - \frac{2\,bcx^2+3\,b\arctan(cx^2)+3\,a}{15\,x^5}$$

input
```
integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="giac")
```

output
```
-1/20*b*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqr
t(abs(c)))/sqrt(abs(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt
(abs(c)))*sqrt(abs(c)))/sqrt(abs(c)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(ab
s(c)) + 1/abs(c))/sqrt(abs(c)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(c))
+ 1/abs(c))/sqrt(abs(c))) - 1/15*(2*b*c*x^2 + 3*b*arctan(c*x^2) + 3*a)/x^5
```

3.73.9 Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.40

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{\frac{2\,b\,c\,x^2}{3}+a}{5\,x^5}-\frac{b\,\mathrm{atan}(c\,x^2)}{5\,x^5}+\frac{(-1)^{1/4}\,b\,c^{5/2}\,\mathrm{atan}\left((-1)^{1/4}\sqrt{c}\,x\right)\,1\mathrm{i}}{5}+\frac{(-1)^{1/4}\,b\,c^{5/2}\,\mathrm{atan}\left((-1)^{1/4}\sqrt{c}\,x\,1\mathrm{i}\right)}{5}$$

input
```
int((a + b*atan(c*x^2))/x^6,x)
```

output
```
((-1)^(1/4)*b*c^(5/2)*atan((-1)^(1/4)*c^(1/2)*x)*1i)/5 - (b*atan(c*x^2))/(
5*x^5) - (a + (2*b*c*x^2)/3)/(5*x^5) + ((-1)^(1/4)*b*c^(5/2)*atan((-1)^(1/
4)*c^(1/2)*x*1i))/5
```

3.73. $\int \frac{a+b\arctan(cx^2)}{x^6}\,dx$

3.74 $\int x^7(a+b\arctan(cx^2))^2\,dx$

3.74.1 Optimal result 494
3.74.2 Mathematica [A] (verified) 494
3.74.3 Rubi [A] (verified) 495
3.74.4 Maple [A] (verified) 498
3.74.5 Fricas [A] (verification not implemented) 498
3.74.6 Sympy [A] (verification not implemented) 499
3.74.7 Maxima [A] (verification not implemented) 499
3.74.8 Giac [A] (verification not implemented) 500
3.74.9 Mupad [B] (verification not implemented) 500

3.74.1 Optimal result

Integrand size = 16, antiderivative size = 124

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\frac{abx^2}{4c^3}+\frac{b^2x^4}{24c^2}+\frac{b^2x^2\arctan\left(cx^2\right)}{4c^3}-\frac{bx^6(a+b\arctan\left(cx^2\right))}{12c}-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{8c^4}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{b^2\log\left(1+c^2x^4\right)}{6c^4}$$

output
```
1/4*a*b*x^2/c^3+1/24*b^2*x^4/c^2+1/4*b^2*x^2*arctan(c*x^2)/c^3-1/12*b*x^6*
(a+b*arctan(c*x^2))/c-1/8*(a+b*arctan(c*x^2))^2/c^4+1/8*x^8*(a+b*arctan(c*
x^2))^2-1/6*b^2*ln(c^2*x^4+1)/c^4
```

3.74.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\frac{cx^2(6ab+b^2cx^2-2abc^2x^4+3a^2c^3x^6)-2b(bcx^2(-3+c^2x^4)+a(3-3c^4x^8))\arctan\left(cx^2\right)+3b^2(-1+c^4x}{24c^4}$$

input
```
Integrate[x^7*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
(c*x^2*(6*a*b + b^2*c*x^2 - 2*a*b*c^2*x^4 + 3*a^2*c^3*x^6) - 2*b*(b*c*x^2*
(-3 + c^2*x^4) + a*(3 - 3*c^4*x^8))*ArcTan[c*x^2] + 3*b^2*(-1 + c^4*x^8)*A
rcTan[c*x^2]^2 - 4*b^2*Log[1 + c^2*x^4])/(24*c^4)
```

3.74.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5363, 5361, 5451, 5361, 243, 49, 2009, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+b \arctan \left(c x^2\right)\right)^2 dx$$

$$\Big\downarrow \text{ 5363}$$

$$\frac{1}{2} \int x^6 \left(a+b \arctan \left(c x^2\right)\right)^2 dx^2$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{2} \left(\frac{1}{4} x^8 \left(a+b \arctan \left(c x^2\right)\right)^2-\frac{1}{2} b c \int \frac{x^8 \left(a+b \arctan \left(c x^2\right)\right)}{c^2 x^4+1} dx^2\right)$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{1}{2} \left(\frac{1}{4} x^8 \left(a+b \arctan \left(c x^2\right)\right)^2-\frac{1}{2} b c \left(\frac{\int x^4 \left(a+b \arctan \left(c x^2\right)\right) dx^2}{c^2}-\frac{\int \frac{x^4 \left(a+b \arctan \left(c x^2\right)\right)}{c^2 x^4+1} dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{2} \left(\frac{1}{4} x^8 \left(a+b \arctan \left(c x^2\right)\right)^2-\frac{1}{2} b c \left(\frac{\frac{1}{3} x^6 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{3} b c \int \frac{x^6}{c^2 x^4+1} dx^2}{c^2}-\frac{\int \frac{x^4 \left(a+b \arctan \left(c x^2\right)\right)}{c^2 x^4+1} dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \left(\frac{1}{4} x^8 \left(a+b \arctan \left(c x^2\right)\right)^2-\frac{1}{2} b c \left(\frac{\frac{1}{3} x^6 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{6} b c \int \frac{x^4}{c^2 x^4+1} dx^4}{c^2}-\frac{\int \frac{x^4 \left(a+b \arctan \left(c x^2\right)\right)}{c^2 x^4+1} dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{6}bc\int\left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^4+1)}\right)dx^4}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx^2))}{c^2x^4+1}}{c^2}\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{6}bc\left(\frac{x^4}{c^2}-\frac{\log(c^2x^4+1)}{c^4}\right)}{c^2}-\frac{\int\frac{x^4(a+b\arctan(cx^2))}{c^2x^4+1}dx^2}{c^2}\right.\right.$$

$\downarrow$ 5451

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{6}bc\left(\frac{x^4}{c^2}-\frac{\log(c^2x^4+1)}{c^4}\right)}{c^2}-\frac{\frac{\int(a+b\arctan(cx^2))dx^2}{c^2}-\int}{c^2}\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{6}bc\left(\frac{x^4}{c^2}-\frac{\log(c^2x^4+1)}{c^4}\right)}{c^2}-\frac{\frac{ax^2+bx^2\arctan(cx^2)-\frac{b\log(c}{2}}{c^2}}{}\right.\right.$$

$\downarrow$ 5419

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{6}bc\left(\frac{x^4}{c^2}-\frac{\log(c^2x^4+1)}{c^4}\right)}{c^2}-\frac{\frac{ax^2+bx^2\arctan(cx^2)-\frac{b\log(c}{2}}{c^2}}{c}\right.\right.$$

input
```
Int[x^7*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
((x^8*(a + b*ArcTan[c*x^2])^2)/4 - (b*c*(((x^6*(a + b*ArcTan[c*x^2]))/3 -
(b*c*(x^4/c^2 - Log[1 + c^2*x^4]/c^4))/6)/c^2 - (-1/2*(a + b*ArcTan[c*x^2]
)^2/(b*c^3) + (a*x^2 + b*x^2*ArcTan[c*x^2] - (b*Log[1 + c^2*x^4])/(2*c))/c
^2)/c^2))/2)/2
```

3.74. $\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$

3.74.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.74. $\int x^7 (a + b \arctan(cx^2))^2 \, dx$

3.74.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

method	result
default	$\frac{a^2x^8}{8}+\frac{b^2x^8\arctan(cx^2)^2}{8}-\frac{b^2\arctan(cx^2)x^6}{12c}+\frac{b^2x^2\arctan(cx^2)}{4c^3}-\frac{b^2\arctan(cx^2)^2}{8c^4}+\frac{b^2x^4}{24c^2}-\frac{b^2\ln(c^2x^4+1)}{6c^4}+$
parts	$\frac{a^2x^8}{8}+\frac{b^2x^8\arctan(cx^2)^2}{8}-\frac{b^2\arctan(cx^2)x^6}{12c}+\frac{b^2x^2\arctan(cx^2)}{4c^3}-\frac{b^2\arctan(cx^2)^2}{8c^4}+\frac{b^2x^4}{24c^2}-\frac{b^2\ln(c^2x^4+1)}{6c^4}+$
parallelrisch	$-\frac{-3b^2\arctan(cx^2)^2x^8c^4-6ab\arctan(cx^2)x^8c^4-3c^4a^2x^8+2b^2\arctan(cx^2)x^6c^3+2abc^3x^6-x^4b^2c^2-6b^2\arctan(cx^2)x^2c-6}{24c^4}$
risch	$-\frac{b^2(c^4x^8-1)\ln(icx^2+1)^2}{32c^4}-\frac{ib(6ac^4x^8+3ibc^4x^8\ln(-icx^2+1)-2bc^3x^6+6bcx^2-3ib\ln(-icx^2+1))\ln(icx^2+1)}{48c^4}-\frac{b^2x^8}{}$

input
```
int(x^7*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^2*x^8+1/8*b^2*x^8*arctan(c*x^2)^2-1/12*b^2*arctan(c*x^2)/c*x^6+1/4*b
^2*x^2*arctan(c*x^2)/c^3-1/8*b^2/c^4*arctan(c*x^2)^2+1/24*b^2*x^4/c^2-1/6*
b^2*ln(c^2*x^4+1)/c^4+1/4*a*b*x^8*arctan(c*x^2)-1/12*a*b/c*x^6+1/4*a*b*x^2
/c^3-1/4*a*b/c^4*arctan(c*x^2)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\frac{3\,a^2c^4x^8-2\,abc^3x^6+b^2c^2x^4+6\,abcx^2+3\,(b^2c^4x^8-b^2)\arctan\left(cx^2\right)^2+6\,ab\arctan\left(\frac{1}{cx^2}\right)-4\,b^2\log\left(c^2x^4\right.}{24\,c^4}$$

input
```
integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
1/24*(3*a^2*c^4*x^8 - 2*a*b*c^3*x^6 + b^2*c^2*x^4 + 6*a*b*c*x^2 + 3*(b^2*c
^4*x^8 - b^2)*arctan(c*x^2)^2 + 6*a*b*arctan(1/(c*x^2)) - 4*b^2*log(c^2*x^
4 + 1) + 2*(3*a*b*c^4*x^8 - b^2*c^3*x^6 + 3*b^2*c*x^2)*arctan(c*x^2))/c^4
```

3.74.6 Sympy [A] (verification not implemented)

Time = 44.49 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.60

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\begin{cases}\frac{a^2x^8}{8}+\frac{abx^8\operatorname{atan}\left(cx^2\right)}{4}-\frac{abx^6}{12c}+\frac{abx^2}{4c^3}-\frac{ab\operatorname{atan}\left(cx^2\right)}{4c^4}+\frac{b^2x^8\operatorname{atan}^2\left(cx^2\right)}{8}-\frac{b^2x^6\operatorname{atan}\left(cx^2\right)}{12c}+\frac{b^2x^4}{24c^2}+\frac{b^2x^2\operatorname{atan}\left(cx^2\right)}{4c^3}-\frac{b^2\sqrt{}}{} \\ \frac{a^2x^8}{8}\end{cases}$$

input
```
integrate(x**7*(a+b*atan(c*x**2))**2,x)
```

output
```
Piecewise((a**2*x**8/8 + a*b*x**8*atan(c*x**2)/4 - a*b*x**6/(12*c) + a*b*x
**2/(4*c**3) - a*b*atan(c*x**2)/(4*c**4) + b**2*x**8*atan(c*x**2)**2/8 - b
**2*x**6*atan(c*x**2)/(12*c) + b**2*x**4/(24*c**2) + b**2*x**2*atan(c*x**2
)/(4*c**3) - b**2*sqrt(-1/c**2)*atan(c*x**2)/(3*c**3) - b**2*log(x**2 + sq
rt(-1/c**2))/(3*c**4) - b**2*atan(c*x**2)**2/(8*c**4), Ne(c, 0)), (a**2*x*
*8/8, True))
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.36

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\frac{1}{8}\,b^2x^8\arctan\left(cx^2\right)^2+\frac{1}{8}\,a^2x^8$$
$$+\frac{1}{12}\left(3\,x^8\arctan\left(cx^2\right)-c\left(\frac{c^2x^6-3\,x^2}{c^4}+\frac{3\,\arctan\left(cx^2\right)}{c^5}\right)\right)ab$$
$$-\frac{1}{24}\left(2\,c\left(\frac{c^2x^6-3\,x^2}{c^4}+\frac{3\,\arctan\left(cx^2\right)}{c^5}\right)\arctan\left(cx^2\right)-\frac{c^2x^4+3\,\arctan\left(cx^2\right)^2-3\,\log\left(12\,c^7x^4+12\,c^5\right)}{c^4}\right.$$

input
```
integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/8*b^2*x^8*arctan(c*x^2)^2 + 1/8*a^2*x^8 + 1/12*(3*x^8*arctan(c*x^2) - c*
((c^2*x^6 - 3*x^2)/c^4 + 3*arctan(c*x^2)/c^5))*a*b - 1/24*(2*c*((c^2*x^6 -
 3*x^2)/c^4 + 3*arctan(c*x^2)/c^5)*arctan(c*x^2) - (c^2*x^4 + 3*arctan(c*x
^2)^2 - 3*log(12*c^7*x^4 + 12*c^5) - log(c^2*x^4 + 1))/c^4)*b^2
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int x^7 \left(a + b \arctan \left(cx^2\right)\right)^2 \, dx = \frac{3\,a^2 c x^8 + 2\left(3\,cx^8 \arctan\left(cx^2\right) - \frac{3\arctan\left(cx^2\right)}{c^3} - \frac{c^9x^6 - 3\,c^7x^2}{c^9}\right)ab + \left(3\,cx^8\arctan\left(cx^2\right)^2 - \frac{2\,c^3x^6\arctan\left(cx^2\right) - c^2x^4 \cdots}{\cdots}\right.}{24\,c}$$

input
```
integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
1/24*(3*a^2*c*x^8 + 2*(3*c*x^8*arctan(c*x^2) - 3*arctan(c*x^2)/c^3 - (c^9*
x^6 - 3*c^7*x^2)/c^9)*a*b + (3*c*x^8*arctan(c*x^2)^2 - (2*c^3*x^6*arctan(c
*x^2) - c^2*x^4 - 6*c*x^2*arctan(c*x^2) + 3*arctan(c*x^2)^2 + 4*log(c^2*x^
4 + 1))/c^3)*b^2)/c
```

3.74.9 Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\begin{aligned}\int x^7 \left(a + b \arctan \left(cx^2\right)\right)^2 \, dx &= \frac{a^2\,x^8}{8} - \frac{b^2\,\mathrm{atan}(c\,x^2)^2}{8\,c^4} + \frac{b^2\,x^8\,\mathrm{atan}(c\,x^2)^2}{8} - \frac{b^2\,\ln\left(c^2\,x^4+1\right)}{6\,c^4} \\ &\quad + \frac{b^2\,x^4}{24\,c^2} + \frac{b^2\,x^2\,\mathrm{atan}(c\,x^2)}{4\,c^3} - \frac{b^2\,x^6\,\mathrm{atan}(c\,x^2)}{12\,c} \\ &\quad + \frac{a\,b\,x^2}{4\,c^3} - \frac{a\,b\,x^6}{12\,c} - \frac{a\,b\,\mathrm{atan}(c\,x^2)}{4\,c^4} + \frac{a\,b\,x^8\,\mathrm{atan}(c\,x^2)}{4}\end{aligned}$$

input
```
int(x^7*(a + b*atan(c*x^2))^2,x)
```

output
```
(a^2*x^8)/8 - (b^2*atan(c*x^2)^2)/(8*c^4) + (b^2*x^8*atan(c*x^2)^2)/8 - (b
^2*log(c^2*x^4 + 1))/(6*c^4) + (b^2*x^4)/(24*c^2) + (b^2*x^2*atan(c*x^2))/
(4*c^3) - (b^2*x^6*atan(c*x^2))/(12*c) + (a*b*x^2)/(4*c^3) - (a*b*x^6)/(12
*c) - (a*b*atan(c*x^2))/(4*c^4) + (a*b*x^8*atan(c*x^2))/4
```

3.75 $\int x^5(a+b\arctan(cx^2))^2\,dx$

3.75.1 Optimal result . . . 501
3.75.2 Mathematica [A] (verified) . . . 501
3.75.3 Rubi [A] (verified) . . . 502
3.75.4 Maple [C] (warning: unable to verify) . . . 505
3.75.5 Fricas [F] . . . 506
3.75.6 Sympy [F] . . . 506
3.75.7 Maxima [F] . . . 507
3.75.8 Giac [F] . . . 507
3.75.9 Mupad [F(-1)] . . . 507

3.75.1 Optimal result

Integrand size = 16, antiderivative size = 154

$$\begin{aligned}\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=&\frac{b^2x^2}{6c^2}-\frac{b^2\arctan\left(cx^2\right)}{6c^3}-\frac{bx^4(a+b\arctan(cx^2))}{6c}\\&-\frac{i(a+b\arctan(cx^2))^2}{6c^3}+\frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)^2\\&-\frac{b(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{3c^3}\\&-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{6c^3}\end{aligned}$$

output
```
1/6*b^2*x^2/c^2-1/6*b^2*arctan(c*x^2)/c^3-1/6*b*x^4*(a+b*arctan(c*x^2))/c-
1/6*I*(a+b*arctan(c*x^2))^2/c^3+1/6*x^6*(a+b*arctan(c*x^2))^2-1/3*b*(a+b*a
rctan(c*x^2))*ln(2/(1+I*c*x^2))/c^3-1/6*I*b^2*polylog(2,1-2/(1+I*c*x^2))/c
^3
```

3.75.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$

$$=\frac{b^2cx^2-abc^2x^4+a^2c^3x^6+b^2(i+c^3x^6)\arctan\left(cx^2\right)^2-b\arctan\left(cx^2\right)\left(b+bc^2x^4-2ac^3x^6+2b\log\left(1+e\right.\right.}{6c^3}$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
(b^2*c*x^2 - a*b*c^2*x^4 + a^2*c^3*x^6 + b^2*(I + c^3*x^6)*ArcTan[c*x^2]^2
 - b*ArcTan[c*x^2]*(b + b*c^2*x^4 - 2*a*c^3*x^6 + 2*b*Log[1 + E^((2*I)*Arc
Tan[c*x^2])]) + a*b*Log[1 + c^2*x^4] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c
*x^2])])/(6*c^3)
```

3.75.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5363, 5361, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b \arctan \left(c x^2\right)\right)^2 \, dx$$

$$\Big\downarrow \text{ 5363}$$

$$\frac{1}{2} \int x^4 \left(a+b \arctan \left(c x^2\right)\right)^2 dx^2$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{2} \left(\frac{1}{3} x^6 \left(a+b \arctan \left(c x^2\right)\right)^2-\frac{2}{3} b c \int \frac{x^6 \left(a+b \arctan \left(c x^2\right)\right)}{c^2 x^4+1}dx^2\right)$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{1}{2} \left(\frac{1}{3} x^6 \left(a+b \arctan \left(c x^2\right)\right)^2-\frac{2}{3} b c \left(\frac{\int x^2 \left(a+b \arctan \left(c x^2\right)\right)dx^2}{c^2}-\frac{\int \frac{x^2 \left(a+b \arctan \left(c x^2\right)\right)}{c^2 x^4+1}dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{2} \left(\frac{1}{3} x^6 \left(a+b \arctan \left(c x^2\right)\right)^2-\frac{2}{3} b c \left(\frac{\frac{1}{2} x^4 \left(a+b \arctan \left(c x^2\right)\right)-\frac{1}{2} b c \int \frac{x^4}{c^2 x^4+1}dx^2}{c^2}-\frac{\int \frac{x^2 \left(a+b \arctan \left(c x^2\right)\right)}{c^2 x^4+1}dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\left(\frac{x^2}{c^2}-\frac{\int\frac{1}{c^2x^4+1}dx^2}{c^2}\right)}{c^2}-\frac{\int\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2}{c^2}\right.\right.$$

$\downarrow$ 216

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)}{c^2}-\frac{\int\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2}{c^2}\right)\right.$$

$\downarrow$ 5455

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)}{c^2}-\frac{-\frac{\int\frac{a+b\arctan\left(cx^2\right)}{i-cx^2}dx^2}{c}-\frac{i(}{}}{c^2}\right.\right.$$

$\downarrow$ 5379

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)}{c^2}-\frac{-\frac{\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right.\right.}{c}}{c}}{}\right.\right.$$

$\downarrow$ 2849

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)}{c^2}-\frac{-\frac{ib\int\frac{\log\left(\frac{2}{icx^2+1}\right)}{1-\frac{2}{icx^2+1}}d\frac{1}{icx^2+1}}{c}+}{}\right.\right.$$

$\downarrow$ 2752

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}bc\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)}{c^2}-\frac{-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^2}-\frac{\log}{}}{}\right.\right.$$

input
```
Int[x^5*(a + b*ArcTan[c*x^2])^2,x]
```

3.75. $\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$

output
```
((x^6*(a + b*ArcTan[c*x^2])^2)/3 - (2*b*c*(((x^4*(a + b*ArcTan[c*x^2]))/2
- (b*c*(x^2/c^2 - ArcTan[c*x^2]/c^3))/2)/c^2 - (((-1/2*I)*(a + b*ArcTan[c*
x^2])^2)/(b*c^2) - (((a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/c + ((I/2
)*b*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c)/c)/c^2))/3)/2
```

3.75.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.75.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.56 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.16

method	result
default	$\frac{x^6a^2}{6}+\frac{b^2x^6\arctan(cx^2)^2}{6}-\frac{b^2\arctan(cx^2)x^4}{6c}+\frac{b^2\arctan(cx^2)\ln(c^2x^4+1)}{6c^3}+\frac{b^2x^2}{6c^2}-\frac{b^2\arctan(cx^2)}{6c^3}-\frac{b^2\left(\sum_{\_\alpha=\mathrm{RootOf}} \right.}{}$
parts	$\frac{x^6a^2}{6}+\frac{b^2x^6\arctan(cx^2)^2}{6}-\frac{b^2\arctan(cx^2)x^4}{6c}+\frac{b^2\arctan(cx^2)\ln(c^2x^4+1)}{6c^3}+\frac{b^2x^2}{6c^2}-\frac{b^2\arctan(cx^2)}{6c^3}-\frac{b^2\left(\sum_{\_\alpha=\mathrm{RootOf}} \right.}{}$
risch	$-\frac{ib^2\ln\left(\frac{1}{2}+\frac{icx^2}{2}\right)\ln\left(\frac{1}{2}-\frac{icx^2}{2}\right)}{6c^3}+\frac{ab\ln(c^2x^4+1)}{6c^3}-\frac{abx^4}{6c}+\frac{ib^2x^4\ln(icx^2+1)}{12c}-\frac{b^2x^6\ln(icx^2+1)^2}{24}-\frac{b^2\arctan(cx^2)}{12c^3}+$

input
```
int(x^5*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a^2+1/6*b^2*x^6*arctan(c*x^2)^2-1/6*b^2*arctan(c*x^2)/c*x^4+1/6*b^
2*arctan(c*x^2)/c^3*ln(c^2*x^4+1)+1/6*b^2*x^2/c^2-1/6*b^2*arctan(c*x^2)/c^
3-1/24*b^2/c^4*sum(1/_alpha^2*(2*ln(x-_alpha)*ln(c^2*x^4+1)-c*(1/c/_alpha^
3*ln(x-_alpha)^2+2/_alpha*ln(x-_alpha)*(_alpha^2*ln(1/2*(x+_alpha)/_alpha)
*c-ln((_alpha^3*c+x)/_alpha/(_alpha^2*c+1))+ln((_alpha^3*c-x)/_alpha/(_alp
ha^2*c-1)))+2/_alpha*(_alpha^2*dilog(1/2*(x+_alpha)/_alpha)*c-dilog((_alph
a^3*c+x)/_alpha/(_alpha^2*c+1))+dilog((_alpha^3*c-x)/_alpha/(_alpha^2*c-1)
)))),_alpha=RootOf(_Z^4*c^2+1))+1/3*a*b*x^6*arctan(c*x^2)-1/6/c*a*b*x^4+1/
6*a*b/c^3*ln(c^2*x^4+1)
```

3.75.5 Fricas [F]

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^2x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^5*arctan(c*x^2)^2 + 2*a*b*x^5*arctan(c*x^2) + a^2*x^5, x)
```

3.75.6 Sympy [F]

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int x^5\left(a+b\operatorname{atan}\left(cx^2\right)\right)^2\,dx$$

input
```
integrate(x**5*(a+b*atan(c*x**2))**2,x)
```

output
```
Integral(x**5*(a + b*atan(c*x**2))**2, x)
```

3.75.7 Maxima [F]

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^2x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/6*a^2*x^6 + 1/6*(2*x^6*arctan(c*x^2) - (x^4/c^2 - log(c^2*x^4 + 1)/c^4)*
c)*a*b + 1/96*(4*x^6*arctan(c*x^2)^2 - x^6*log(c^2*x^4 + 1)^2 + 96*integra
te(1/48*(4*c^2*x^9*log(c^2*x^4 + 1) - 8*c*x^7*arctan(c*x^2) + 36*(c^2*x^9
+ x^5)*arctan(c*x^2)^2 + 3*(c^2*x^9 + x^5)*log(c^2*x^4 + 1)^2)/(c^2*x^4 +
1), x))*b^2
```

3.75.8 Giac [F]

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^2x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2*x^5, x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int x^5\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,dx$$

input
```
int(x^5*(a + b*atan(c*x^2))^2,x)
```

output
```
int(x^5*(a + b*atan(c*x^2))^2, x)
```

3.76 $\int x^3(a+b\arctan(cx^2))^2\,dx$

3.76.1 Optimal result 508
3.76.2 Mathematica [A] (verified) 508
3.76.3 Rubi [A] (verified) 509
3.76.4 Maple [A] (verified) 510
3.76.5 Fricas [A] (verification not implemented) 511
3.76.6 Sympy [A] (verification not implemented) 511
3.76.7 Maxima [A] (verification not implemented) 512
3.76.8 Giac [A] (verification not implemented) 512
3.76.9 Mupad [B] (verification not implemented) 513

3.76.1 Optimal result

Integrand size = 16, antiderivative size = 90

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=-\frac{abx^2}{2c}-\frac{b^2x^2\arctan\left(cx^2\right)}{2c}+\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{4c^2}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)^2+\frac{b^2\log\left(1+c^2x^4\right)}{4c^2}$$

output
```
-1/2*a*b*x^2/c-1/2*b^2*x^2*arctan(c*x^2)/c+1/4*(a+b*arctan(c*x^2))^2/c^2+1
/4*x^4*(a+b*arctan(c*x^2))^2+1/4*b^2*ln(c^2*x^4+1)/c^2
```

3.76.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\frac{acx^2(-2b+acx^2)+2b(a-bcx^2+ac^2x^4)\arctan(cx^2)+b^2(1+c^2x^4)\arctan(cx^2)^2+b^2\log(1+c^2x^4)}{4c^2}$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
(a*c*x^2*(-2*b + a*c*x^2) + 2*b*(a - b*c*x^2 + a*c^2*x^4)*ArcTan[c*x^2] +
b^2*(1 + c^2*x^4)*ArcTan[c*x^2]^2 + b^2*Log[1 + c^2*x^4])/(4*c^2)
```

3.76.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5363, 5361, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\arctan\left(cx^2\right)\right)^2 dx$$
$$\Big\downarrow \text{5363}$$
$$\frac{1}{2}\int x^2 \left(a+b\arctan\left(cx^2\right)\right)^2 dx^2$$
$$\Big\downarrow \text{5361}$$
$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^2-bc\int \frac{x^4\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2\right)$$
$$\Big\downarrow \text{5451}$$
$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^2-bc\left(\frac{\int \left(a+b\arctan\left(cx^2\right)\right)dx^2}{c^2}-\frac{\int \frac{a+b\arctan(cx^2)}{c^2x^4+1}dx^2}{c^2}\right)\right)$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^2-bc\left(\frac{ax^2+bx^2\arctan\left(cx^2\right)-\frac{b\log(c^2x^4+1)}{2c}}{c^2}-\frac{\int \frac{a+b\arctan(cx^2)}{c^2x^4+1}dx^2}{c^2}\right)\right)$$
$$\Big\downarrow \text{5419}$$
$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^2-bc\left(\frac{ax^2+bx^2\arctan\left(cx^2\right)-\frac{b\log(c^2x^4+1)}{2c}}{c^2}-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^3}\right)\right)$$

input
```
Int[x^3*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
((x^4*(a + b*ArcTan[c*x^2])^2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x^2])^2/(b*c^
3) + (a*x^2 + b*x^2*ArcTan[c*x^2] - (b*Log[1 + c^2*x^4])/(2*c))/c^2))/2
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.76.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

method	result
parallelrisch	$\frac{b^2x^4\arctan(cx^2)^2c^2+2abx^4\arctan(cx^2)c^2+a^2c^2x^4-2b^2\arctan(cx^2)x^2c-2abcx^2+b^2\arctan(cx^2)^2+b^2\ln(c^2x^4+1)+2ab\,\mathrm{ar}}{4c^2}$
default	$\frac{a^2x^4}{4}+\frac{b^2x^4\arctan(cx^2)^2}{4}-\frac{b^2x^2\arctan(cx^2)}{2c}+\frac{b^2\arctan(cx^2)^2}{4c^2}+\frac{b^2\ln(c^2x^4+1)}{4c^2}+\frac{abx^4\arctan(cx^2)}{2}-\frac{abx^2}{2c}-$
parts	$\frac{a^2x^4}{4}+\frac{b^2x^4\arctan(cx^2)^2}{4}-\frac{b^2x^2\arctan(cx^2)}{2c}+\frac{b^2\arctan(cx^2)^2}{4c^2}+\frac{b^2\ln(c^2x^4+1)}{4c^2}+\frac{abx^4\arctan(cx^2)}{2}-\frac{abx^2}{2c}-$
risch	$-\frac{b^2(c^2x^4+1)\ln(icx^2+1)^2}{16c^2}-\frac{ib(4a^2c^2x^4+2ix^4b\ln(-icx^2+1)ac^2-4abcx^2+b^2+2ib\ln(-icx^2+1)a)\ln(icx^2+1)}{16ac^2}+\frac{iabx^4}{}$

input
```
int(x^3*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b^2*x^4*arctan(c*x^2)^2*c^2+2*a*b*x^4*arctan(c*x^2)*c^2+a^2*c^2*x^4-2
*b^2*arctan(c*x^2)*x^2*c-2*a*b*c*x^2+b^2*arctan(c*x^2)^2+b^2*ln(c^2*x^4+1)
+2*a*b*arctan(c*x^2))/c^2
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\frac{a^2c^2x^4-2\,abcx^2+\left(b^2c^2x^4+b^2\right)\arctan\left(cx^2\right)^2-2\,ab\arctan\left(\frac{1}{cx^2}\right)+b^2\log\left(c^2x^4+1\right)+2\left(abc^2x^4-b^2cx^2\right)}{4\,c^2}$$

input
```
integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
1/4*(a^2*c^2*x^4 - 2*a*b*c*x^2 + (b^2*c^2*x^4 + b^2)*arctan(c*x^2)^2 - 2*a
*b*arctan(1/(c*x^2)) + b^2*log(c^2*x^4 + 1) + 2*(a*b*c^2*x^4 - b^2*c*x^2)*
arctan(c*x^2))/c^2
```

3.76.6 Sympy [A] (verification not implemented)

Time = 16.83 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.72

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\begin{cases}\frac{a^2x^4}{4}+\frac{abx^4\operatorname{atan}\left(cx^2\right)}{2}-\frac{abx^2}{2c}+\frac{ab\operatorname{atan}\left(cx^2\right)}{2c^2}+\frac{b^2x^4\operatorname{atan}^2\left(cx^2\right)}{4}-\frac{b^2x^2\operatorname{atan}\left(cx^2\right)}{2c}+\frac{b^2\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^2\right)}{2c}+\frac{b^2\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right.}{2c^2} \\ \frac{a^2x^4}{4}\end{cases}$$

input
```
integrate(x**3*(a+b*atan(c*x**2))**2,x)
```

output
```
Piecewise((a**2*x**4/4 + a*b*x**4*atan(c*x**2)/2 - a*b*x**2/(2*c) + a*b*at
an(c*x**2)/(2*c**2) + b**2*x**4*atan(c*x**2)**2/4 - b**2*x**2*atan(c*x**2)
/(2*c) + b**2*sqrt(-1/c**2)*atan(c*x**2)/(2*c) + b**2*log(x**2 + sqrt(-1/c
**2))/(2*c**2) + b**2*atan(c*x**2)**2/(4*c**2), Ne(c, 0)), (a**2*x**4/4, T
rue))
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.40

$$\int x^3(a+b\arctan(cx^2))^2\,dx$$
$$=\frac{1}{4}b^2x^4\arctan(cx^2)^2+\frac{1}{4}a^2x^4+\frac{1}{2}\left(x^4\arctan(cx^2)-c\left(\frac{x^2}{c^2}-\frac{\arctan(cx^2)}{c^3}\right)\right)ab$$
$$-\frac{1}{4}\left(2c\left(\frac{x^2}{c^2}-\frac{\arctan(cx^2)}{c^3}\right)\arctan(cx^2)+\frac{\arctan(cx^2)^2-\log(4c^5x^4+4c^3)}{c^2}\right)b^2$$

input
```
integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arctan(c*x^2)^2 + 1/4*a^2*x^4 + 1/2*(x^4*arctan(c*x^2) - c*(x^
2/c^2 - arctan(c*x^2)/c^3))*a*b - 1/4*(2*c*(x^2/c^2 - arctan(c*x^2)/c^3)*a
rctan(c*x^2) + (arctan(c*x^2)^2 - log(4*c^5*x^4 + 4*c^3))/c^2)*b^2
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int x^3(a+b\arctan(cx^2))^2\,dx$$
$$=\frac{a^2cx^4+\frac{2\left(c^2x^4\arctan(cx^2)-cx^2+\arctan(cx^2)\right)ab}{c}+\frac{\left(c^2x^4\arctan(cx^2)^2-2\,cx^2\arctan(cx^2)+\arctan(cx^2)^2+\log(c^2x^4+1)\right)b^2}{c}}{4\,c}$$

input
```
integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
1/4*(a^2*c*x^4 + 2*(c^2*x^4*arctan(c*x^2) - c*x^2 + arctan(c*x^2))*a*b/c +
 (c^2*x^4*arctan(c*x^2)^2 - 2*c*x^2*arctan(c*x^2) + arctan(c*x^2)^2 + log(
c^2*x^4 + 1))*b^2/c)/c
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.24

$$\int x^3(a+b\arctan(cx^2))^2\,dx=\frac{a^2\,x^4}{4}+\frac{b^2\,\mathrm{atan}(c\,x^2)^2}{4\,c^2}+\frac{b^2\,x^4\,\mathrm{atan}(c\,x^2)^2}{4}+\frac{b^2\,\ln\left(c^2\,x^4+1\right)}{4\,c^2}-\frac{b^2\,x^2\,\mathrm{atan}(c\,x^2)}{2\,c}-\frac{a\,b\,x^2}{2\,c}+\frac{a\,b\,\mathrm{atan}(c\,x^2)}{2\,c^2}+\frac{a\,b\,x^4\,\mathrm{atan}(c\,x^2)}{2}$$

input
```
int(x^3*(a + b*atan(c*x^2))^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*atan(c*x^2)^2)/(4*c^2) + (b^2*x^4*atan(c*x^2)^2)/4 + (b
^2*log(c^2*x^4 + 1))/(4*c^2) - (b^2*x^2*atan(c*x^2))/(2*c) - (a*b*x^2)/(2*
c) + (a*b*atan(c*x^2))/(2*c^2) + (a*b*x^4*atan(c*x^2))/2
```

3.77 $\int x(a+b\arctan(cx^2))^2\,dx$

3.77.1 Optimal result . . . 514
3.77.2 Mathematica [A] (verified) . . . 514
3.77.3 Rubi [A] (verified) . . . 515
3.77.4 Maple [A] (verified) . . . 517
3.77.5 Fricas [F] . . . 518
3.77.6 Sympy [F] . . . 518
3.77.7 Maxima [F] . . . 518
3.77.8 Giac [F] . . . 519
3.77.9 Mupad [F(-1)] . . . 519

3.77.1 Optimal result

Integrand size = 14, antiderivative size = 101

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\frac{i(a+b\arctan(cx^2))^2}{2c}+\frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)^2+\frac{b(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{c}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{2c}$$

output
```
1/2*I*(a+b*arctan(c*x^2))^2/c+1/2*x^2*(a+b*arctan(c*x^2))^2+b*(a+b*arctan(
c*x^2))*ln(2/(1+I*c*x^2))/c+1/2*I*b^2*polylog(2,1-2/(1+I*c*x^2))/c
```

3.77.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\frac{b^2(-i+cx^2)\arctan\left(cx^2\right)^2+2b\arctan\left(cx^2\right)\left(acx^2+b\log\left(1+e^{2i\arctan(cx^2)}\right)\right)+a(acx^2-b\log(1+c^2x^4}{2c}$$

input
```
Integrate[x*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
(b^2*(-I + c*x^2)*ArcTan[c*x^2]^2 + 2*b*ArcTan[c*x^2]*(a*c*x^2 + b*Log[1 +
 E^((2*I)*ArcTan[c*x^2])]) + a*(a*c*x^2 - b*Log[1 + c^2*x^4]) - I*b^2*Poly
Log[2, -E^((2*I)*ArcTan[c*x^2])])/(2*c)
```

3.77.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5363, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2}\int \left(a+b\arctan\left(cx^2\right)\right)^2dx^2$$

$$\Big\downarrow \text{5345}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\int \frac{x^2\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2\right)$$

$$\Big\downarrow \text{5455}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{\int \frac{a+b\arctan\left(cx^2\right)}{i-cx^2}dx^2}{c}-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{5379}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)}{c}-b\int \frac{\log\left(\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2}{c}-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{\frac{ib\int \frac{\log\left(\frac{2}{icx^2+1}\right)}{1-\frac{2}{icx^2+1}}d\frac{1}{icx^2+1}}{c}+\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)}{c}}{c}-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)}{c}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)}{2c}}{c}\right)\right)$$

input
```
Int[x*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
(x^2*(a + b*ArcTan[c*x^2])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^2])^2)/(
b*c^2) - (((a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/c + ((I/2)*b*PolyLo
g[2, 1 - 2/(1 + I*c*x^2)])/c)/c))/2
```

3.77.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.77.4 Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.39

method	result
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\arctan(cx^2)^2(cx^2+i)+2\arctan(cx^2)\ln\left(1+\frac{(icx^2+1)^2}{c^2x^4+1}\right)-2i\arctan(cx^2)^2-i\,\mathrm{polylog}\left(2,-\frac{(icx^2+1)^2}{c^2x^4+1}\right)\cdots\right.}{2c}$
derivativedivides	$\frac{a^2cx^2-i\arctan(cx^2)^2b^2+\arctan(cx^2)^2b^2cx^2-i\,\mathrm{polylog}\left(2,-\frac{(icx^2+1)^2}{c^2x^4+1}\right)b^2+2\arctan(cx^2)\ln\left(1+\frac{(icx^2+1)^2}{c^2x^4+1}\right)b^2+2\cdots}{2c}$
default	$\frac{a^2cx^2-i\arctan(cx^2)^2b^2+\arctan(cx^2)^2b^2cx^2-i\,\mathrm{polylog}\left(2,-\frac{(icx^2+1)^2}{c^2x^4+1}\right)b^2+2\arctan(cx^2)\ln\left(1+\frac{(icx^2+1)^2}{c^2x^4+1}\right)b^2+2\cdots}{2c}$
risch	$\frac{ia^2}{2c}-\frac{b^2\arctan(cx^2)}{2c}+\frac{i\ln(-icx^2+1)abx^2}{2}-\frac{\ln(-icx^2+1)ab}{2c}-\frac{i\ln(-icx^2+1)^2b^2}{8c}+\frac{ib^2\ln(icx^2+1)^2}{8c}-\frac{ba\cdots}{}$

input
```
int(x*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+1/2*b^2/c*(arctan(c*x^2)^2*(c*x^2+I)+2*arctan(c*x^2)*ln(1+(1+I
*c*x^2)^2/(c^2*x^4+1))-2*I*arctan(c*x^2)^2-I*polylog(2,-(1+I*c*x^2)^2/(c^2
*x^4+1)))+a*b/c*(c*x^2*arctan(c*x^2)-1/2*ln(c^2*x^4+1))
```

3.77.5 Fricas [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x*arctan(c*x^2)^2 + 2*a*b*x*arctan(c*x^2) + a^2*x, x)
```

3.77.6 Sympy [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int x\left(a+b\,\mathrm{atan}\left(cx^2\right)\right)^2\,dx$$

input
```
integrate(x*(a+b*atan(c*x**2))**2,x)
```

output
```
Integral(x*(a + b*atan(c*x**2))**2, x)
```

3.77.7 Maxima [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*x^2 + 1/32*(4*x^2*arctan(c*x^2)^2 - x^2*log(c^2*x^4 + 1)^2 + 384*c
^2*integrate(1/16*x^5*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 32*c^2*integrate
(1/16*x^5*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 128*c^2*integrate(1/16*x^
5*log(c^2*x^4 + 1)/(c^2*x^4 + 1), x) + 4*arctan(c*x^2)^3/c - 256*c*integra
te(1/16*x^3*arctan(c*x^2)/(c^2*x^4 + 1), x) + 32*integrate(1/16*x*log(c^2*
x^4 + 1)^2/(c^2*x^4 + 1), x))*b^2 + 1/2*(2*c*x^2*arctan(c*x^2) - log(c^2*x
^4 + 1))*a*b/c
```

3.77.8 Giac [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2*x, x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int x\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,dx$$

input
```
int(x*(a + b*atan(c*x^2))^2,x)
```

output
```
int(x*(a + b*atan(c*x^2))^2, x)
```

3.78 $\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx$

3.78.1 Optimal result 520
3.78.2 Mathematica [A] (verified) 521
3.78.3 Rubi [A] (verified) 521
3.78.4 Maple [F] 523
3.78.5 Fricas [F] 524
3.78.6 Sympy [F] 524
3.78.7 Maxima [F] 524
3.78.8 Giac [F] 525
3.78.9 Mupad [F(-1)] 525

3.78.1 Optimal result

Integrand size = 16, antiderivative size = 151

$$\begin{aligned}\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = {} & (a+b\arctan(cx^2))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right)\\ & -\frac{1}{2}ib(a+b\arctan(cx^2))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)\\ & +\frac{1}{2}ib(a+b\arctan(cx^2))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right)\\ & -\frac{1}{4}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right)\\ & +\frac{1}{4}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^2}\right)\end{aligned}$$

output
```
-(a+b*arctan(c*x^2))^2*arctanh(-1+2/(1+I*c*x^2))-1/2*I*b*(a+b*arctan(c*x^2
))*polylog(2,1-2/(1+I*c*x^2))+1/2*I*b*(a+b*arctan(c*x^2))*polylog(2,-1+2/(
1+I*c*x^2))-1/4*b^2*polylog(3,1-2/(1+I*c*x^2))+1/4*b^2*polylog(3,-1+2/(1+I
*c*x^2))
```

3.78.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.33

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = a^2\log(x)+\frac{1}{2}iab(\text{PolyLog}(2,-icx^2)-\text{PolyLog}(2,icx^2)) + \frac{1}{48}b^2\Big(-i\pi^3+16i\arctan(cx^2)^3 + 24\arctan(cx^2)^2\log\left(1-e^{-2i\arctan(cx^2)}\right) - 24\arctan(cx^2)^2\log\left(1+e^{2i\arctan(cx^2)}\right) + 24i\arctan(cx^2)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx^2)}\right) + 24i\arctan(cx^2)\,\text{PolyLog}\left(2,-e^{2i\arctan(cx^2)}\right) + 12\,\text{PolyLog}\left(3,e^{-2i\arctan(cx^2)}\right) - 12\,\text{PolyLog}\left(3,-e^{2i\arctan(cx^2)}\right)\Big)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2/x,x]
```

output
```
a^2*Log[x] + (I/2)*a*b*(PolyLog[2, (-I)*c*x^2] - PolyLog[2, I*c*x^2]) + (b
^2*((-I)*Pi^3 + (16*I)*ArcTan[c*x^2]^3 + 24*ArcTan[c*x^2]^2*Log[1 - E^((-2
*I)*ArcTan[c*x^2])] - 24*ArcTan[c*x^2]^2*Log[1 + E^((2*I)*ArcTan[c*x^2])]
+ (24*I)*ArcTan[c*x^2]*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] + (24*I)*ArcTa
n[c*x^2]*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])] + 12*PolyLog[3, E^((-2*I)*Ar
cTan[c*x^2])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x^2])]))/48
```

3.78.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5359, 5357, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^2))^2}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{1}{2}\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}dx^2$$

$$\Big\downarrow \; 5357$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^2-4bc\int \frac{\left(a+b\arctan\left(cx^2\right)\right)\text{arctanh}\left(1-\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2\right)$$

$$\Big\downarrow \; 5523$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^2-4bc\left(\frac{1}{2}\int \frac{\left(a+b\arctan\left(cx^2\right)\right)\log\left(2-\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2-\frac{1}{2}\int \right.\right.$$

$$\Big\downarrow \; 5529$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)\left(a+b\arctan\left(cx^2\right)\right)}{2c}-\frac{1}{2}\right.\right.\right.$$

$$\Big\downarrow \; 7164$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)\left(a+b\arctan\left(cx^2\right)\right)}{2c}+\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^2])^2/x,x]
```

output
```
(2*(a + b*ArcTan[c*x^2])^2*ArcTanh[1 - 2/(1 + I*c*x^2)] - 4*b*c*((((I/2)*(
a + b*ArcTan[c*x^2])*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c + (b*PolyLog[3, 1
- 2/(1 + I*c*x^2)])/(4*c))/2 + (((-1/2*I)*(a + b*ArcTan[c*x^2])*PolyLog[2,
 -1 + 2/(1 + I*c*x^2)])/c - (b*PolyLog[3, -1 + 2/(1 + I*c*x^2)])/(4*c))/2)
)/2
```

3.78. $\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x}\,dx$

3.78.3.1 Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.78.4 Maple [F]

$$\int \frac{(a+b\arctan(c\,x^2))^2}{x}dx$$

input
```
int((a+b*arctan(c*x^2))^2/x,x)
```

output
```
int((a+b*arctan(c*x^2))^2/x,x)
```

3.78.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x, x)
```

3.78.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx^2))^2}{x}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**2/x,x)
```

output
```
Integral((a + b*atan(c*x**2))**2/x, x)
```

3.78.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x^2)^2 + b^2*log(c^2*x^4 + 1)
^2 + 32*a*b*arctan(c*x^2))/x, x)
```

3.78.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2/x, x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^2))^2}{x}\,dx$$

input
```
int((a + b*atan(c*x^2))^2/x,x)
```

output
```
int((a + b*atan(c*x^2))^2/x, x)
```

3.79 $\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx$

3.79.1 Optimal result . . . 526
3.79.2 Mathematica [A] (verified) . . . 526
3.79.3 Rubi [A] (verified) . . . 527
3.79.4 Maple [C] (warning: unable to verify) . . . 529
3.79.5 Fricas [F] . . . 530
3.79.6 Sympy [F] . . . 530
3.79.7 Maxima [F] . . . 530
3.79.8 Giac [F] . . . 531
3.79.9 Mupad [F(-1)] . . . 531

3.79.1 Optimal result

Integrand size = 16, antiderivative size = 97

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = -\frac{1}{2}ic(a+b\arctan(cx^2))^2 - \frac{(a+b\arctan(cx^2))^2}{2x^2} + bc(a+b\arctan(cx^2))\log\left(2-\frac{2}{1-icx^2}\right) - \frac{1}{2}ib^2c\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right)$$

output
```
-1/2*I*c*(a+b*arctan(c*x^2))^2-1/2*(a+b*arctan(c*x^2))^2/x^2+b*c*(a+b*arct
an(c*x^2))*ln(2-2/(1-I*c*x^2))-1/2*I*b^2*c*polylog(2,-1+2/(1-I*c*x^2))
```

3.79.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = -\frac{a^2}{2x^2} + abc\left(-\frac{\arctan(cx^2)}{cx^2} + \log(cx^2) - \frac{1}{2}\log(1+c^2x^4)\right) + \frac{1}{2}b^2c\left(-\frac{\arctan(cx^2)^2}{cx^2} + 2\arctan(cx^2)\log\left(1-e^{2i\arctan(cx^2)}\right) - i\left(\arctan(cx^2)^2 + \mathrm{PolyLog}\left(2,e^{2i\arctan(cx^2)}\right)\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^3,x]
```

output
```
-1/2*a^2/x^2 + a*b*c*(-(ArcTan[c*x^2]/(c*x^2)) + Log[c*x^2] - Log[1 + c^2*
x^4]/2) + (b^2*c*(-(ArcTan[c*x^2]^2/(c*x^2)) + 2*ArcTan[c*x^2]*Log[1 - E^(
(2*I)*ArcTan[c*x^2])] - I*(ArcTan[c*x^2]^2 + PolyLog[2, E^((2*I)*ArcTan[c*
x^2])])))/2
```

3.79.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5363, 5361, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2}\int \frac{(a+b\arctan(cx^2))^2}{x^4}dx^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(2bc\int \frac{a+b\arctan(cx^2)}{x^2(c^2x^4+1)}dx^2-\frac{(a+b\arctan(cx^2))^2}{x^2}\right)$$

$$\Big\downarrow \text{5459}$$

$$\frac{1}{2}\left(-\frac{(a+b\arctan(cx^2))^2}{x^2}+2bc\left(i\int \frac{a+b\arctan(cx^2)}{x^2(cx^2+i)}dx^2-\frac{i(a+b\arctan(cx^2))^2}{2b}\right)\right)$$

$$\Big\downarrow \text{5403}$$

$$\frac{1}{2}\left(-\frac{(a+b\arctan(cx^2))^2}{x^2}+2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx^2}\right)}{c^2x^4+1}dx^2-i\log\left(2-\frac{2}{1-icx^2}\right)(a+b\arctan(cx^2))\right)-\right.\right.$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{2}\left(-\frac{(a+b\arctan(cx^2))^2}{x^2}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx^2}\right)(a+b\arctan(cx^2))-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{1-icx^2}-1\right)\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^2])^2/x^3,x]
```

output
```
(-((a + b*ArcTan[c*x^2])^2/x^2) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^2])^2
)/b + I*((-I)*(a + b*ArcTan[c*x^2])*Log[2 - 2/(1 - I*c*x^2)] - (b*PolyLog[
2, -1 + 2/(1 - I*c*x^2)])/2)))/2
```

3.79.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.79.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.33 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.49

method	result
default	$-\frac{a^2}{2x^2}-\frac{b^2 \arctan\left(c\,x^2\right)^2}{2x^2}+2b^2 c\arctan\left(c\,x^2\right)\ln\left(x\right)-\frac{b^2\arctan\left(c\,x^2\right)\ln\left(c^2x^4+1\right)c}{2}+\frac{b^2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(c^2\_Z^4+1\right)} \frac{2\ln(\cdots}{\cdots}\right.}{\cdots}$
parts	$-\frac{a^2}{2x^2}-\frac{b^2 \arctan\left(c\,x^2\right)^2}{2x^2}+2b^2 c\arctan\left(c\,x^2\right)\ln\left(x\right)-\frac{b^2\arctan\left(c\,x^2\right)\ln\left(c^2x^4+1\right)c}{2}+\frac{b^2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(c^2\_Z^4+1\right)} \frac{2\ln(\cdots}{\cdots}\right.}{\cdots}$

input
```
int((a+b*arctan(c*x^2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2-1/2*b^2/x^2*arctan(c*x^2)^2+2*b^2*c*arctan(c*x^2)*ln(x)-1/2*b
^2*arctan(c*x^2)*ln(c^2*x^4+1)*c+1/8*b^2*sum(1/_alpha^2*(2*ln(x-_alpha)*ln
(c^2*x^4+1)-c*(1/c/_alpha^3*ln(x-_alpha)^2+2/_alpha*ln(x-_alpha)*(_alpha^2
*ln(1/2*(x+_alpha)/_alpha)*c-ln((_alpha^3*c+x)/_alpha/(_alpha^2*c+1))+ln((
_alpha^3*c-x)/_alpha/(_alpha^2*c-1)))+2/_alpha*(_alpha^2*dilog(1/2*(x+_alp
ha)/_alpha)*c-dilog((_alpha^3*c+x)/_alpha/(_alpha^2*c+1))+dilog((_alpha^3*
c-x)/_alpha/(_alpha^2*c-1))))),_alpha=RootOf(_Z^4*c^2+1))-b^2*sum(1/_R1^2*
(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1)),_R1=RootOf(_Z^4*c^2+1))+2*a*b*(
-1/2/x^2*arctan(c*x^2)+c*(ln(x)-1/4*ln(c^2*x^4+1)))
```

3.79.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^3, x)
```

3.79.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx^2))^2}{x^3}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**2/x**3,x)
```

output
```
Integral((a + b*atan(c*x**2))**2/x**3, x)
```

3.79.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*a*b + 1/32*(3
2*x^2*integrate(-1/16*(4*c^2*x^4*log(c^2*x^4 + 1) - 8*c*x^2*arctan(c*x^2)
- 12*(c^2*x^4 + 1)*arctan(c*x^2)^2 - (c^2*x^4 + 1)*log(c^2*x^4 + 1)^2)/(c^
2*x^7 + x^3), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2)*b^2/x^2 - 1/2*a
^2/x^2
```

3.79.8 Giac [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(b\arctan(cx^2) + a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2/x^3, x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(a + b\,\mathrm{atan}(c\,x^2))^2}{x^3}\,dx$$

input
```
int((a + b*atan(c*x^2))^2/x^3,x)
```

output
```
int((a + b*atan(c*x^2))^2/x^3, x)
```

3.80 $\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx$

3.80.1 Optimal result 532
3.80.2 Mathematica [A] (verified) 532
3.80.3 Rubi [A] (verified) 533
3.80.4 Maple [A] (verified) 535
3.80.5 Fricas [A] (verification not implemented) 536
3.80.6 Sympy [B] (verification not implemented) 536
3.80.7 Maxima [A] (verification not implemented) 537
3.80.8 Giac [F] 537
3.80.9 Mupad [B] (verification not implemented) 537

3.80.1 Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = -\frac{bc(a+b\arctan(cx^2))}{2x^2} - \frac{1}{4}c^2(a+b\arctan(cx^2))^2 - \frac{(a+b\arctan(cx^2))^2}{4x^4} + b^2c^2\log(x) - \frac{1}{4}b^2c^2\log(1+c^2x^4)$$

output
```
-1/2*b*c*(a+b*arctan(c*x^2))/x^2-1/4*c^2*(a+b*arctan(c*x^2))^2-1/4*(a+b*ar
ctan(c*x^2))^2/x^4+b^2*c^2*ln(x)-1/4*b^2*c^2*ln(c^2*x^4+1)
```

3.80.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = -\frac{a^2+2abcx^2+2b(a+bcx^2+ac^2x^4)\arctan(cx^2)+b^2(1+c^2x^4)\arctan(cx^2)^2-4b^2c^2x^4\log(x)+b^2c^2x^4}{4x^4}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^5,x]
```

output
```
-1/4*(a^2 + 2*a*b*c*x^2 + 2*b*(a + b*c*x^2 + a*c^2*x^4)*ArcTan[c*x^2] + b^
2*(1 + c^2*x^4)*ArcTan[c*x^2]^2 - 4*b^2*c^2*x^4*Log[x] + b^2*c^2*x^4*Log[1
 + c^2*x^4])/x^4
```

3.80.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5363, 5361, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^5}dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2}\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^6}dx^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(bc\int \frac{a+b\arctan\left(cx^2\right)}{x^4\left(c^2x^4+1\right)}dx^2-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{2x^4}\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{2}\left(bc\left(\int \frac{a+b\arctan\left(cx^2\right)}{x^4}dx^2-c^2\int \frac{a+b\arctan\left(cx^2\right)}{c^2x^4+1}dx^2\right)-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{2x^4}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^2\right)}{c^2x^4+1}dx^2\right)+bc\int \frac{1}{x^2\left(c^2x^4+1\right)}dx^2-\frac{a+b\arctan\left(cx^2\right)}{x^2}\right)-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{2x^4}\right.$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^2\right)}{c^2x^4+1}dx^2\right)+\frac{1}{2}bc\int \frac{1}{x^2\left(c^2x^4+1\right)}dx^4-\frac{a+b\arctan\left(cx^2\right)}{x^2}\right)-\frac{\left(a+b\arctan\left(cx^2\right)\right)}{2x^4}\right.$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^2\right)}{c^2x^4+1}dx^2\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^4-c^2\int \frac{1}{c^2x^4+1}dx^4\right)-\frac{a+b\arctan\left(cx^2\right)}{x^2}\right)-(a+b\right.$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^2\right)}{c^2x^4+1}dx^2\right)+\frac{1}{2}bc\left(\log\left(x^4\right)-c^2\int \frac{1}{c^2x^4+1}dx^4\right)-\frac{a+b\arctan\left(cx^2\right)}{x^2}\right)-\frac{(a+b\ar$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^2\right)}{c^2x^4+1}dx^2\right)-\frac{a+b\arctan\left(cx^2\right)}{x^2}+\frac{1}{2}bc\left(\log\left(x^4\right)-\log\left(c^2x^4+1\right)\right)\right)-\frac{(a+b\arctan}{2x^4}$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{2}\left(bc\left(-\frac{c\left(a+b\arctan\left(cx^2\right)\right)^2}{2b}-\frac{a+b\arctan\left(cx^2\right)}{x^2}+\frac{1}{2}bc\left(\log\left(x^4\right)-\log\left(c^2x^4+1\right)\right)\right)-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{2x^4}$$

input
```
Int[(a + b*ArcTan[c*x^2])^2/x^5,x]
```

output
```
(-1/2*(a + b*ArcTan[c*x^2])^2/x^4 + b*c*(-((a + b*ArcTan[c*x^2])/x^2) - (c
*(a + b*ArcTan[c*x^2])^2)/(2*b) + (b*c*(Log[x^4] - Log[1 + c^2*x^4]))/2))/
2
```

3.80.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.80.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

method	result
default	$-\frac{a^2}{4x^4}-\frac{b^2\arctan(c\,x^2)^2}{4x^4}-\frac{b^2c\arctan(c\,x^2)}{2x^2}-\frac{b^2\arctan(c\,x^2)^2c^2}{4}+b^2c^2\ln(x)-\frac{b^2c^2\ln(c^2x^4+1)}{4}-\frac{ab\arctan(}{2x^4}$
parts	$-\frac{a^2}{4x^4}-\frac{b^2\arctan(c\,x^2)^2}{4x^4}-\frac{b^2c\arctan(c\,x^2)}{2x^2}-\frac{b^2\arctan(c\,x^2)^2c^2}{4}+b^2c^2\ln(x)-\frac{b^2c^2\ln(c^2x^4+1)}{4}-\frac{ab\arctan(}{2x^4}$
parallelrisch	$\frac{-b^2x^4\arctan(c\,x^2)^2c^2+4b^2c^2\ln(x)x^4-b^2c^2\ln(c^2x^4+1)x^4-2ab\,x^4\arctan(c\,x^2)c^2+a^2c^2x^4-2b^2\arctan(c\,x^2)x^2c-2abc\,x^2-b^2}{4x^4}$
risch	$\frac{b^2(c^2x^4+1)\ln(ic\,x^2+1)^2}{16x^4}+\frac{ib(ib\,c^2x^4\ln(-ic\,x^2+1)+2bc\,x^2+2a+ib\ln(-ic\,x^2+1))\ln(ic\,x^2+1)}{8x^4}-\frac{-4i\ln((5ibc+ac)x^2+5b}{}$

input
```
int((a+b*arctan(c*x^2))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4*a^2-1/4*b^2/x^4*arctan(c*x^2)^2-1/2*b^2*c*arctan(c*x^2)/x^2-1/4*b
^2*arctan(c*x^2)^2*c^2+b^2*c^2*ln(x)-1/4*b^2*c^2*ln(c^2*x^4+1)-1/2*a*b/x^4
*arctan(c*x^2)-1/2*a*b*c/x^2-1/2*a*b*arctan(c*x^2)*c^2
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx$$
$$= \frac{2\,abc^2x^4\arctan\left(\frac{1}{cx^2}\right) - b^2c^2x^4\log\left(c^2x^4+1\right) + 4\,b^2c^2x^4\log(x) - 2\,abcx^2 - \left(b^2c^2x^4+b^2\right)\arctan\left(cx^2\right)^2 - a}{4\,x^4}$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="fricas")
```

output
```
1/4*(2*a*b*c^2*x^4*arctan(1/(c*x^2)) - b^2*c^2*x^4*log(c^2*x^4 + 1) + 4*b^
2*c^2*x^4*log(x) - 2*a*b*c*x^2 - (b^2*c^2*x^4 + b^2)*arctan(c*x^2)^2 - a^2
 - 2*(b^2*c*x^2 + a*b)*arctan(c*x^2))/x^4
```

3.80.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(80) = 160.

Time = 22.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.92

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx$$
$$= \begin{cases} -\frac{a^2}{4x^4} - \frac{abc^2\operatorname{atan}(cx^2)}{2} - \frac{abc}{2x^2} - \frac{ab\operatorname{atan}(cx^2)}{2x^4} + b^2c^2\log(x) - \frac{b^2c^2\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2} - \frac{b^2c^2\operatorname{atan}^2(cx^2)}{4} + \frac{b^2c\operatorname{atan}(cx^2)}{2\sqrt{-\frac{1}{c^2}}} - \\ -\frac{a^2}{4x^4} \end{cases}$$

input
```
integrate((a+b*atan(c*x**2))**2/x**5,x)
```

output
```
Piecewise((-a**2/(4*x**4) - a*b*c**2*atan(c*x**2)/2 - a*b*c/(2*x**2) - a*b
*atan(c*x**2)/(2*x**4) + b**2*c**2*log(x) - b**2*c**2*log(x**2 + sqrt(-1/c
**2))/2 - b**2*c**2*atan(c*x**2)**2/4 + b**2*c*atan(c*x**2)/(2*sqrt(-1/c**
2)) - b**2*c*atan(c*x**2)/(2*x**2) - b**2*atan(c*x**2)**2/(4*x**4), Ne(c,
0)), (-a**2/(4*x**4), True))
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = -\frac{1}{2}\left(\left(c\arctan(cx^2)+\frac{1}{x^2}\right)c+\frac{\arctan(cx^2)}{x^4}\right)ab + \frac{1}{4}\left(\left(\arctan(cx^2)^2-\log(c^2x^4+1)+4\log(x)\right)c^2-2\left(c\arctan(cx^2)+\frac{1}{x^2}\right)c\arctan(cx^2)\right)b^2 - \frac{b^2\arctan(cx^2)^2}{4x^4}-\frac{a^2}{4x^4}$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="maxima")
```

output
```
-1/2*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*a*b + 1/4*((arctan(
c*x^2)^2 - log(c^2*x^4 + 1) + 4*log(x))*c^2 - 2*(c*arctan(c*x^2) + 1/x^2)*
c*arctan(c*x^2))*b^2 - 1/4*b^2*arctan(c*x^2)^2/x^4 - 1/4*a^2/x^4
```

3.80.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2/x^5, x)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.75

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = b^2\,c^2\,\ln(x) - \frac{b^2\,c^2\,\mathrm{atan}(c\,x^2)^2}{4} - \frac{b^2\,\mathrm{atan}(c\,x^2)^2}{4\,x^4} - \frac{b^2\,c^2\,\ln(c^2\,x^4+1)}{4} - \frac{a^2}{4\,x^4} - \frac{b^2\,c\,\mathrm{atan}(c\,x^2)}{2\,x^2} - \frac{a\,b\,c}{2\,x^2} - \frac{a\,b\,c^2\,\mathrm{atan}\left(\frac{a^2\,c\,x^2}{a^2+25\,b^2}+\frac{25\,b^2\,c\,x^2}{a^2+25\,b^2}\right)}{2} - \frac{a\,b\,\mathrm{atan}(c\,x^2)}{2\,x^4}$$

input
```
int((a + b*atan(c*x^2))^2/x^5,x)
```

output
```
b^2*c^2*log(x) - (b^2*c^2*atan(c*x^2)^2)/4 - (b^2*atan(c*x^2)^2)/(4*x^4) -
 (b^2*c^2*log(c^2*x^4 + 1))/4 - a^2/(4*x^4) - (b^2*c*atan(c*x^2))/(2*x^2)
- (a*b*c)/(2*x^2) - (a*b*c^2*atan((a^2*c*x^2)/(a^2 + 25*b^2) + (25*b^2*c*x
^2)/(a^2 + 25*b^2)))/2 - (a*b*atan(c*x^2))/(2*x^4)
```

3.80. $\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx$

3.81 $\int x^2(a+b\arctan(cx^2))^2\,dx$

3.81.1 Optimal result . . . 539
3.81.2 Mathematica [F] . . . 540
3.81.3 Rubi [A] (verified) . . . 540
3.81.4 Maple [F] . . . 543
3.81.5 Fricas [F] . . . 543
3.81.6 Sympy [F] . . . 543
3.81.7 Maxima [F] . . . 544
3.81.8 Giac [F] . . . 544
3.81.9 Mupad [F(-1)] . . . 544

3.81.1 Optimal result

Integrand size = 16, antiderivative size = 1393

$$\int x^2(a+b\arctan(cx^2))^2\,dx = \text{Too large to display}$$

output
```
1/12*x^3*(2*a+I*b*ln(1-I*c*x^2))^2-1/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x
*c^(1/2))^2/c^(3/2)+1/6*b^2*x^3*ln(1-I*c*x^2)*ln(1+I*c*x^2)+1/3*(-1)^(1/4)
*b^2*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))/c^(3/2)+1/3*(-1)^(1/4)*b^2*po
lylog(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)-1/6*(-1)^(1/4)*b^2*polylog(2
,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)+1/3*(-
1)^(3/4)*b^2*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))/c^(3/2)+1/3*(-1)^(3/4
)*b^2*polylog(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2)-1/6*(-1)^(3/4)*b^2*p
olylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2
)-1/6*(-1)^(3/4)*b^2*polylog(2,1-(1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3
/4)*x*c^(1/2)))/c^(3/2)-1/6*(-1)^(1/4)*b^2*polylog(2,1+(-1+I)*(1+(-1)^(3/4
)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)-1/9*I*b*x^3*(2*a+I*b*ln(1-I
*c*x^2))+4/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))/c^(3/2)+1/3*(-1)^
(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))^2/c^(3/2)-4/3*(-1)^(3/4)*b^2*arctan
h((-1)^(3/4)*x*c^(1/2))/c^(3/2)+2/3*I*b^2*x*ln(1+I*c*x^2)/c-1/9*b^2*x^3*ln
(1-I*c*x^2)-1/12*b^2*x^3*ln(1+I*c*x^2)^2-2/3*(-1)^(1/4)*a*b*arctanh((-1)^(
3/4)*x*c^(1/2))/c^(3/2)-1/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*l
n(1-I*c*x^2)/c^(3/2)-1/3*(-1)^(1/4)*b*arctan((-1)^(3/4)*x*c^(1/2))*(2*a+I*
b*ln(1-I*c*x^2))/c^(3/2)+1/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*l
n(1+I*c*x^2)/c^(3/2)+1/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1
+I*c*x^2)/c^(3/2)-2/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/...
```

3.81.2 Mathematica [F]

$$\int x^2 \left(a+b \arctan \left(c x^2\right)\right)^2 \, dx=\int x^2 \left(a+b \arctan \left(c x^2\right)\right)^2 \, dx$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
Integrate[x^2*(a + b*ArcTan[c*x^2])^2, x]
```

3.81.3 Rubi [A] (verified)

Time = 2.56 (sec) , antiderivative size = 1393, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5365, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \arctan \left(c x^2\right)\right)^2 \, dx$$

$$\Big\downarrow \text{5365}$$

$$\int \left(\frac{1}{4} x^2 \left(2 a+i b \log \left(1-i c x^2\right)\right)^2+\frac{1}{2} b x^2 \log \left(1+i c x^2\right) \left(b \log \left(1-i c x^2\right)-2 i a\right)-\frac{1}{4} b^2 x^2 \log ^2\left(1+i c x^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
\frac{1}{12}\left(2a+ib\log\left(1-icx^2\right)\right)^2x^3-\frac{1}{12}b^2\log^2\left(icx^2+1\right)x^3+\frac{2}{9}iabx^3-\frac{1}{9}b^2\log\left(1-icx^2\right)x^3-\frac{1}{9}ib\left(2a+ib\log\left(1-icx^2\right)\right)x^3-\frac{1}{3}iab\log\left(icx^2+1\right)x^3+\frac{1}{6}b^2\log\left(1-icx^2\right)\log\left(icx^2+1\right)x^3-\frac{2ib^2\log\left(1-icx^2\right)x}{3c}+\frac{2ib^2\log\left(icx^2+1\right)x}{3c}-\frac{4abx}{3c}+\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}+\frac{4(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}-\frac{4(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}-\frac{2\sqrt[4]{-1}ab\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}-\frac{2(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}}+\frac{2(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{3c^{3/2}}-\frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{\sqrt{2}\left(\sqrt{c}x+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{3c^{3/2}}+\frac{2(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}}-\frac{2(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{(-1)^{3/4}\sqrt{c}x+1}\right)}{3c^{3/2}}+\frac{(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(-\frac{\sqrt{2}\left(\sqrt{c}x+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)}{3c^{3/2}}+\frac{(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{c}x+1\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)}{3c^{3/2}}-\frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1-i)\left((-1)^{3/4}\sqrt{c}x+1\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{3c^{3/2}}-\frac{(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{3c^{3/2}}-\frac{\sqrt[4]{-1}b\arctan\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log\left(1-icx^2\right)\right)}{3c^{3/2}}+\frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(icx^2+1\right)}{3c^{3/2}}+\frac{(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(icx^2+1\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}b^2\operatorname{PolyLog}\left(2,1-\frac{2}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{3c^{3/2}}-\frac{\sqrt[4]{-1}b^2\operatorname{PolyLog}\left(2,1-\frac{\sqrt{2}\left(\sqrt{c}x+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{6c^{3/2}}+\frac{(-1)^{3/4}b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}}+\frac{(-1)^{3/4}b^2\operatorname{PolyLog}\left(2,1-\frac{2}{(-1)^{3/4}\sqrt{c}x+1}\right)}{3c^{3/2}}-\frac{(-1)^{3/4}b^2\operatorname{PolyLog}\left(2,\frac{\sqrt{2}\left(\sqrt{c}x+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{c}x+1}+1\right)}{6c^{3/2}}-
$$

$$
(-1)^{3/4}b^2\operatorname{PolyLog}\left(2,1-\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{c}x+1\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)\qquad\sqrt[4]{-1}b^2\operatorname{PolyLog}\left(2,1-\frac{(1-i)\left((-1)^{3/4}\sqrt{c}x+1\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)
$$

3.81. $\int x^2\left(a+b\arctan\left(cx^2\right)\right)^2dx$

input
```
Int[x^2*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
(-4*a*b*x)/(3*c) + ((2*I)/9)*a*b*x^3 + (4*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)
*Sqrt[c]*x])/(3*c^(3/2)) + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)
/(3*c^(3/2)) - (2*(-1)^(1/4)*a*b*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)
) - (4*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)) - ((-1)^(
3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*
ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)
) + (2*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*S
qrt[c]*x)])/(3*c^(3/2)) - ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log
[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2
)) + (2*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)
*Sqrt[c]*x)])/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x
]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh
[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^
(3/4)*Sqrt[c]*x))])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[
c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)]
)/(3*c^(3/2)) - ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*
(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) - (((
2*I)/3)*b^2*x*Log[1 - I*c*x^2])/c - (b^2*x^3*Log[1 - I*c*x^2])/9 - ((-1)^(
3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/(3*c^(3/2)) - (I/
9)*b*x^3*(2*a + I*b*Log[1 - I*c*x^2]) - ((-1)^(1/4)*b*ArcTan[(-1)^(3/4)...
```

3.81.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5365
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

3.81.4 Maple [F]

$$\int x^2 (a + b \arctan(c x^2))^2 dx$$

input
```
int(x^2*(a+b*arctan(c*x^2))^2,x)
```

output
```
int(x^2*(a+b*arctan(c*x^2))^2,x)
```

3.81.5 Fricas [F]

$$\int x^2 (a + b \arctan(cx^2))^2 \, dx = \int (b \arctan(cx^2) + a)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctan(c*x^2)^2 + 2*a*b*x^2*arctan(c*x^2) + a^2*x^2, x)
```

3.81.6 Sympy [F]

$$\int x^2 (a + b \arctan(cx^2))^2 \, dx = \int x^2 (a + b \operatorname{atan}(cx^2))^2 \, dx$$

input
```
integrate(x**2*(a+b*atan(c*x**2))**2,x)
```

output
```
Integral(x**2*(a + b*atan(c*x**2))**2, x)
```

3.81.7 Maxima [F]

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/6*(4*x^3*arctan(c*x^2) - c*(8*x/c^2 - (2*sqrt(2)*arctan(1/
2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + 2*sqrt(2)*arctan(1/
2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + sqrt(2)*log(c*x^2 +
 sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x +
1)/sqrt(c))/c^2))*a*b + 1/48*(4*x^3*arctan(c*x^2)^2 - x^3*log(c^2*x^4 + 1)
^2 + 48*integrate(1/48*(8*c^2*x^6*log(c^2*x^4 + 1) - 16*c*x^4*arctan(c*x^2
) + 36*(c^2*x^6 + x^2)*arctan(c*x^2)^2 + 3*(c^2*x^6 + x^2)*log(c^2*x^4 + 1
)^2)/(c^2*x^4 + 1), x))*b^2
```

3.81.8 Giac [F]

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2*x^2, x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int x^2\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,dx$$

input
```
int(x^2*(a + b*atan(c*x^2))^2,x)
```

output
```
int(x^2*(a + b*atan(c*x^2))^2, x)
```

3.82 $\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx$

3.82.1 Optimal result 545
3.82.2 Mathematica [B] (warning: unable to verify) 546
3.82.3 Rubi [A] (verified) 546
3.82.4 Maple [F] 550
3.82.5 Fricas [F] 550
3.82.6 Sympy [F] 550
3.82.7 Maxima [F] 551
3.82.8 Giac [F] 551
3.82.9 Mupad [F(-1)] 551

3.82.1 Optimal result

Integrand size = 12, antiderivative size = 1191

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\text{Too large to display}$$

output
```
-1/2*(-1)^(3/4)*b^2*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1
/4)*x*c^(1/2)))/c^(1/2)+1/2*b^2*x*ln(1-I*c*x^2)*ln(1+I*c*x^2)-1/2*(-1)^(3/
4)*b^2*polylog(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2))
)/c^(1/2)-1/2*(-1)^(1/4)*b^2*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1
+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-1/2*(-1)^(1/4)*b^2*polylog(2,1-(1+I)*(1+(-
1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)+a^2*x-1/4*b^2*x*ln(1
-I*c*x^2)^2-1/4*b^2*x*ln(1+I*c*x^2)^2-2*(-1)^(3/4)*a*b*arctan((-1)^(3/4)*x
*c^(1/2))/c^(1/2)+2*(-1)^(3/4)*a*b*arctanh((-1)^(3/4)*x*c^(1/2))/c^(1/2)+2
*(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(1/4)*x*c^(1/2))
)/c^(1/2)-2*(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(1/4)
*x*c^(1/2)))/c^(1/2)+2*(-1)^(1/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(
1-(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-2*(-1)^(1/4)*b^2*arctanh((-1)^(3/4)*x*c^(
1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-I*a*b*x*ln(1+I*c*x^2)+(-1)^(3
/4)*b^2*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+(-1)^(3/4)*b^2*pol
ylog(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2/
(1-(-1)^(3/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2/(1+(-1)^(3/
4)*x*c^(1/2)))/c^(1/2)+(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))^2/c^(1/
2)-(-1)^(1/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))^2/c^(1/2)+(-1)^(1/4)*b^2*a
rctan((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)/c^(1/2)-(-1)^(1/4)*b^2*arctanh((
-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)/c^(1/2)-(-1)^(1/4)*b^2*arctan((-1)^(...
```

3.82.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4697 vs. 2(1191) = 2382.

Time = 38.10 (sec) , antiderivative size = 4697, normalized size of antiderivative = 3.94

$$\int (a+b\arctan(cx^2))^2\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2,x]
```

output
```
a^2*x + (a*b*Sqrt[c*x^2]*(2*Sqrt[c*x^2]*ArcTan[c*x^2] - Sqrt[2]*(ArcTan[(-
1 + c*x^2)/(Sqrt[2]*Sqrt[c*x^2])] - ArcTanh[(Sqrt[2]*Sqrt[c*x^2])/(1 + c*x
^2)])))/(c*x) + (b^2*Sqrt[c*x^2]*(2*Sqrt[c*x^2]*ArcTan[c*x^2]^2 - 4*((ArcT
an[c*x^2]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[
c*x^2]] + Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]] - Log[1 + c*x^2 + Sqrt[2]*S
qrt[c*x^2]]))/(2*Sqrt[2]) - (-((ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + ArcTan[1
 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]]) + (ArcTan[1
 - Sqrt[2]*Sqrt[c*x^2]] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 + c*x^2 +
 Sqrt[2]*Sqrt[c*x^2]] - (Sqrt[c*x^2]*(1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2)^(3/
2)*(2*(-5*ArcTan[2 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 4*ArcTan[1 - Sqr
t[2]*Sqrt[c*x^2]]^2 + ((1 + 2*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2
]]^2)/E^(I*ArcTan[2 + I]) + ((1 - 2*I)*Sqrt[1 - I]*ArcTan[1 - Sqrt[2]*Sqrt
[c*x^2]]^2)/E^ArcTanh[1 + 2*I] - (5*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*Arc
Tanh[1 + 2*I] + (5*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]])*L
og[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + 5*(
(-I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + ArcTanh[1 + 2*I])*Log[1 - E^((2*I)*
ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (5*I)*ArcTan[2 +
I]*Log[-Sin[ArcTan[2 + I] - ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]] - 5*ArcTanh[
1 + 2*I]*Log[Sin[ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + I*ArcTanh[1 + 2*I]]]) +
 5*PolyLog[2, E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]...
```

3.82.3 Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 1191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5347, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\, dx$$

$$\Big\downarrow \text{5347}$$

$$\int \Big(a^2+iab\log\left(1-icx^2\right)-iab\log\left(1+icx^2\right)-\frac{1}{4}b^2\log^2\left(1-icx^2\right)-\frac{1}{4}b^2\log^2\left(1+icx^2\right)+\frac{1}{2}b^2\log\left(1-icx^2\right)\text{lo}$$

$$\Big\downarrow \text{2009}$$

3.82. $\int \left(a+b\arctan\left(cx^2\right)\right)^2\, dx$

$$xa^2-\frac{2(-1)^{3/4}b\arctan\left((-1)^{3/4}\sqrt{c}x\right)a}{\sqrt{c}}+\frac{2(-1)^{3/4}b\,\mathrm{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)a}{\sqrt{c}}+ibx\log\left(1-icx^2\right)a-$$

$$ibx\log\left(icx^2+1\right)a+\frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2\mathrm{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}-$$

$$\frac{1}{4}b^2x\log^2\left(1-icx^2\right)-\frac{1}{4}b^2x\log^2\left(icx^2+1\right)+\frac{2\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{\sqrt{c}}-$$

$$\frac{2\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{\sqrt{c}}+$$

$$\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{\sqrt{2}\left(\sqrt{c}x+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{\sqrt{c}}+$$

$$\frac{2\sqrt[4]{-1}b^2\mathrm{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{\sqrt{c}}-$$

$$\frac{2\sqrt[4]{-1}b^2\mathrm{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{(-1)^{3/4}\sqrt{c}x+1}\right)}{\sqrt{c}}+$$

$$\frac{\sqrt[4]{-1}b^2\mathrm{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(-\frac{\sqrt{2}\left(\sqrt{c}x+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)}{\sqrt{c}}+$$

$$\frac{\sqrt[4]{-1}b^2\mathrm{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{c}x+1\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)}{\sqrt{c}}+$$

$$\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1-i)\left((-1)^{3/4}\sqrt{c}x+1\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{\sqrt{c}}+$$

$$\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2\mathrm{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{\sqrt{c}}-$$

$$\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(icx^2+1\right)}{\sqrt{c}}+\frac{\sqrt[4]{-1}b^2\mathrm{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(icx^2+1\right)}{\sqrt{c}}+$$

$$\frac{1}{2}b^2x\log\left(1-icx^2\right)\log\left(icx^2+1\right)+\frac{(-1)^{3/4}b^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{\sqrt{c}}+$$

$$\frac{(-1)^{3/4}b^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{\sqrt{c}}-\frac{(-1)^{3/4}b^2\,\mathrm{PolyLog}\left(2,1-\frac{\sqrt{2}\left(\sqrt{c}x+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{2\sqrt{c}}+$$

$$\frac{\sqrt[4]{-1}b^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{\sqrt{c}}+\frac{\sqrt[4]{-1}b^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{(-1)^{3/4}\sqrt{c}x+1}\right)}{\sqrt{c}}-$$

$$\frac{\sqrt[4]{-1}b^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt{2}\left(\sqrt{c}x+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{c}x+1}+1\right)}{2\sqrt{c}}-\frac{\sqrt[4]{-1}b^2\,\mathrm{PolyLog}\left(2,1-\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{c}x+1\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)}{2\sqrt{c}}-$$

$$\frac{(-1)^{3/4}b^2\,\mathrm{PolyLog}\left(2,1-\frac{(1-i)\left((-1)^{3/4}\sqrt{c}x+1\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{2\sqrt{c}}$$

3.82. $\int\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$

input
```
Int[(a + b*ArcTan[c*x^2])^2,x]
```

output
```
a^2*x - (2*(-1)^(3/4)*a*b*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] + ((-1)^(3
/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/Sqrt[c] + (2*(-1)^(3/4)*a*b*ArcTan
h[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt
[c]*x]^2)/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(
1 - (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*
Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*Ar
cTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1
)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c
]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTan
h[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)
^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt
[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTanh[(-
1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/
4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log
[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c]
 + I*a*b*x*Log[1 - I*c*x^2] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]
*Log[1 - I*c*x^2])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]
*Log[1 - I*c*x^2])/Sqrt[c] - (b^2*x*Log[1 - I*c*x^2]^2)/4 - I*a*b*x*Log[1
+ I*c*x^2] - ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2]
)/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x...
```

3.82.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5347
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[ExpandIn
tegrand[(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n])/2)^p, x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

3.82.4 Maple [F]

$$\int \left(a+b\arctan \left(c\,x^{2}\right)\right)^{2}dx$$

input
```
int((a+b*arctan(c*x^2))^2,x)
```

output
```
int((a+b*arctan(c*x^2))^2,x)
```

3.82.5 Fricas [F]

$$\int \left(a+b\arctan \left(cx^2\right)\right)^2\,dx=\int \left(b\arctan \left(cx^2\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2, x)
```

3.82.6 Sympy [F]

$$\int \left(a+b\arctan \left(cx^2\right)\right)^2\,dx=\int \left(a+b\operatorname{atan}\left(cx^2\right)\right)^2\,dx$$

input
```
integrate((a+b*atan(c*x**2))**2,x)
```

output
```
Integral((a + b*atan(c*x**2))**2, x)
```

3.82.7 Maxima [F]

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c
^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c
^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(
c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) - 4*x*arctan(c*x^2))*a*b + 1/16*(4
*x*arctan(c*x^2)^2 - x*log(c^2*x^4 + 1)^2 + 16*integrate(1/16*(8*c^2*x^4*l
og(c^2*x^4 + 1) - 16*c*x^2*arctan(c*x^2) + 12*(c^2*x^4 + 1)*arctan(c*x^2)^
2 + (c^2*x^4 + 1)*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1), x))*b^2 + a^2*x
```

3.82.8 Giac [F]

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2, x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,dx$$

input
```
int((a + b*atan(c*x^2))^2,x)
```

output
```
int((a + b*atan(c*x^2))^2, x)
```

3.83 $\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx$

3.83.1 Optimal result . . . 552
3.83.2 Mathematica [B] (warning: unable to verify) . . . 553
3.83.3 Rubi [A] (verified) . . . 553
3.83.4 Maple [F] . . . 557
3.83.5 Fricas [F] . . . 557
3.83.6 Sympy [F] . . . 557
3.83.7 Maxima [F] . . . 558
3.83.8 Giac [F] . . . 558
3.83.9 Mupad [F(-1)] . . . 558

3.83.1 Optimal result

Integrand size = 16, antiderivative size = 1164

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx = \text{Too large to display}$$

output
```
-1/4*(2*a+I*b*ln(1-I*c*x^2))^2/x-1/2*b^2*ln(1-I*c*x^2)*ln(1+I*c*x^2)/x-1/2
*(-1)^(1/4)*b^2*polylog(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x
*c^(1/2)))*c^(1/2)-1/2*(-1)^(3/4)*b^2*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^
(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)-1/2*(-1)^(3/4)*b^2*polylog(2,1-(1
+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)-1/2*(-1)^(1
/4)*b^2*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2
)))*c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))*c^(1/2)
+(-1)^(1/4)*b^2*polylog(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))*c^(1/2)+(-1)^(3/4)
*b^2*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))*c^(1/2)+(-1)^(3/4)*b^2*polylo
g(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)+(-1)^(1/4)*b^2*arctan((-1)^(3/4)
*x*c^(1/2))^2*c^(1/2)-(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))^2*c^(1/
2)+1/4*b^2*ln(1+I*c*x^2)^2/x-2*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2)
)*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)-2*(-1)^(1/4)*a*b*arctanh((-1)^(3/
4)*x*c^(1/2))*c^(1/2)-2*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(
1-(-1)^(1/4)*x*c^(1/2)))*c^(1/2)+2*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1
/2))*ln(2/(1+(-1)^(1/4)*x*c^(1/2)))*c^(1/2)+2*(-1)^(3/4)*b^2*arctanh((-1)^
(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))*c^(1/2)+I*a*b*ln(1+I*c*x^2
)/x-(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)*c^(1/2)-(-1
)^(1/4)*b*arctan((-1)^(3/4)*x*c^(1/2))*(2*a+I*b*ln(1-I*c*x^2))*c^(1/2)+(-1
)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)*c^(1/2)+(-1)^(3/...
```

3.83.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4697 vs. 2(1164) = 2328.

Time = 35.92 (sec) , antiderivative size = 4697, normalized size of antiderivative = 4.04

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^2,x]
```

output
```
-(a^2/x) + (a*b*(c*x^2)^(3/2)*((-2*ArcTan[c*x^2])/Sqrt[c*x^2] + Sqrt[2]*(A
rcTan[(-1 + c*x^2)/(Sqrt[2]*Sqrt[c*x^2])] + ArcTanh[(Sqrt[2]*Sqrt[c*x^2])/
(1 + c*x^2)])))/(c*x^3) + (b^2*(c*x^2)^(3/2)*((-2*ArcTan[c*x^2]^2)/Sqrt[c*
x^2] + 4*((ArcTan[c*x^2]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 2*ArcTan[1
+ Sqrt[2]*Sqrt[c*x^2]] - Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]] + Log[1 + c*
x^2 + Sqrt[2]*Sqrt[c*x^2]]))/(2*Sqrt[2]) - ((ArcTan[1 - Sqrt[2]*Sqrt[c*x^2
]] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]]
 - (ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log
[1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2]] - (Sqrt[c*x^2]*(1 + (1 - Sqrt[2]*Sqrt[c*
x^2])^2)^(3/2)*(2*(-5*ArcTan[2 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 4*Ar
cTan[1 - Sqrt[2]*Sqrt[c*x^2]]^2 + ((1 + 2*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2
]*Sqrt[c*x^2]]^2)/E^(I*ArcTan[2 + I]) + ((1 - 2*I)*Sqrt[1 - I]*ArcTan[1 -
Sqrt[2]*Sqrt[c*x^2]]^2)/E^ArcTanh[1 + 2*I] - (5*I)*ArcTan[1 - Sqrt[2]*Sqrt
[c*x^2]]*ArcTanh[1 + 2*I] + (5*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqr
t[c*x^2]])*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^
2]]))] + 5*((-I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + ArcTanh[1 + 2*I])*Log[1
 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (5*I)
*ArcTan[2 + I]*Log[-Sin[ArcTan[2 + I] - ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]]
- 5*ArcTanh[1 + 2*I]*Log[Sin[ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + I*ArcTanh[1
 + 2*I]]]) + 5*PolyLog[2, E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]...
```

3.83.3 Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 1164, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5365, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.83. $\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx$

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}\,dx$$

$$\Big\downarrow \; 5365$$

$$\int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^2}+\frac{b\log\left(1+icx^2\right)\left(b\log\left(1-icx^2\right)-2ia\right)}{2x^2}-\frac{b^2\log^2\left(1+icx^2\right)}{4x^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\sqrt[4]{-1}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 b^2-(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2 b^2+\frac{\log^2\left(icx^2+1\right)b^2}{4x}-$$
$$2(-1)^{3/4}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)b^2+$$
$$2(-1)^{3/4}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{\sqrt[4]{-1}\sqrt{c}x+1}\right)b^2-$$
$$(-1)^{3/4}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{\sqrt{2}(\sqrt{c}x+\sqrt[4]{-1})}{\sqrt[4]{-1}\sqrt{c}x+1}\right)b^2+$$
$$2(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)b^2-$$
$$2(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{(-1)^{3/4}\sqrt{c}x+1}\right)b^2+$$
$$(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(-\frac{\sqrt{2}(\sqrt{c}x+(-1)^{3/4})}{(-1)^{3/4}\sqrt{c}x+1}\right)b^2+$$
$$(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{c}x+1\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)b^2-$$
$$(-1)^{3/4}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1-i)\left((-1)^{3/4}\sqrt{c}x+1\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)b^2-$$
$$(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)b^2+$$
$$(-1)^{3/4}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(icx^2+1\right)b^2+$$
$$(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(icx^2+1\right)b^2-\frac{\log\left(1-icx^2\right)\log\left(icx^2+1\right)b^2}{2x}+$$
$$\sqrt[4]{-1}\sqrt{c}\operatorname{PolyLog}\left(2,1-\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)b^2+\sqrt[4]{-1}\sqrt{c}\operatorname{PolyLog}\left(2,1-\frac{2}{\sqrt[4]{-1}\sqrt{c}x+1}\right)b^2-$$
$$\frac{1}{2}\sqrt[4]{-1}\sqrt{c}\operatorname{PolyLog}\left(2,1-\frac{\sqrt{2}(\sqrt{c}x+\sqrt[4]{-1})}{\sqrt[4]{-1}\sqrt{c}x+1}\right)b^2+$$
$$(-1)^{3/4}\sqrt{c}\operatorname{PolyLog}\left(2,1-\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)b^2+$$
$$(-1)^{3/4}\sqrt{c}\operatorname{PolyLog}\left(2,1-\frac{2}{(-1)^{3/4}\sqrt{c}x+1}\right)b^2-$$
$$\frac{1}{2}(-1)^{3/4}\sqrt{c}\operatorname{PolyLog}\left(2,\frac{\sqrt{2}(\sqrt{c}x+(-1)^{3/4})}{(-1)^{3/4}\sqrt{c}x+1}+1\right)b^2-$$
$$\frac{1}{2}(-1)^{3/4}\sqrt{c}\operatorname{PolyLog}\left(2,1-\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{c}x+1\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)b^2-$$
$$\frac{1}{2}\sqrt[4]{-1}\sqrt{c}\operatorname{PolyLog}\left(2,1-\frac{(1-i)\left((-1)^{3/4}\sqrt{c}x+1\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)b^2-2\sqrt[4]{-1}a\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)b-$$
$$\sqrt[4]{-1}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log\left(1-icx^2\right)\right)b+\frac{ia\log\left(icx^2+1\right)b}{x}-$$
$$\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x}$$

3.83. $\int\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}\,dx$

input
```
Int[(a + b*ArcTan[c*x^2])^2/x^2,x]
```

output
```
(-1)^(1/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2 - 2*(-1)^(1/4)*a*b*S
qrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x] - (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)
^(3/4)*Sqrt[c]*x]^2 - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x
]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)
^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[
c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1
+ (-1)^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqr
t[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)] - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTa
nh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)] + (-1)^(3/4)*b^
2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[
c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))] + (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^
(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*
Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1
- I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*
b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2] - (-1)^(1/4)*b*
Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*Log[1 - I*c*x^2]) - (2*a +
 I*b*Log[1 - I*c*x^2])^2/(4*x) + (I*a*b*Log[1 + I*c*x^2])/x + (-1)^(3/4)*b
^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] + (-1)^(3/4)*b^2*
Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] - (b^2*Log[1 - I*c*
x^2]*Log[1 + I*c*x^2])/(2*x) + (b^2*Log[1 + I*c*x^2]^2)/(4*x) + (-1)^(1...
```

3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5365
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

3.83.4 Maple [F]

$$\int \frac{(a+b\arctan(c\,x^2))^2}{x^2}dx$$

input
```
int((a+b*arctan(c*x^2))^2/x^2,x)
```

output
```
int((a+b*arctan(c*x^2))^2/x^2,x)
```

3.83.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^2, x)
```

3.83.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx^2))^2}{x^2}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**2/x**2,x)
```

output
```
Integral((a + b*atan(c*x**2))**2/x**2, x)
```

3.83.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx=\int \frac{(b\arctan(cx^2)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="maxima")
```

output
```
1/2*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sq
rt(c) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sq
rt(c) + sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c
*x^2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c)) - 4*arctan(c*x^2)/x)*a*b - 1/16*(4*
arctan(c*x^2)^2 - 16*x*integrate(-1/16*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*
x^2*arctan(c*x^2) - 12*(c^2*x^4 + 1)*arctan(c*x^2)^2 - (c^2*x^4 + 1)*log(c
^2*x^4 + 1)^2)/(c^2*x^6 + x^2), x) - log(c^2*x^4 + 1)^2)*b^2/x - a^2/x
```

3.83.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx=\int \frac{(b\arctan(cx^2)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2/x^2, x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x^2))^2}{x^2}\,dx$$

input
```
int((a + b*atan(c*x^2))^2/x^2,x)
```

output
```
int((a + b*atan(c*x^2))^2/x^2, x)
```

3.84 $\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx$

3.84.1 Optimal result 559
3.84.2 Mathematica [F] 560
3.84.3 Rubi [A] (verified) 560
3.84.4 Maple [F] 563
3.84.5 Fricas [F] 563
3.84.6 Sympy [F] 563
3.84.7 Maxima [F] 564
3.84.8 Giac [F] 564
3.84.9 Mupad [F(-1)] 564

3.84.1 Optimal result

Integrand size = 16, antiderivative size = 1360

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx = \text{Too large to display}$$

output
```
-1/12*(2*a+I*b*ln(1-I*c*x^2))^2/x^3-1/3*b*c*(2*a+I*b*ln(1-I*c*x^2))/x-1/6*
b^2*ln(1-I*c*x^2)*ln(1+I*c*x^2)/x^3+1/3*(-1)^(3/4)*b^2*c^(3/2)*polylog(2,1
-2/(1-(-1)^(1/4)*x*c^(1/2)))+1/3*(-1)^(3/4)*b^2*c^(3/2)*polylog(2,1-2/(1+(
-1)^(1/4)*x*c^(1/2)))-1/6*(-1)^(3/4)*b^2*c^(3/2)*polylog(2,1-2^(1/2)*((-1)
^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*pol
ylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*polylog(2,
1-2/(1+(-1)^(3/4)*x*c^(1/2)))-1/6*(-1)^(1/4)*b^2*c^(3/2)*polylog(2,1+2^(1/
2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))-1/6*(-1)^(1/4)*b^2*c^(
3/2)*polylog(2,1-(1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))-
1/6*(-1)^(3/4)*b^2*c^(3/2)*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+
(-1)^(1/4)*x*c^(1/2)))-2/3*a*b*c/x-4/3*(-1)^(1/4)*b^2*c^(3/2)*arctan((-1)^
(3/4)*x*c^(1/2))+1/3*(-1)^(3/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*x*c^(1/2))^2
-4/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))-1/3*(-1)^(1/4)*b
^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))^2+1/12*b^2*ln(1+I*c*x^2)^2/x^3+1/
3*I*a*b*ln(1+I*c*x^2)/x^3+2/3*I*b^2*c*ln(1+I*c*x^2)/x+1/3*(-1)^(1/4)*b^2*c
^(3/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(
-1)^(1/4)*x*c^(1/2)))+2/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1
/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))-2/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1
)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/
2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+...
```

3.84.2 Mathematica [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx=\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^4,x]
```

output
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^4, x]
```

3.84.3 Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 1360, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5365, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx$$

$$\Big\downarrow \text{5365}$$

$$\int \left(\frac{(2a+ib\log(1-icx^2))^2}{4x^4}+\frac{b\log(1+icx^2)\,(b\log(1-icx^2)-2ia)}{2x^4}-\frac{b^2\log^2(1+icx^2)}{4x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}(-1)^{3/4}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)^2 b^2-\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)^2 b^2+\frac{\log^2\left(icx^2+1\right)b^2}{12x^3}-\frac{4}{3}\sqrt[4]{-1}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)b^2-\frac{4}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)b^2+\frac{2}{3}\sqrt[4]{-1}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{cx}}\right)b^2-\frac{2}{3}\sqrt[4]{-1}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{\sqrt[4]{-1}\sqrt{cx}+1}\right)b^2+\frac{1}{3}\sqrt[4]{-1}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{\sqrt{2}\left(\sqrt{cx}+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)b^2+\frac{2}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{cx}}\right)b^2-\frac{2}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{(-1)^{3/4}\sqrt{cx}+1}\right)b^2+\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(-\frac{\sqrt{2}\left(\sqrt{cx}+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{cx}+1}\right)b^2+\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{cx}+1\right)}{(-1)^{3/4}\sqrt{cx}+1}\right)b^2+\frac{1}{3}\sqrt[4]{-1}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{(1-i)\left((-1)^{3/4}\sqrt{cx}+1\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)b^2-\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(1-icx^2\right)b^2-\frac{ic\log\left(1-icx^2\right)b^2}{3x}-\frac{1}{3}\sqrt[4]{-1}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(icx^2+1\right)b^2+\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(icx^2+1\right)b^2-\frac{\log\left(1-icx^2\right)\log\left(icx^2+1\right)b^2}{6x^3}+\frac{2ic\log\left(icx^2+1\right)b^2}{3x}+\frac{1}{3}(-1)^{3/4}c^{3/2}\text{PolyLog}\left(2,1-\frac{2}{1-\sqrt[4]{-1}\sqrt{cx}}\right)b^2+\frac{1}{3}(-1)^{3/4}c^{3/2}\text{PolyLog}\left(2,1-\frac{2}{\sqrt[4]{-1}\sqrt{cx}+1}\right)b^2-\frac{1}{6}(-1)^{3/4}c^{3/2}\text{PolyLog}\left(2,1-\frac{\sqrt{2}\left(\sqrt{cx}+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)b^2+\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{PolyLog}\left(2,1-\frac{2}{1-(-1)^{3/4}\sqrt{cx}}\right)b^2+\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{PolyLog}\left(2,1-\frac{2}{(-1)^{3/4}\sqrt{cx}+1}\right)b^2-\frac{1}{6}\sqrt[4]{-1}c^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{2}\left(\sqrt{cx}+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{cx}+1}+1\right)b^2-\frac{1}{6}\sqrt[4]{-1}c^{3/2}\text{PolyLog}\left(2,1-\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{cx}+1\right)}{(-1)^{3/4}\sqrt{cx}+1}\right)b^2-\frac{1}{6}(-1)^{3/4}c^{3/2}\text{PolyLog}\left(2,1-\frac{(1-i)\left((-1)^{3/4}\sqrt{cx}+1\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)b^2+\frac{2}{3}(-1)^{3/4}ac^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)b-\frac{1}{3}(-1)^{3/4}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{cx}\right)\left(2a+ib\log\left(1-icx^2\right)\right)b-\frac{c\left(2a+ib\log\left(1-icx^2\right)\right)b}{3x}+\frac{ia\log\left(icx^2+1\right)b}{3x^3}-\frac{2acb}{3x}-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{12x^3}$$

3.84. $\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx$

input
```
Int[(a + b*ArcTan[c*x^2])^2/x^4,x]
```

output
```
(-2*a*b*c)/(3*x) - (4*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])
/3 + ((-1)^(3/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(
3/4)*a*b*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - (4*(-1)^(1/4)*b^2*c^(3
/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1
)^(3/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt
[c]*x]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1/4)*b^2*c^(3/2)*Ar
cTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + ((-1)^(1
/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sq
rt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTa
nh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1
/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c
]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sq
rt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/3 + ((-1)^(1
/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)
*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*ArcT
an[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1
)^(1/4)*Sqrt[c]*x)])/3 - ((I/3)*b^2*c*Log[1 - I*c*x^2])/x - ((-1)^(1/4)*b^
2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/3 - (b*c*(2*a +
I*b*Log[1 - I*c*x^2]))/(3*x) - ((-1)^(3/4)*b*c^(3/2)*ArcTan[(-1)^(3/4)*Sqr
t[c]*x]*(2*a + I*b*Log[1 - I*c*x^2]))/3 - (2*a + I*b*Log[1 - I*c*x^2])^...
```

3.84.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5365
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

3.84.4 Maple [F]

$$\int \frac{(a+b\arctan(c\,x^2))^2}{x^4}dx$$

input
```
int((a+b*arctan(c*x^2))^2/x^4,x)
```

output
```
int((a+b*arctan(c*x^2))^2/x^4,x)
```

3.84.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^4, x)
```

3.84.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx^2))^2}{x^4}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**2/x**4,x)
```

output
```
Integral((a + b*atan(c*x**2))**2/x**4, x)
```

3.84.7 Maxima [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x^4}\,dx = \int \frac{(b\arctan(cx^2) + a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="maxima")
```

output
```
-1/6*((c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c)
)/c^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c)
)/c^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*l
og(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) + 8/x)*c + 4*arctan(c*x^2)/x^3)
*a*b + 1/48*(48*x^3*integrate(-1/48*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x^2
*arctan(c*x^2) - 36*(c^2*x^4 + 1)*arctan(c*x^2)^2 - 3*(c^2*x^4 + 1)*log(c^
2*x^4 + 1)^2)/(c^2*x^8 + x^4), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2
)*b^2/x^3 - 1/3*a^2/x^3
```

3.84.8 Giac [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x^4}\,dx = \int \frac{(b\arctan(cx^2) + a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2/x^4, x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx^2))^2}{x^4}\,dx = \int \frac{(a + b\,\mathrm{atan}(c\,x^2))^2}{x^4}\,dx$$

input
```
int((a + b*atan(c*x^2))^2/x^4,x)
```

output
```
int((a + b*atan(c*x^2))^2/x^4, x)
```

3.85 $\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx$

3.85.1 Optimal result 565
3.85.2 Mathematica [F] 566
3.85.3 Rubi [A] (verified) 566
3.85.4 Maple [F] 569
3.85.5 Fricas [F] 569
3.85.6 Sympy [F] 569
3.85.7 Maxima [F] 570
3.85.8 Giac [F] 570
3.85.9 Mupad [F(-1)] 570

3.85.1 Optimal result

Integrand size = 16, antiderivative size = 1444

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx = \text{Too large to display}$$

output
```
-1/20*(2*a+I*b*ln(1-I*c*x^2))^2/x^5-2/15*a*b*c/x^3-4/15*(-1)^(3/4)*b^2*c^(
5/2)*arctan((-1)^(3/4)*x*c^(1/2))-1/5*(-1)^(1/4)*b^2*c^(5/2)*arctan((-1)^(
3/4)*x*c^(1/2))^2+4/15*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2)
)+1/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))^2-1/5*b^2*c^2*l
n(1-I*c*x^2)/x-1/15*b*c*(2*a+I*b*ln(1-I*c*x^2))/x^3-1/10*b^2*ln(1-I*c*x^2)
*ln(1+I*c*x^2)/x^5-1/5*(-1)^(1/4)*b^2*c^(5/2)*polylog(2,1-2/(1-(-1)^(1/4)*
x*c^(1/2)))+1/20*b^2*ln(1+I*c*x^2)^2/x^5+1/5*I*a*b*ln(1+I*c*x^2)/x^5+2/15*
I*b^2*c*ln(1+I*c*x^2)/x^3+2/5*I*a*b*c^2/x-1/5*(-1)^(1/4)*b^2*c^(5/2)*polyl
og(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))+1/10*(-1)^(1/4)*b^2*c^(5/2)*polylog(2,1
-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))-1/5*(-1)^(3/4)*b
^2*c^(5/2)*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))-1/5*(-1)^(3/4)*b^2*c^(5
/2)*polylog(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))+1/10*(-1)^(3/4)*b^2*c^(5/2)*po
lylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))+1/10*(-
1)^(3/4)*b^2*c^(5/2)*polylog(2,1-(1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3
/4)*x*c^(1/2)))+1/10*(-1)^(1/4)*b^2*c^(5/2)*polylog(2,1+(-1+I)*(1+(-1)^(3/
4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))-8/15*b^2*c^2/x-1/5*(-1)^(3/4)*b^2*
c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(
1+(-1)^(3/4)*x*c^(1/2)))-1/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*x*c
^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))+1/5*(-
1)^(3/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln((1-I)*(1+(-1)^(3/4...
```

3.85.2 Mathematica [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx=\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^6,x]
```

output
```
Integrate[(a + b*ArcTan[c*x^2])^2/x^6, x]
```

3.85.3 Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 1444, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5365, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx$$

$$\Big\downarrow \; 5365$$

$$\int \left(\frac{(2a+ib\log(1-icx^2))^2}{4x^6}+\frac{b\log(1+icx^2)\,(b\log(1-icx^2)-2ia)}{2x^6}-\frac{b^2\log^2(1+icx^2)}{4x^6}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{5}\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)^2 c^{5/2}+\frac{1}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)^2 c^{5/2}-$$
$$\frac{4}{15}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)c^{5/2}+\frac{4}{15}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)c^{5/2}+$$
$$\frac{2}{5}\sqrt[4]{-1}ab\,\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)c^{5/2}+\frac{2}{5}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{cx}}\right)c^{5/2}-$$
$$\frac{2}{5}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{\sqrt[4]{-1}\sqrt{cx}+1}\right)c^{5/2}+$$
$$\frac{1}{5}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{\sqrt{2}\left(\sqrt{cx}+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)c^{5/2}-$$
$$\frac{2}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{cx}}\right)c^{5/2}+$$
$$\frac{2}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{2}{(-1)^{3/4}\sqrt{cx}+1}\right)c^{5/2}-$$
$$\frac{1}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(-\frac{\sqrt{2}\left(\sqrt{cx}+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{cx}+1}\right)c^{5/2}-$$
$$\frac{1}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{cx}+1\right)}{(-1)^{3/4}\sqrt{cx}+1}\right)c^{5/2}+$$
$$\frac{1}{5}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(\frac{(1-i)\left((-1)^{3/4}\sqrt{cx}+1\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)c^{5/2}+$$
$$\frac{1}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(1-icx^2\right)c^{5/2}+$$
$$\frac{1}{5}\sqrt[4]{-1}b\arctan\left((-1)^{3/4}\sqrt{cx}\right)\left(2a+ib\log\left(1-icx^2\right)\right)c^{5/2}-$$
$$\frac{1}{5}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{cx}\right)\log\left(icx^2+1\right)c^{5/2}-$$
$$\frac{1}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{cx}\right)\log\left(icx^2+1\right)c^{5/2}-$$
$$\frac{1}{5}\sqrt[4]{-1}b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\sqrt[4]{-1}\sqrt{cx}}\right)c^{5/2}-\frac{1}{5}\sqrt[4]{-1}b^2\,\text{PolyLog}\left(2,1-\frac{2}{\sqrt[4]{-1}\sqrt{cx}+1}\right)c^{5/2}+$$
$$\frac{1}{10}\sqrt[4]{-1}b^2\,\text{PolyLog}\left(2,1-\frac{\sqrt{2}\left(\sqrt{cx}+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)c^{5/2}-$$
$$\frac{1}{5}(-1)^{3/4}b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-(-1)^{3/4}\sqrt{cx}}\right)c^{5/2}-$$
$$\frac{1}{5}(-1)^{3/4}b^2\,\text{PolyLog}\left(2,1-\frac{2}{(-1)^{3/4}\sqrt{cx}+1}\right)c^{5/2}+$$
$$\frac{1}{10}(-1)^{3/4}b^2\,\text{PolyLog}\left(2,\frac{\sqrt{2}\left(\sqrt{cx}+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{cx}+1}+1\right)c^{5/2}+$$
$$\frac{1}{10}(-1)^{3/4}b^2\,\text{PolyLog}\left(2,1-\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{cx}+1\right)}{(-1)^{3/4}\sqrt{cx}+1}\right)c^{5/2}+$$
$$\frac{1}{10}\sqrt[4]{-1}b^2\,\text{PolyLog}\left(2,1-\frac{(1-i)\left((-1)^{3/4}\sqrt{cx}+1\right)}{\sqrt[4]{-1}\sqrt{cx}+1}\right)c^{5/2}-\frac{b^2\log\left(1-icx^2\right)c^2}{5x}-$$
$$\frac{ib\left(2a+ib\log\left(1-icx^2\right)\right)c^2}{5x}-\frac{8b^2c^2}{15x}+\frac{2iabc^2}{5x}-\frac{ib^2\log\left(1-icx^2\right)c}{15x^3}-\frac{b\left(2a+ib\log\left(1-icx^2\right)\right)c}{15x^3}+$$
$$\frac{2ib^2\log\left(icx^2+1\right)c}{15x^3}-\frac{2abc}{15x^3}-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{20x^5}+\frac{b^2\log^2\left(icx^2+1\right)}{20x^5}-$$
$$\frac{b^2\log\left(1-icx^2\right)\log\left(icx^2+1\right)}{10x^5}+\frac{iab\log\left(icx^2+1\right)}{5x^5}$$

3.85. $\int\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^6}\,dx$

input
```
Int[(a + b*ArcTan[c*x^2])^2/x^6,x]
```

output
```
(-2*a*b*c)/(15*x^3) + (((2*I)/5)*a*b*c^2)/x - (8*b^2*c^2)/(15*x) - (4*(-1)
^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/15 - ((-1)^(1/4)*b^2*c^(5
/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/5 + (2*(-1)^(1/4)*a*b*c^(5/2)*ArcTanh[
(-1)^(3/4)*Sqrt[c]*x])/5 + (4*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sq
rt[c]*x])/15 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/5
+ (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(
1/4)*Sqrt[c]*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*
x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4
)*Sqrt[c]*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]
*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/5 + (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[
(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/5 - ((-1)^(3/4)*b
^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt
[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/5 - ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[
(-1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(
3/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]
*Log[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 -
 ((I/15)*b^2*c*Log[1 - I*c*x^2])/x^3 - (b^2*c^2*Log[1 - I*c*x^2])/(5*x) +
((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/5
- (b*c*(2*a + I*b*Log[1 - I*c*x^2]))/(15*x^3) - ((I/5)*b*c^2*(2*a + I*b...
```

3.85.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5365
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

3.85.4 Maple [F]

$$\int \frac{\left(a+b\arctan\left(c\,x^{2}\right)\right)^{2}}{x^{6}}dx$$

input
```
int((a+b*arctan(c*x^2))^2/x^6,x)
```

output
```
int((a+b*arctan(c*x^2))^2/x^6,x)
```

3.85.5 Fricas [F]

$$\int \frac{\left(a+b\arctan\left(cx^{2}\right)\right)^{2}}{x^{6}}\,dx=\int \frac{\left(b\arctan\left(cx^{2}\right)+a\right)^{2}}{x^{6}}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^6, x)
```

3.85.6 Sympy [F]

$$\int \frac{\left(a+b\arctan\left(cx^{2}\right)\right)^{2}}{x^{6}}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(cx^{2}\right)\right)^{2}}{x^{6}}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**2/x**6,x)
```

output
```
Integral((a + b*atan(c*x**2))**2/x**6, x)
```

3.85.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^6}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="maxima")
```

output
```
-1/30*((6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqr
t(c)) + 6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqr
t(c)) + 3*sqrt(2)*c^(3/2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1) - 3*sqrt(2)*c
^(3/2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1) + 8/x^3)*c + 12*arctan(c*x^2)/x^
5)*a*b + 1/80*(80*x^5*integrate(-1/80*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x
^2*arctan(c*x^2) - 60*(c^2*x^4 + 1)*arctan(c*x^2)^2 - 5*(c^2*x^4 + 1)*log(
c^2*x^4 + 1)^2)/(c^2*x^10 + x^6), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1
)^2)*b^2/x^5 - 1/5*a^2/x^5
```

3.85.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^6}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2/x^6, x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^2))^2}{x^6}\,dx$$

input
```
int((a + b*atan(c*x^2))^2/x^6,x)
```

output
```
int((a + b*atan(c*x^2))^2/x^6, x)
```

3.86 $\int x^3(a+b\arctan(cx^2))^3\,dx$

3.86.1 Optimal result . . . 571
3.86.2 Mathematica [A] (verified) . . . 571
3.86.3 Rubi [A] (verified) . . . 572
3.86.4 Maple [C] (warning: unable to verify) . . . 575
3.86.5 Fricas [F] . . . 577
3.86.6 Sympy [F] . . . 577
3.86.7 Maxima [F] . . . 577
3.86.8 Giac [F] . . . 578
3.86.9 Mupad [F(-1)] . . . 578

3.86.1 Optimal result

Integrand size = 16, antiderivative size = 149

$$\begin{aligned}\int x^3(a+b\arctan(cx^2))^3\,dx &= -\frac{3ib(a+b\arctan(cx^2))^2}{4c^2}-\frac{3bx^2(a+b\arctan(cx^2))^2}{4c}\\&\quad+\frac{(a+b\arctan(cx^2))^3}{4c^2}+\frac{1}{4}x^4(a+b\arctan(cx^2))^3\\&\quad-\frac{3b^2(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{2c^2}\\&\quad-\frac{3ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{4c^2}\end{aligned}$$

output
```
-3/4*I*b*(a+b*arctan(c*x^2))^2/c^2-3/4*b*x^2*(a+b*arctan(c*x^2))^2/c+1/4*(
a+b*arctan(c*x^2))^3/c^2+1/4*x^4*(a+b*arctan(c*x^2))^3-3/2*b^2*(a+b*arctan
(c*x^2))*ln(2/(1+I*c*x^2))/c^2-3/4*I*b^3*polylog(2,1-2/(1+I*c*x^2))/c^2
```

3.86.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.14

$$\int x^3(a+b\arctan(cx^2))^3\,dx$$

$$= \frac{3b^2(a+ac^2x^4+b(i-cx^2))\arctan(cx^2)^2+b^3(1+c^2x^4)\arctan(cx^2)^3+3b\arctan(cx^2)\left(a(a-2bcx^2+a\right.}{}$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x^2])^3,x]
```

output
```
(3*b^2*(a + a*c^2*x^4 + b*(I - c*x^2))*ArcTan[c*x^2]^2 + b^3*(1 + c^2*x^4)
*ArcTan[c*x^2]^3 + 3*b*ArcTan[c*x^2]*(a*(a - 2*b*c*x^2 + a*c^2*x^4) - 2*b^
2*Log[1 + E^((2*I)*ArcTan[c*x^2])]) + a*(a*c*x^2*(-3*b + a*c*x^2) + 3*b^2*
Log[1 + c^2*x^4]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])])/(4*c^2
)
```

3.86.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5363, 5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b \arctan \left(c x^2\right)\right)^3 \, dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2} \int x^2 \left(a+b \arctan \left(c x^2\right)\right)^3 dx^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2} \left(\frac{1}{2} x^4 \left(a+b \arctan \left(c x^2\right)\right)^3-\frac{3}{2} b c \int \frac{x^4 \left(a+b \arctan \left(c x^2\right)\right)^2}{c^2 x^4+1}dx^2\right)$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{2} \left(\frac{1}{2} x^4 \left(a+b \arctan \left(c x^2\right)\right)^3-\frac{3}{2} b c \left(\frac{\int \left(a+b \arctan \left(c x^2\right)\right)^2 dx^2}{c^2}-\frac{\int \frac{\left(a+b \arctan \left(c x^2\right)\right)^2}{c^2 x^4+1}dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{5345}$$

$$\frac{1}{2} \left(\frac{1}{2} x^4 \left(a+b \arctan \left(c x^2\right)\right)^3-\frac{3}{2} b c \left(\frac{x^2 \left(a+b \arctan \left(c x^2\right)\right)^2-2 b c \int \frac{x^2 \left(a+b \arctan \left(c x^2\right)\right)}{c^2 x^4+1}dx^2}{c^2}-\frac{\int \frac{\left(a+b \arctan \left(c x^2\right)\right)^2}{c^2 x^4+1}d\right.\right.$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\int\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)}{c^2x^4+1}dx^2}{c^2}-\frac{\left(a+b\arctan\left(cx^2\right)\right)}{3bc^3}\right.\right.$$

$\downarrow$ 5455

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{3bc^3}+\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{\int\frac{a+b\arctan\left(cx^2\right)}{i-cx^2}dx}{c}\right.}{c^2}\right.\right.$$

$\downarrow$ 5379

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{3bc^3}+\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\,\mathrm{ar}\right.}{c}\right.}{c^2}\right.\right.$$

$\downarrow$ 2849

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{3bc^3}+\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{ib\int\frac{\log\left(\frac{2}{icx^2+1}\right)}{1-\frac{2}{icx^2+1}}d\frac{}{ic}}{c}\right.}{}\right.\right.$$

$\downarrow$ 2752

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{3bc^3}+\frac{x^2\left(a+b\arctan\left(cx^2\right)\right)^2-2bc\left(-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2bc^2}\right.}{c}\right.\right.$$

3.86. $\int x^3\left(a+b\arctan\left(cx^2\right)\right)^3\,dx$

input
```
Int[x^3*(a + b*ArcTan[c*x^2])^3,x]
```

output
```
((x^4*(a + b*ArcTan[c*x^2])^3)/2 - (3*b*c*(-1/3*(a + b*ArcTan[c*x^2])^3/(b
*c^3) + (x^2*(a + b*ArcTan[c*x^2])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^
2])^2)/(b*c^2) - (((a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/c + ((I/2)*
b*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c)/c))/c^2))/2)/2
```

3.86.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.86.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.66 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.68

method	result
default	$\frac{a^3x^4}{4}+\frac{b^3x^4\arctan(cx^2)^3}{4}-\frac{3b^3\arctan(cx^2)^2x^2}{4c}+\frac{b^3\arctan(cx^2)^3}{4c^2}+\frac{3b^3\ln(c^2x^4+1)\arctan(cx^2)}{4c^2}-\frac{3b^3\left(\sum_{\_\_\alpha=\mathrm{RootOf}(c^2\ldots}\right.}{\ldots}$
parts	$\frac{a^3x^4}{4}+\frac{b^3x^4\arctan(cx^2)^3}{4}-\frac{3b^3\arctan(cx^2)^2x^2}{4c}+\frac{b^3\arctan(cx^2)^3}{4c^2}+\frac{3b^3\ln(c^2x^4+1)\arctan(cx^2)}{4c^2}-\frac{3b^3\left(\sum_{\_\_\alpha=\mathrm{RootOf}(c^2\ldots}\right.}{\ldots}$
risch	$\frac{3a\,b^2\ln(c^2x^4+1)}{4c^2}+\frac{3ib^2\left(\sum_{\_\_\alpha=\mathrm{RootOf}\left(c\_\_Z^2-\mathrm{RootOf}\left(\_\_Z^2+1,\mathrm{index}=1\right)\right)}\frac{\ln(x-\_\_\alpha)\ln\left(-icx^2+1\right)+2c\left(-\frac{\ln(x-\_\_\alpha)\left(\ln\left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(i\sqrt{\frac{i}{c}}\ldots\right.}{\sqrt{\ldots}}\right.\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$

input

```
int(x^3*(a+b*arctan(c*x^2))^3,x,method=_RETURNVERBOSE)
```

output

```
1/4*a^3*x^4+1/4*b^3*x^4*arctan(c*x^2)^3-3/4*b^3*arctan(c*x^2)^2/c*x^2+1/4*
b^3*arctan(c*x^2)^3/c^2+3/4*b^3/c^2*ln(c^2*x^4+1)*arctan(c*x^2)-3/16*b^3/c
^3*sum(1/_alpha^2*(2*ln(x-_alpha)*ln(c^2*x^4+1)-c*(1/c/_alpha^3*ln(x-_alph
a)^2+2/_alpha*ln(x-_alpha)*(_alpha^2*ln(1/2*(x+_alpha)/_alpha)*c-ln((_alph
a^3*c+x)/_alpha/(_alpha^2*c+1))+ln((_alpha^3*c-x)/_alpha/(_alpha^2*c-1)))+
2/_alpha*(_alpha^2*dilog(1/2*(x+_alpha)/_alpha)*c-dilog((_alpha^3*c+x)/_al
pha/(_alpha^2*c+1))+dilog((_alpha^3*c-x)/_alpha/(_alpha^2*c-1))))),_alpha=
RootOf(_Z^4*c^2+1))+3/4*a*b^2*x^4*arctan(c*x^2)^2-3/2*a*b^2*arctan(c*x^2)/
c*x^2+3/4*a*b^2/c^2*arctan(c*x^2)^2+3/4*a*b^2/c^2*ln(c^2*x^4+1)+3/4*a^2*b*
x^4*arctan(c*x^2)-3/4*a^2*b/c*x^2+3/4*a^2*b/c^2*arctan(c*x^2)
```

3.86. $\int x^3(a+b\arctan(cx^2))^3\,dx$

3.86.5 Fricas [F]

$$\int x^3 \left(a + b\arctan\left(cx^2\right)\right)^3 \, dx = \int \left(b\arctan\left(cx^2\right) + a\right)^3 x^3 \, dx$$

input
```
integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arctan(c*x^2)^3 + 3*a*b^2*x^3*arctan(c*x^2)^2 + 3*a^2*b*x
^3*arctan(c*x^2) + a^3*x^3, x)
```

3.86.6 Sympy [F]

$$\int x^3 \left(a + b\arctan\left(cx^2\right)\right)^3 \, dx = \int x^3 \left(a + b\,\mathrm{atan}\left(cx^2\right)\right)^3 \, dx$$

input
```
integrate(x**3*(a+b*atan(c*x**2))**3,x)
```

output
```
Integral(x**3*(a + b*atan(c*x**2))**3, x)
```

3.86.7 Maxima [F]

$$\int x^3 \left(a + b\arctan\left(cx^2\right)\right)^3 \, dx = \int \left(b\arctan\left(cx^2\right) + a\right)^3 x^3 \, dx$$

input
```
integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

output
```
3/4*a*b^2*x^4*arctan(c*x^2)^2 + 1/4*a^3*x^4 + 3/4*(x^4*arctan(c*x^2) - c*(
x^2/c^2 - arctan(c*x^2)/c^3))*a^2*b - 3/4*(2*c*(x^2/c^2 - arctan(c*x^2)/c^
3)*arctan(c*x^2) + (arctan(c*x^2)^2 - log(4*c^5*x^4 + 4*c^3))/c^2)*a*b^2 +
 1/128*(4*x^4*arctan(c*x^2)^3 - 3*x^4*arctan(c*x^2)*log(c^2*x^4 + 1)^2 + 1
28*integrate(1/64*(12*c^2*x^7*arctan(c*x^2)*log(c^2*x^4 + 1) - 12*c*x^5*ar
ctan(c*x^2)^2 + 56*(c^2*x^7 + x^3)*arctan(c*x^2)^3 + 3*(c*x^5 + 2*(c^2*x^7
 + x^3)*arctan(c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1), x))*b^3
```

3.86.8 Giac [F]

$$\int x^3 \left(a + b \arctan\left(cx^2\right)\right)^3 \, dx = \int \left(b \arctan\left(cx^2\right) + a\right)^3 x^3 \, dx$$

input
```
integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^3*x^3, x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \arctan\left(cx^2\right)\right)^3 \, dx = \int x^3 \left(a + b \operatorname{atan}\left(c x^2\right)\right)^3 \, dx$$

input
```
int(x^3*(a + b*atan(c*x^2))^3,x)
```

output
```
int(x^3*(a + b*atan(c*x^2))^3, x)
```

3.87 $\int x(a+b\arctan(cx^2))^3\,dx$

3.87.1 Optimal result 579
3.87.2 Mathematica [A] (verified) 579
3.87.3 Rubi [A] (verified) 580
3.87.4 Maple [B] (verified) 582
3.87.5 Fricas [F] 583
3.87.6 Sympy [F] 583
3.87.7 Maxima [F] 583
3.87.8 Giac [F] 584
3.87.9 Mupad [F(-1)] 584

3.87.1 Optimal result

Integrand size = 14, antiderivative size = 144

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\frac{i\left(a+b\arctan\left(cx^2\right)\right)^3}{2c}+\frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)^3+\frac{3b\left(a+b\arctan\left(cx^2\right)\right)^2\log\left(\frac{2}{1+icx^2}\right)}{2c}+\frac{3ib^2\left(a+b\arctan\left(cx^2\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{2c}+\frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right)}{4c}$$

output
```
1/2*I*(a+b*arctan(c*x^2))^3/c+1/2*x^2*(a+b*arctan(c*x^2))^3+3/2*b*(a+b*arc
tan(c*x^2))^2*ln(2/(1+I*c*x^2))/c+3/2*I*b^2*(a+b*arctan(c*x^2))*polylog(2,
1-2/(1+I*c*x^2))/c+3/4*b^3*polylog(3,1-2/(1+I*c*x^2))/c
```

3.87.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.56

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx$$

$$=\frac{2a^3cx^2+6a^2bcx^2\arctan\left(cx^2\right)-6iab^2\arctan\left(cx^2\right)^2+6ab^2cx^2\arctan\left(cx^2\right)^2-2ib^3\arctan\left(cx^2\right)^3+2b^3cx^2\cdots}{\cdots}$$

input
```
Integrate[x*(a + b*ArcTan[c*x^2])^3,x]
```

output
```
(2*a^3*c*x^2 + 6*a^2*b*c*x^2*ArcTan[c*x^2] - (6*I)*a*b^2*ArcTan[c*x^2]^2 +
 6*a*b^2*c*x^2*ArcTan[c*x^2]^2 - (2*I)*b^3*ArcTan[c*x^2]^3 + 2*b^3*c*x^2*A
rcTan[c*x^2]^3 + 12*a*b^2*ArcTan[c*x^2]*Log[1 + E^((2*I)*ArcTan[c*x^2])] +
 6*b^3*ArcTan[c*x^2]^2*Log[1 + E^((2*I)*ArcTan[c*x^2])] - 3*a^2*b*Log[1 +
c^2*x^4] - (6*I)*b^2*(a + b*ArcTan[c*x^2])*PolyLog[2, -E^((2*I)*ArcTan[c*x
^2])] + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x^2])])/(4*c)
```

3.87.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5363, 5345, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2}\int \left(a+b\arctan\left(cx^2\right)\right)^3dx^2$$

$$\Big\downarrow \text{5345}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^3-3bc\int \frac{x^2\left(a+b\arctan\left(cx^2\right)\right)^2}{c^2x^4+1}dx^2\right)$$

$$\Big\downarrow \text{5455}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^3-3bc\left(-\frac{\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{i-cx^2}dx^2}{c}-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^3}{3bc^2}\right)\right)$$

$$\Big\downarrow \text{5379}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^2}{c}-2b\int \frac{\left(a+b\arctan\left(cx^2\right)\right)\log\left(\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2}{c}-\frac{i\left(a+b\arc\right.}{3}\right.\right.$$

$$\Big\downarrow \text{5529}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\arctan(cx^2)\right)^2}{c}-2b\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2-\frac{i\,\text{PolyLog}\left(2,1-\right.}{}\right.}{c}\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$\frac{1}{2}\left(x^2\left(a+b\arctan\left(cx^2\right)\right)^3-3bc\left(-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^3}{3bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx^2}\right)\left(a+b\arctan(cx^2)\right)^2}{c}-2b\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^2}\right.}{}\right.}{c}\right.\right.$$

input
```
Int[x*(a + b*ArcTan[c*x^2])^3,x]
```

output
```
(x^2*(a + b*ArcTan[c*x^2])^3 - 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x^2])^3)/(
b*c^2) - (((a + b*ArcTan[c*x^2])^2*Log[2/(1 + I*c*x^2)])/c - 2*b*(((-1/2*I
)*(a + b*ArcTan[c*x^2])*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c - (b*PolyLog[3,
 1 - 2/(1 + I*c*x^2)])/(4*c)))/c))/2
```

3.87.3.1 Defintions of rubi rules used

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.87.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(129) = 258.

Time = 9.39 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.91

method	result
derivativedivides	$\frac{a^3 c x^2+b^3\left(\arctan(c x^2)^3 (c x^2+i)-2i\arctan(c x^2)^3+3\arctan(c x^2)^2\ln\left(1+\frac{(ic x^2+1)^2}{c^2x^4+1}\right)-3i\arctan(c x^2)\,\mathrm{polylog}\left(2,\right.\right.}{}$
default	$\frac{a^3 c x^2+b^3\left(\arctan(c x^2)^3 (c x^2+i)-2i\arctan(c x^2)^3+3\arctan(c x^2)^2\ln\left(1+\frac{(ic x^2+1)^2}{c^2x^4+1}\right)-3i\arctan(c x^2)\,\mathrm{polylog}\left(2,\right.\right.}{}$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\arctan(c x^2)^3 (c x^2+i)-2i\arctan(c x^2)^3+3\arctan(c x^2)^2\ln\left(1+\frac{(ic x^2+1)^2}{c^2x^4+1}\right)-3i\arctan(c x^2)\,\mathrm{polylog}\left(2\right.\right.}{2c}$

input
```
int(x*(a+b*arctan(c*x^2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/c*(a^3*c*x^2+b^3*(arctan(c*x^2)^3*(c*x^2+I)-2*I*arctan(c*x^2)^3+3*arct
an(c*x^2)^2*ln(1+(1+I*c*x^2)^2/(c^2*x^4+1))-3*I*arctan(c*x^2)*polylog(2,-(
1+I*c*x^2)^2/(c^2*x^4+1))+3/2*polylog(3,-(1+I*c*x^2)^2/(c^2*x^4+1)))+3*a*b
^2*(arctan(c*x^2)^2*(c*x^2+I)+2*arctan(c*x^2)*ln(1+(1+I*c*x^2)^2/(c^2*x^4+
1))-2*I*arctan(c*x^2)^2-I*polylog(2,-(1+I*c*x^2)^2/(c^2*x^4+1)))+3*a^2*b*(
c*x^2*arctan(c*x^2)-1/2*ln(c^2*x^4+1)))
```

3.87.5 Fricas [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arctan(c*x^2)^3 + 3*a*b^2*x*arctan(c*x^2)^2 + 3*a^2*b*x*arc
tan(c*x^2) + a^3*x, x)
```

3.87.6 Sympy [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int x\left(a+b\operatorname{atan}\left(cx^2\right)\right)^3\,dx$$

input
```
integrate(x*(a+b*atan(c*x**2))**3,x)
```

output
```
Integral(x*(a + b*atan(c*x**2))**3, x)
```

3.87.7 Maxima [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

output
```
1/16*b^3*x^2*arctan(c*x^2)^3 - 3/64*b^3*x^2*arctan(c*x^2)*log(c^2*x^4 + 1)
^2 + 7/64*b^3*arctan(c*x^2)^4/c + 28*b^3*c^2*integrate(1/32*x^5*arctan(c*x
^2)^3/(c^2*x^4 + 1), x) + 3*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2)*log(c
^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^5*arctan(c
*x^2)^2/(c^2*x^4 + 1), x) + 12*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2)*lo
g(c^2*x^4 + 1)/(c^2*x^4 + 1), x) + 1/2*a^3*x^2 + 1/2*a*b^2*arctan(c*x^2)^3
/c - 12*b^3*c*integrate(1/32*x^3*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 3*b^3
*c*integrate(1/32*x^3*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 3*b^3*integra
te(1/32*x*arctan(c*x^2)*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 3/4*(2*c*x^
2*arctan(c*x^2) - log(c^2*x^4 + 1))*a^2*b/c
```

3.87.8 Giac [F]

$$\int x(a+b\arctan(cx^2))^3\,dx=\int (b\arctan(cx^2)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^3*x, x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int x(a+b\arctan(cx^2))^3\,dx=\int x\,(a+b\,\mathrm{atan}(c\,x^2))^3\,dx$$

input
```
int(x*(a + b*atan(c*x^2))^3,x)
```

output
```
int(x*(a + b*atan(c*x^2))^3, x)
```

3.88 $\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx$

3.88.1 Optimal result 585
3.88.2 Mathematica [A] (verified) 586
3.88.3 Rubi [A] (verified) 587
3.88.4 Maple [F] 589
3.88.5 Fricas [F] 590
3.88.6 Sympy [F] 590
3.88.7 Maxima [F] 590
3.88.8 Giac [F] 591
3.88.9 Mupad [F(-1)] 591

3.88.1 Optimal result

Integrand size = 16, antiderivative size = 229

$$\begin{aligned}\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx =\ & (a+b\arctan(cx^2))^3\operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right) \\ & -\frac{3}{4}ib(a+b\arctan(cx^2))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right) \\ & +\frac{3}{4}ib(a+b\arctan(cx^2))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right) \\ & -\frac{3}{4}b^2(a+b\arctan(cx^2))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right) \\ & +\frac{3}{4}b^2(a+b\arctan(cx^2))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^2}\right) \\ & +\frac{3}{8}ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx^2}\right) \\ & -\frac{3}{8}ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx^2}\right)\end{aligned}$$

output
```
-(a+b*arctan(c*x^2))^3*arctanh(-1+2/(1+I*c*x^2))-3/4*I*b*(a+b*arctan(c*x^2
))^2*polylog(2,1-2/(1+I*c*x^2))+3/4*I*b*(a+b*arctan(c*x^2))^2*polylog(2,-1
+2/(1+I*c*x^2))-3/4*b^2*(a+b*arctan(c*x^2))*polylog(3,1-2/(1+I*c*x^2))+3/4
*b^2*(a+b*arctan(c*x^2))*polylog(3,-1+2/(1+I*c*x^2))+3/8*I*b^3*polylog(4,1
-2/(1+I*c*x^2))-3/8*I*b^3*polylog(4,-1+2/(1+I*c*x^2))
```

3.88.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.82

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx = a^3\log(x)+\frac{3}{4}ia^2b\left(\text{PolyLog}(2,-icx^2)-\text{PolyLog}(2,icx^2)\right)+\frac{1}{16}ab^2\Big(-i\pi^3+16i\arctan(cx^2)^3+24\arctan(cx^2)^2\log\left(1-e^{-2i\arctan(cx^2)}\right)-24\arctan(cx^2)^2\log\left(1+e^{2i\arctan(cx^2)}\right)+24i\arctan(cx^2)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx^2)}\right)+24i\arctan(cx^2)\,\text{PolyLog}\left(2,-e^{2i\arctan(cx^2)}\right)+12\,\text{PolyLog}\left(3,e^{-2i\arctan(cx^2)}\right)-12\,\text{PolyLog}\left(3,-e^{2i\arctan(cx^2)}\right)\Big)-\frac{1}{128}ib^3\Big(\pi^4-32\arctan(cx^2)^4+64i\arctan(cx^2)^3\log\left(1-e^{-2i\arctan(cx^2)}\right)-64i\arctan(cx^2)^3\log\left(1+e^{2i\arctan(cx^2)}\right)-96\arctan(cx^2)^2\,\text{PolyLog}\left(2,e^{-2i\arctan(cx^2)}\right)-96\arctan(cx^2)^2\,\text{PolyLog}\left(2,-e^{2i\arctan(cx^2)}\right)+96i\arctan(cx^2)\,\text{PolyLog}\left(3,e^{-2i\arctan(cx^2)}\right)-96i\arctan(cx^2)\,\text{PolyLog}\left(3,-e^{2i\arctan(cx^2)}\right)+48\,\text{PolyLog}\left(4,e^{-2i\arctan(cx^2)}\right)+48\,\text{PolyLog}\left(4,-e^{2i\arctan(cx^2)}\right)\Big)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^3/x,x]
```

output
```
a^3*Log[x] + ((3*I)/4)*a^2*b*(PolyLog[2, (-I)*c*x^2] - PolyLog[2, I*c*x^2]
) + (a*b^2*((-I)*Pi^3 + (16*I)*ArcTan[c*x^2]^3 + 24*ArcTan[c*x^2]^2*Log[1
- E^((-2*I)*ArcTan[c*x^2])] - 24*ArcTan[c*x^2]^2*Log[1 + E^((2*I)*ArcTan[c
*x^2])] + (24*I)*ArcTan[c*x^2]*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] + (24*
I)*ArcTan[c*x^2]*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])] + 12*PolyLog[3, E^((
-2*I)*ArcTan[c*x^2])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x^2])]))/16 - (I/
128)*b^3*(Pi^4 - 32*ArcTan[c*x^2]^4 + (64*I)*ArcTan[c*x^2]^3*Log[1 - E^((-
2*I)*ArcTan[c*x^2])] - (64*I)*ArcTan[c*x^2]^3*Log[1 + E^((2*I)*ArcTan[c*x^
2])] - 96*ArcTan[c*x^2]^2*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] - 96*ArcTan
[c*x^2]^2*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])] + (96*I)*ArcTan[c*x^2]*Poly
Log[3, E^((-2*I)*ArcTan[c*x^2])] - (96*I)*ArcTan[c*x^2]*PolyLog[3, -E^((2*
I)*ArcTan[c*x^2])] + 48*PolyLog[4, E^((-2*I)*ArcTan[c*x^2])] + 48*PolyLog[
4, -E^((2*I)*ArcTan[c*x^2])])
```

3.88.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5359, 5357, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^2))^3}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{1}{2}\int \frac{(a+b\arctan(cx^2))^3}{x^2}dx^2$$

$$\Big\downarrow \text{5357}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)(a+b\arctan(cx^2))^3-6bc\int \frac{(a+b\arctan(cx^2))^2\text{arctanh}\left(1-\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2\right)$$

$$\Big\downarrow \text{5523}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)(a+b\arctan(cx^2))^3-6bc\left(\frac{1}{2}\int \frac{(a+b\arctan(cx^2))^2\log\left(2-\frac{2}{icx^2+1}\right)}{c^2x^4+1}dx^2-\frac{1}{2}\int\right.\right.$$

$\big\downarrow$ 5529

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)\left(a+b\arctan\left(cx^2\right)\right)^2}{2c}-i\right.\right.\right.$$

$\big\downarrow$ 5533

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)\left(a+b\arctan\left(cx^2\right)\right)^2}{2c}-i\right.\right.\right.$$

$\big\downarrow$ 7164

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)\left(a+b\arctan\left(cx^2\right)\right)^2}{2c}-i\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^2])^3/x,x]
```

output
```
(2*(a + b*ArcTan[c*x^2])^3*ArcTanh[1 - 2/(1 + I*c*x^2)] - 6*b*c*((((I/2)*(
a + b*ArcTan[c*x^2])^2*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c - I*b*(((I/2)*(a
 + b*ArcTan[c*x^2])*PolyLog[3, 1 - 2/(1 + I*c*x^2)])/c + (b*PolyLog[4, 1 -
 2/(1 + I*c*x^2)])/(4*c)))/2 + (((-1/2*I)*(a + b*ArcTan[c*x^2])^2*PolyLog[
2, -1 + 2/(1 + I*c*x^2)])/c + I*b*(((I/2)*(a + b*ArcTan[c*x^2])*PolyLog[3,
 -1 + 2/(1 + I*c*x^2)])/c + (b*PolyLog[4, -1 + 2/(1 + I*c*x^2)])/(4*c)))/2
))/2
```

3.88.3.1 Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.88.4 Maple [F]

$$\int \frac{(a+b\arctan(c\,x^2))^3}{x}dx$$

input
```
int((a+b*arctan(c*x^2))^3/x,x)
```

output
```
int((a+b*arctan(c*x^2))^3/x,x)
```

3.88.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx=\int \frac{(b\arctan(cx^2)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c
*x^2) + a^3)/x, x)
```

3.88.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx=\int \frac{(a+b\,\mathrm{atan}(cx^2))^3}{x}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**3/x,x)
```

output
```
Integral((a + b*atan(c*x**2))**3/x, x)
```

3.88.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx=\int \frac{(b\arctan(cx^2)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x^2)^3 + 3*b^3*arctan(c*x^2)*
log(c^2*x^4 + 1)^2 + 96*a*b^2*arctan(c*x^2)^2 + 96*a^2*b*arctan(c*x^2))/x,
 x)
```

3.88.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx = \int \frac{(b\arctan(cx^2)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^3/x, x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^2))^3}{x}\,dx$$

input
```
int((a + b*atan(c*x^2))^3/x,x)
```

output
```
int((a + b*atan(c*x^2))^3/x, x)
```

3.89 $\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx$

3.89.1 Optimal result . . . 592
3.89.2 Mathematica [A] (verified) . . . 593
3.89.3 Rubi [A] (verified) . . . 593
3.89.4 Maple [F] . . . 596
3.89.5 Fricas [F] . . . 596
3.89.6 Sympy [F] . . . 596
3.89.7 Maxima [F] . . . 597
3.89.8 Giac [F] . . . 597
3.89.9 Mupad [F(-1)] . . . 597

3.89.1 Optimal result

Integrand size = 16, antiderivative size = 138

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = -\frac{1}{2}ic(a+b\arctan(cx^2))^3 - \frac{(a+b\arctan(cx^2))^3}{2x^2} + \frac{3}{2}bc(a+b\arctan(cx^2))^2\log\left(2-\frac{2}{1-icx^2}\right) - \frac{3}{2}ib^2c(a+b\arctan(cx^2))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right) + \frac{3}{4}b^3c\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx^2}\right)$$

output
```
-1/2*I*c*(a+b*arctan(c*x^2))^3-1/2*(a+b*arctan(c*x^2))^3/x^2+3/2*b*c*(a+b*
arctan(c*x^2))^2*ln(2-2/(1-I*c*x^2))-3/2*I*b^2*c*(a+b*arctan(c*x^2))*polyl
og(2,-1+2/(1-I*c*x^2))+3/4*b^3*c*polylog(3,-1+2/(1-I*c*x^2))
```

3.89.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.73

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx$$
$$=\frac{1}{4}\left(-\frac{2a^3}{x^2}-\frac{6a^2b\arctan(cx^2)}{x^2}+12a^2bc\log(x)-3a^2bc\log(1+c^2x^4)+6ab^2c\left(\arctan(cx^2)\left(\left(-i-\frac{1}{cx^2}\right)\arctan(cx^2)+2\log\left(1-e^{2i\arctan(cx^2)}\right)\right)-i\,\text{PolyLog}\left(2,e^{2i\arctan(cx^2)}\right)\right)+2b^3c\left(-\frac{i\pi^3}{8}+i\arctan(cx^2)^3-\frac{\arctan(cx^2)^3}{cx^2}+3\arctan(cx^2)^2\log\left(1-e^{-2i\arctan(cx^2)}\right)+3i\arctan(cx^2)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx^2)}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx^2)}\right)\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^3/x^3,x]
```

output
```
((-2*a^3)/x^2 - (6*a^2*b*ArcTan[c*x^2])/x^2 + 12*a^2*b*c*Log[x] - 3*a^2*b*
c*Log[1 + c^2*x^4] + 6*a*b^2*c*(ArcTan[c*x^2]*((-I - 1/(c*x^2))*ArcTan[c*x
^2] + 2*Log[1 - E^((2*I)*ArcTan[c*x^2])]) - I*PolyLog[2, E^((2*I)*ArcTan[c
*x^2])]) + 2*b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x^2]^3 - ArcTan[c*x^2]^3/(c
*x^2) + 3*ArcTan[c*x^2]^2*Log[1 - E^((-2*I)*ArcTan[c*x^2])] + (3*I)*ArcTan
[c*x^2]*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] + (3*PolyLog[3, E^((-2*I)*Arc
Tan[c*x^2])])/2))/4
```

3.89.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5363, 5361, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2}\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^4}dx^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(3bc\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2\left(c^2x^4+1\right)}dx^2-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^2}\right)$$

$$\Big\downarrow \text{5459}$$

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^2}+3bc\left(i\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2\left(cx^2+i\right)}dx^2-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^3}{3b}\right)\right)$$

$$\Big\downarrow \text{5403}$$

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^2}+3bc\left(i\left(2ibc\int \frac{\left(a+b\arctan\left(cx^2\right)\right)\log\left(2-\frac{2}{1-icx^2}\right)}{c^2x^4+1}dx^2-i\log\left(2-\frac{2}{1-icx^2}\right)(a\right.\right.\right.$$

$$\Big\downarrow \text{5527}$$

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^2}+3bc\left(i\left(2ibc\left(\frac{i\operatorname{PolyLog}\left(2,\frac{2}{1-icx^2}-1\right)\left(a+b\arctan\left(cx^2\right)\right)}{2c}-\frac{1}{2}ib\int \frac{\operatorname{PolyLog}\left(2,\right.}{c^2x^4}\right.\right.\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^2}+3bc\left(i\left(2ibc\left(\frac{i\operatorname{PolyLog}\left(2,\frac{2}{1-icx^2}-1\right)\left(a+b\arctan\left(cx^2\right)\right)}{2c}-\frac{b\operatorname{PolyLog}\left(3,\frac{2}{1-icx^2}\right.}{4c}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^2])^3/x^3,x]
```

output
```
(-((a + b*ArcTan[c*x^2])^3/x^2) + 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x^2])^3
)/b + I*((-I)*(a + b*ArcTan[c*x^2])^2*Log[2 - 2/(1 - I*c*x^2)] + (2*I)*b*c
*(((I/2)*(a + b*ArcTan[c*x^2])*PolyLog[2, -1 + 2/(1 - I*c*x^2)])/c - (b*Po
lyLog[3, -1 + 2/(1 - I*c*x^2)])/(4*c)))))/2
```

3.89. $\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^3}\,dx$

3.89.3.1 Defintions of rubi rules used

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.89. $\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx$

3.89.4 Maple [F]

$$\int \frac{(a+b\arctan(c\,x^2))^3}{x^3}dx$$

input
```
int((a+b*arctan(c*x^2))^3/x^3,x)
```

output
```
int((a+b*arctan(c*x^2))^3/x^3,x)
```

3.89.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = \int \frac{(b\arctan(cx^2)+a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c
*x^2) + a^3)/x^3, x)
```

3.89.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = \int \frac{(a+b\operatorname{atan}(cx^2))^3}{x^3}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**3/x**3,x)
```

output
```
Integral((a + b*atan(c*x**2))**3/x**3, x)
```

3.89.7 Maxima [F]

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\arctan\left(cx^2\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="maxima")
```

output
```
-3/4*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*a^2*b - 1/2*a
^3/x^2 - 1/64*(4*b^3*arctan(c*x^2)^3 - 3*b^3*arctan(c*x^2)*log(c^2*x^4 + 1
)^2 - 64*x^2*integrate(-1/32*(12*b^3*c^2*x^4*arctan(c*x^2)*log(c^2*x^4 + 1
) - 28*(b^3*c^2*x^4 + b^3)*arctan(c*x^2)^3 - 12*(8*a*b^2*c^2*x^4 + b^3*c*x
^2 + 8*a*b^2)*arctan(c*x^2)^2 + 3*(b^3*c*x^2 - (b^3*c^2*x^4 + b^3)*arctan(
c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^7 + x^3), x))/x^2
```

3.89.8 Giac [F]

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\arctan\left(cx^2\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^3/x^3, x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*atan(c*x^2))^3/x^3,x)
```

output
```
int((a + b*atan(c*x^2))^3/x^3, x)
```

3.90 $\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx$

3.90.1 Optimal result . . . 598
3.90.2 Mathematica [A] (verified) . . . 598
3.90.3 Rubi [A] (verified) . . . 599
3.90.4 Maple [C] (warning: unable to verify) . . . 601
3.90.5 Fricas [F] . . . 602
3.90.6 Sympy [F] . . . 603
3.90.7 Maxima [F] . . . 603
3.90.8 Giac [F] . . . 604
3.90.9 Mupad [F(-1)] . . . 604

3.90.1 Optimal result

Integrand size = 16, antiderivative size = 149

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx = -\frac{3}{4}ibc^2(a+b\arctan(cx^2))^2 - \frac{3bc(a+b\arctan(cx^2))^2}{4x^2} - \frac{1}{4}c^2(a+b\arctan(cx^2))^3 - \frac{(a+b\arctan(cx^2))^3}{4x^4} + \frac{3}{2}b^2c^2(a+b\arctan(cx^2))\log\left(2-\frac{2}{1-icx^2}\right) - \frac{3}{4}ib^3c^2\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right)$$

output
```
-3/4*I*b*c^2*(a+b*arctan(c*x^2))^2-3/4*b*c*(a+b*arctan(c*x^2))^2/x^2-1/4*c
^2*(a+b*arctan(c*x^2))^3-1/4*(a+b*arctan(c*x^2))^3/x^4+3/2*b^2*c^2*(a+b*ar
ctan(c*x^2))*ln(2-2/(1-I*c*x^2))-3/4*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x^2))
```

3.90.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx = -\frac{3b^2(a+ac^2x^4+bcx^2(1+icx^2))\arctan(cx^2)^2 + b^3(1+c^2x^4)\arctan(cx^2)^3 + 3b\arctan(cx^2)\left(a(a+2bc\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x^2])^3/x^5,x]
```

output
```
-1/4*(3*b^2*(a + a*c^2*x^4 + b*c*x^2*(1 + I*c*x^2))*ArcTan[c*x^2]^2 + b^3*
(1 + c^2*x^4)*ArcTan[c*x^2]^3 + 3*b*ArcTan[c*x^2]*(a*(a + 2*b*c*x^2 + a*c^
2*x^4) - 2*b^2*c^2*x^4*Log[1 - E^((2*I)*ArcTan[c*x^2])]) + a*(a*(a + 3*b*c
*x^2) - 6*b^2*c^2*x^4*Log[(c*x^2)/Sqrt[1 + c^2*x^4]]) + (3*I)*b^3*c^2*x^4*
PolyLog[2, E^((2*I)*ArcTan[c*x^2])])/x^4
```

3.90.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5363, 5361, 5453, 5361, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{2}\int \frac{(a+b\arctan(cx^2))^3}{x^6}dx^2$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\frac{3}{2}bc\int \frac{(a+b\arctan(cx^2))^2}{x^4(c^2x^4+1)}dx^2-\frac{(a+b\arctan(cx^2))^3}{2x^4}\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{2}\left(\frac{3}{2}bc\left(\int \frac{(a+b\arctan(cx^2))^2}{x^4}dx^2-c^2\int \frac{(a+b\arctan(cx^2))^2}{c^2x^4+1}dx^2\right)-\frac{(a+b\arctan(cx^2))^3}{2x^4}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\frac{3}{2}bc\left(c^2\left(-\int \frac{(a+b\arctan(cx^2))^2}{c^2x^4+1}dx^2\right)+2bc\int \frac{a+b\arctan(cx^2)}{x^2(c^2x^4+1)}dx^2-\frac{(a+b\arctan(cx^2))^2}{x^2}\right)-\frac{(a+b\text{a}}{}$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{2}\left(\frac{3}{2}bc\left(2bc\int\frac{a+b\arctan\left(cx^2\right)}{x^2\left(c^2x^4+1\right)}dx^2-\frac{c\left(a+b\arctan\left(cx^2\right)\right)^3}{3b}-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}\right)-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{2x^4}$$

$\Big\downarrow$ 5459

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{2x^4}+\frac{3}{2}bc\left(2bc\left(i\int\frac{a+b\arctan\left(cx^2\right)}{x^2\left(cx^2+i\right)}dx^2-\frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{2b}\right)-\frac{c\left(a+b\arctan\left(c\right.\right.}{3b}\right.\right.$$

$\Big\downarrow$ 5403

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{2x^4}+\frac{3}{2}bc\left(2bc\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-icx^2}\right)}{c^2x^4+1}dx^2-i\log\left(2-\frac{2}{1-icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 2897

$$\frac{1}{2}\left(-\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{2x^4}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx^2}\right)\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx^2}\right.\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^2])^3/x^5,x]
```

output
```
(-1/2*(a + b*ArcTan[c*x^2])^3/x^4 + (3*b*c*(-((a + b*ArcTan[c*x^2])^2/x^2)
 - (c*(a + b*ArcTan[c*x^2])^3)/(3*b) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^
2])^2)/b + I*((-I)*(a + b*ArcTan[c*x^2])*Log[2 - 2/(1 - I*c*x^2)] - (b*Pol
yLog[2, -1 + 2/(1 - I*c*x^2)])/2))))/2)/2
```

3.90.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.90.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.42 (sec) , antiderivative size = 465, normalized size of antiderivative = 3.12

method	result
default	$-\frac{a^3}{4x^4}-\frac{b^3\arctan(c\,x^2)^3}{4x^4}-\frac{3b^3c\arctan(c\,x^2)^2}{4x^2}-\frac{b^3\arctan(c\,x^2)^3c^2}{4}+3b^3c^2\arctan(c\,x^2)\ln(x)-\frac{3b^3c^2\arctan(c\,x}{4}$
parts	$-\frac{a^3}{4x^4}-\frac{b^3\arctan(c\,x^2)^3}{4x^4}-\frac{3b^3c\arctan(c\,x^2)^2}{4x^2}-\frac{b^3\arctan(c\,x^2)^3c^2}{4}+3b^3c^2\arctan(c\,x^2)\ln(x)-\frac{3b^3c^2\arctan(c\,x}{4}$

input
```
int((a+b*arctan(c*x^2))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^3/x^4-1/4*b^3/x^4*arctan(c*x^2)^3-3/4*b^3*c*arctan(c*x^2)^2/x^2-1/4
*b^3*arctan(c*x^2)^3*c^2+3*b^3*c^2*arctan(c*x^2)*ln(x)-3/4*b^3*c^2*arctan(
c*x^2)*ln(c^2*x^4+1)+3/16*b^3*c*sum(1/_alpha^2*(2*ln(x-_alpha)*ln(c^2*x^4+
1)-c*(1/c/_alpha^3*ln(x-_alpha)^2+2/_alpha*ln(x-_alpha)*(_alpha^2*ln(1/2*(
x+_alpha)/_alpha)*c-ln((_alpha^3*c+x)/_alpha/(_alpha^2*c+1))+ln((_alpha^3*
c-x)/_alpha/(_alpha^2*c-1)))+2/_alpha*(_alpha^2*dilog(1/2*(x+_alpha)/_alph
a)*c-dilog((_alpha^3*c+x)/_alpha/(_alpha^2*c+1))+dilog((_alpha^3*c-x)/_alp
ha/(_alpha^2*c-1))))),_alpha=RootOf(_Z^4*c^2+1))-3/2*b^3*c*sum(1/_R1^2*(ln
(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1)),_R1=RootOf(_Z^4*c^2+1))-3/4*a*b^2/
x^4*arctan(c*x^2)^2-3/2*a*b^2*c*arctan(c*x^2)/x^2-3/4*a*b^2*arctan(c*x^2)^
2*c^2+3*a*b^2*c^2*ln(x)-3/4*a*b^2*c^2*ln(c^2*x^4+1)-3/4*a^2*b/x^4*arctan(c
*x^2)-3/4*a^2*b*c/x^2-3/4*a^2*b*c^2*arctan(c*x^2)
```

3.90.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx=\int \frac{(b\arctan(cx^2)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c
*x^2) + a^3)/x^5, x)
```

3.90.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx = \int \frac{(a+b\operatorname{atan}(cx^2))^3}{x^5}\,dx$$

input
```
integrate((a+b*atan(c*x**2))**3/x**5,x)
```

output
```
Integral((a + b*atan(c*x**2))**3/x**5, x)
```

3.90.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx = \int \frac{(b\arctan(cx^2)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="maxima")
```

output
```
-3/4*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*a^2*b + 3/4*((arcta
n(c*x^2)^2 - log(c^2*x^4 + 1) + 4*log(x))*c^2 - 2*(c*arctan(c*x^2) + 1/x^2
)*c*arctan(c*x^2))*a*b^2 - 3/4*a*b^2*arctan(c*x^2)^2/x^4 + 1/128*(128*x^4*
integrate(-1/64*(12*c^2*x^4*arctan(c*x^2)*log(c^2*x^4 + 1) - 12*c*x^2*arct
an(c*x^2)^2 - 56*(c^2*x^4 + 1)*arctan(c*x^2)^3 + 3*(c*x^2 - 2*(c^2*x^4 + 1
)*arctan(c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^9 + x^5), x) - 4*arctan(c*x^2)
^3 + 3*arctan(c*x^2)*log(c^2*x^4 + 1)^2)*b^3/x^4 - 1/4*a^3/x^4
```

3.90.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx = \int \frac{(b\arctan(cx^2)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^3/x^5, x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^2))^3}{x^5}\,dx$$

input
```
int((a + b*atan(c*x^2))^3/x^5,x)
```

output
```
int((a + b*atan(c*x^2))^3/x^5, x)
```

3.91 $\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 dx$

3.91.1 Optimal result 605
3.91.2 Mathematica [N/A] 605
3.91.3 Rubi [N/A] 606
3.91.4 Maple [N/A] (verified) 606
3.91.5 Fricas [N/A] 607
3.91.6 Sympy [F(-1)] 607
3.91.7 Maxima [N/A] 607
3.91.8 Giac [N/A] 608
3.91.9 Mupad [N/A] 608

3.91.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 dx = \operatorname{Int}\left((dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

3.91.2 Mathematica [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 dx = \int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^3, x]
```

3.91.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 dx$$

$$\Big\downarrow \; 5377$$

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x^2])^3,x]
```

output
```
$Aborted
```

3.91.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.91.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^2\right)\right)^3 dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

3.91.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx = \int \left(b\arctan\left(cx^2\right)+a\right)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c
*x^2) + a^3)*(d*x)^m, x)
```

3.91.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atan(c*x**2))**3,x)
```

output
```
Timed out
```

3.91.7 Maxima [N/A]

Not integrable

Time = 3.97 (sec) , antiderivative size = 406, normalized size of antiderivative = 22.56

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx = \int \left(b\arctan\left(cx^2\right)+a\right)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x^2)^3 - 3*
b^3*d^m*x*x^m*arctan(c*x^2)*log(c^2*x^4 + 1)^2 + 32*(m + 1)*integrate(1/32
*(24*b^3*c^2*d^m*x^4*x^m*arctan(c*x^2)*log(c^2*x^4 + 1) + 28*(b^3*d^m*m +
(b^3*c^2*d^m*m + b^3*c^2*d^m)*x^4 + b^3*d^m)*x^m*arctan(c*x^2)^3 - 24*(b^3
*c*d^m*x^2 - 4*a*b^2*d^m*m - 4*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^4 - 4*a
*b^2*d^m)*x^m*arctan(c*x^2)^2 + 96*(a^2*b*d^m*m + (a^2*b*c^2*d^m*m + a^2*b
*c^2*d^m)*x^4 + a^2*b*d^m)*x^m*arctan(c*x^2) + 3*(2*b^3*c*d^m*x^2*x^m + (b
^3*d^m*m + (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^4 + b^3*d^m)*x^m*arctan(c*x^2))
*log(c^2*x^4 + 1)^2)/((c^2*m + c^2)*x^4 + m + 1), x))/(m + 1)
```

3.91.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^3(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^3*(d*x)^m, x)
```

3.91.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^3\,dx$$

input
```
int((d*x)^m*(a + b*atan(c*x^2))^3,x)
```

output
```
int((d*x)^m*(a + b*atan(c*x^2))^3, x)
```

3.92 $\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx$

3.92.1 Optimal result 609
3.92.2 Mathematica [N/A] 609
3.92.3 Rubi [N/A] 610
3.92.4 Maple [N/A] (verified) 610
3.92.5 Fricas [N/A] 611
3.92.6 Sympy [N/A] 611
3.92.7 Maxima [N/A] 611
3.92.8 Giac [N/A] 612
3.92.9 Mupad [N/A] 612

3.92.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx = \operatorname{Int}\left((dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

3.92.2 Mathematica [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx = \int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^2, x]
```

3.92.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 dx$$
$$\Big\downarrow \text{ 5377}$$
$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x^2])^2,x]
```

output
```
$Aborted
```

3.92.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.92.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^2\right)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

3.92.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)*(d*x)^m, x)
```

3.92.6 Sympy [N/A]

Not integrable

Time = 111.97 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int (dx)^m \left(a+b\,\mathrm{atan}\left(cx^2\right)\right)^2\,dx$$

input
```
integrate((d*x)**m*(a+b*atan(c*x**2))**2,x)
```

output
```
Integral((d*x)**m*(a + b*atan(c*x**2))**2, x)
```

3.92.7 Maxima [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 303, normalized size of antiderivative = 16.83

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x^2)^2 - b^
2*d^m*x*x^m*log(c^2*x^4 + 1)^2 + 16*(m + 1)*integrate(1/16*(8*b^2*c^2*d^m*
x^4*x^m*log(c^2*x^4 + 1) + 12*((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^4 + b^2*d^m
*m + b^2*d^m)*x^m*arctan(c*x^2)^2 + ((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^4 + b
^2*d^m*m + b^2*d^m)*x^m*log(c^2*x^4 + 1)^2 - 16*(b^2*c*d^m*x^2 - 2*(a*b*c^
2*d^m*m + a*b*c^2*d^m)*x^4 - 2*a*b*d^m*m - 2*a*b*d^m)*x^m*arctan(c*x^2))/(
(c^2*m + c^2)*x^4 + m + 1), x))/(m + 1)
```

3.92.8 Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2(dx)^m \,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)^2*(d*x)^m, x)
```

3.92.9 Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \,dx=\int {\left(d\,x\right)}^m\,{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)}^2 \,dx$$

input
```
int((d*x)^m*(a + b*atan(c*x^2))^2,x)
```

output
```
int((d*x)^m*(a + b*atan(c*x^2))^2, x)
```

3.93 $\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)\,dx$

3.93.1 Optimal result 613
3.93.2 Mathematica [A] (verified) 613
3.93.3 Rubi [A] (verified) 614
3.93.4 Maple [F] 615
3.93.5 Fricas [F] 615
3.93.6 Sympy [F] 615
3.93.7 Maxima [F] 616
3.93.8 Giac [F] 616
3.93.9 Mupad [F(-1)] 616

3.93.1 Optimal result

Integrand size = 16, antiderivative size = 75

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{(dx)^{1+m}\left(a+b\arctan\left(cx^2\right)\right)}{d(1+m)}-\frac{2bc(dx)^{3+m}\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{4},\frac{7+m}{4},-c^2x^4\right)}{d^3(1+m)(3+m)}$$

output
```
(d*x)^(1+m)*(a+b*arctan(c*x^2))/d/(1+m)-2*b*c*(d*x)^(3+m)*hypergeom([1, 3/
4+1/4*m],[7/4+1/4*m],-c^2*x^4)/d^3/(1+m)/(3+m)
```

3.93.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)\,dx = -\frac{x(dx)^m\left(-\left((3+m)\left(a+b\arctan\left(cx^2\right)\right)\right)+2bcx^2\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{4},\frac{7+m}{4},-c^2x^4\right)\right)}{(1+m)(3+m)}$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^2]),x]
```

output
```
-((x*(d*x)^m*(-((3 + m)*(a + b*ArcTan[c*x^2])) + 2*b*c*x^2*Hypergeometric2
F1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)]))/((1 + m)*(3 + m)))
```

3.93.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5373, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right) \, dx$$

$$\Big\downarrow \text{ 5373}$$

$$\frac{(dx)^{m+1}\left(a+b\arctan\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc\int \frac{(dx)^{m+2}}{c^2x^4+1}dx}{d^2(m+1)}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(dx)^{m+1}\left(a+b\arctan\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\,\text{Hypergeometric2F1}\left(1,\frac{m+3}{4},\frac{m+7}{4},-c^2x^4\right)}{d^3(m+1)(m+3)}$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x^2]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcTan[c*x^2]))/(d*(1 + m)) - (2*b*c*(d*x)^(3 + m)*H
ypergeometric2F1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)])/(d^3*(1 + m)*(3 + m
))
```

3.93.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5373
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n/(
d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.93.4 Maple [F]

$$\int (dx)^m \left(a+b \arctan \left(c\,x^2\right)\right) dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x^2)),x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x^2)),x)
```

3.93.5 Fricas [F]

$$\int (dx)^m \left(a+b \arctan \left(cx^2\right)\right)\, dx = \int \left(b \arctan \left(cx^2\right)+a\right)(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x^2) + a)*(d*x)^m, x)
```

3.93.6 Sympy [F]

$$\int (dx)^m \left(a+b \arctan \left(cx^2\right)\right)\, dx = \int (dx)^m \left(a+b\,\text{atan}\left(cx^2\right)\right)\, dx$$

input
```
integrate((d*x)**m*(a+b*atan(c*x**2)),x)
```

output
```
Integral((d*x)**m*(a + b*atan(c*x**2)), x)
```

3.93.7 Maxima [F]

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
(d^m*x*x^m*arctan(c*x^2) - 2*(c*d^m*m + c*d^m)*integrate(x^2*x^m/((c^2*m +
 c^2)*x^4 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

3.93.8 Giac [F]

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^2) + a)*(d*x)^m, x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)\,dx=\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)\,dx$$

input
```
int((d*x)^m*(a + b*atan(c*x^2)),x)
```

output
```
int((d*x)^m*(a + b*atan(c*x^2)), x)
```

3.94 $\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx$

3.94.1 Optimal result 617
3.94.2 Mathematica [N/A] 617
3.94.3 Rubi [N/A] 618
3.94.4 Maple [N/A] (verified) 618
3.94.5 Fricas [N/A] 619
3.94.6 Sympy [F(-1)] 619
3.94.7 Maxima [N/A] 619
3.94.8 Giac [N/A] 620
3.94.9 Mupad [N/A] 620

3.94.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx=\text{Int}\left(\frac{(dx)^m}{a+b\arctan(cx^2)},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctan(c*x^2)),x)
```

3.94.2 Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx=\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^2]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^2]), x]
```

3.94.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\arctan\left(cx^2\right)}dx$$

$$\Big\downarrow \text{ 5377}$$

$$\int \frac{(dx)^m}{a+b\arctan\left(cx^2\right)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTan[c*x^2]),x]
```

output
```
$Aborted
```

3.94.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.94.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\arctan\left(c\,x^2\right)}dx$$

input
```
int((d*x)^m/(a+b*arctan(c*x^2)),x)
```

output
```
int((d*x)^m/(a+b*arctan(c*x^2)),x)
```

3.94.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx = \int \frac{(dx)^m}{b\arctan(cx^2)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arctan(c*x^2) + a), x)
```

3.94.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atan(c*x**2)),x)
```

output
```
Timed out
```

3.94.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx = \int \frac{(dx)^m}{b\arctan(cx^2)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arctan(c*x^2) + a), x)
```

3.94.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx = \int \frac{(dx)^m}{b\arctan(cx^2)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctan(c*x^2) + a), x)
```

3.94.9 Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx = \int \frac{(d\,x)^m}{a+b\,\mathrm{atan}\left(c\,x^2\right)}\,dx$$

input
```
int((d*x)^m/(a + b*atan(c*x^2)),x)
```

output
```
int((d*x)^m/(a + b*atan(c*x^2)), x)
```

3.95 $\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2} dx$

3.95.1 Optimal result 621
3.95.2 Mathematica [N/A] 621
3.95.3 Rubi [N/A] 622
3.95.4 Maple [N/A] (verified) 622
3.95.5 Fricas [N/A] 623
3.95.6 Sympy [F(-1)] 623
3.95.7 Maxima [N/A] 623
3.95.8 Giac [N/A] 624
3.95.9 Mupad [N/A] 624

3.95.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2} dx = \text{Int}\left(\frac{(dx)^m}{(a+b\arctan(cx^2))^2}, x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

3.95.2 Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2} dx = \int \frac{(dx)^m}{(a+b\arctan(cx^2))^2} dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^2])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^2])^2, x]
```

3.95.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}dx$$
$$\Big\downarrow \text{5377}$$
$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTan[c*x^2])^2,x]
```

output
```
$Aborted
```

3.95.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.95.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b\arctan(c\,x^2))^2}dx$$

input
```
int((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

output
```
int((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

3.95.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^2)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2), x)
```

3.95.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atan(c*x**2))**2,x)
```

output
```
Timed out
```

3.95.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 6.89

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^2)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

output
```
-1/2*((c^2*d^m*x^4 + d^m)*x^m - 2*(b^2*c*x*arctan(c*x^2) + a*b*c*x)*integr
ate(1/2*((c^2*d^m*m + 3*c^2*d^m)*x^4 + d^m*m - d^m)*x^m/(b^2*c*x^2*arctan(
c*x^2) + a*b*c*x^2), x))/(b^2*c*x*arctan(c*x^2) + a*b*c*x)
```

3.95.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^2)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctan(c*x^2) + a)^2, x)
```

3.95.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \int \frac{(d\,x)^m}{(a+b\,\mathrm{atan}(c\,x^2))^2}\,dx$$

input
```
int((d*x)^m/(a + b*atan(c*x^2))^2,x)
```

output
```
int((d*x)^m/(a + b*atan(c*x^2))^2, x)
```

3.96 $\int x^{11}(a+b\arctan(cx^3))\,dx$

3.96.1 Optimal result . . . 625
3.96.2 Mathematica [A] (verified) . . . 625
3.96.3 Rubi [A] (verified) . . . 626
3.96.4 Maple [A] (verified) . . . 627
3.96.5 Fricas [A] (verification not implemented) . . . 628
3.96.6 Sympy [A] (verification not implemented) . . . 628
3.96.7 Maxima [A] (verification not implemented) . . . 628
3.96.8 Giac [A] (verification not implemented) . . . 629
3.96.9 Mupad [B] (verification not implemented) . . . 629

3.96.1 Optimal result

Integrand size = 14, antiderivative size = 54

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{bx^3}{12c^3}-\frac{bx^9}{36c}-\frac{b\arctan\left(cx^3\right)}{12c^4}+\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)$$

output
```
1/12*b*x^3/c^3-1/36*b*x^9/c-1/12*b*arctan(c*x^3)/c^4+1/12*x^12*(a+b*arctan
(c*x^3))
```

3.96.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{bx^3}{12c^3}-\frac{bx^9}{36c}+\frac{ax^{12}}{12}-\frac{b\arctan\left(cx^3\right)}{12c^4}+\frac{1}{12}bx^{12}\arctan\left(cx^3\right)$$

input
```
Integrate[x^11*(a + b*ArcTan[c*x^3]),x]
```

output
```
(b*x^3)/(12*c^3) - (b*x^9)/(36*c) + (a*x^12)/12 - (b*ArcTan[c*x^3])/(12*c^
4) + (b*x^12*ArcTan[c*x^3])/12
```

3.96.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 807, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{4}bc\int \frac{x^{14}}{c^2x^6+1}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{12}bc\int \frac{x^{12}}{c^2x^6+1}dx^3$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{12}bc\int \left(\frac{x^6}{c^2}+\frac{1}{c^4\left(c^2x^6+1\right)}-\frac{1}{c^4}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{12}bc\left(\frac{\arctan\left(cx^3\right)}{c^5}-\frac{x^3}{c^4}+\frac{x^9}{3c^2}\right)$$

input
```
Int[x^11*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^12*(a + b*ArcTan[c*x^3]))/12 - (b*c*(-(x^3/c^4) + x^9/(3*c^2) + ArcTan[
c*x^3]/c^5))/12
```

3.96.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.96.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{a\,x^{12}}{12}+\frac{b\,x^{12}\arctan\left(c\,x^{3}\right)}{12}-\frac{b\,x^{9}}{36c}+\frac{b\,x^{3}}{12c^{3}}-\frac{b\arctan\left(c\,x^{3}\right)}{12c^{4}}$	50
parts	$\frac{a\,x^{12}}{12}+\frac{b\,x^{12}\arctan\left(c\,x^{3}\right)}{12}-\frac{b\,x^{9}}{36c}+\frac{b\,x^{3}}{12c^{3}}-\frac{b\arctan\left(c\,x^{3}\right)}{12c^{4}}$	50
parallelrisch	$\frac{3b\arctan\left(c\,x^{3}\right)x^{12}c^{4}+3a\,c^{4}x^{12}-b\,c^{3}x^{9}+3bc\,x^{3}-3b\arctan\left(c\,x^{3}\right)}{36c^{4}}$	56
risch	$-\frac{i x^{12} b\ln\left(i c\,x^{3}+1\right)}{24}+\frac{i x^{12} b\ln\left(-i c\,x^{3}+1\right)}{24}+\frac{a\,x^{12}}{12}-\frac{b\,x^{9}}{36c}+\frac{b\,x^{3}}{12c^{3}}-\frac{b\arctan\left(c\,x^{3}\right)}{12c^{4}}$	72

input
```
int(x^11*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/12*a*x^12+1/12*b*x^12*arctan(c*x^3)-1/36*b*x^9/c+1/12*b*x^3/c^3-1/12*b*a
rctan(c*x^3)/c^4
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int x^{11}(a+b\arctan(cx^3))\,dx=\frac{3\,ac^4x^{12}-bc^3x^9+3\,bcx^3+3\,(bc^4x^{12}-b)\arctan(cx^3)}{36\,c^4}$$

input
```
integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/36*(3*a*c^4*x^12 - b*c^3*x^9 + 3*b*c*x^3 + 3*(b*c^4*x^12 - b)*arctan(c*x
^3))/c^4
```

3.96.6 Sympy [A] (verification not implemented)

Time = 129.73 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int x^{11}(a+b\arctan(cx^3))\,dx=\begin{cases}\frac{ax^{12}}{12}+\frac{bx^{12}\operatorname{atan}(cx^3)}{12}-\frac{bx^9}{36c}+\frac{bx^3}{12c^3}-\frac{b\operatorname{atan}(cx^3)}{12c^4} & \text{for } c\neq 0\\ \frac{ax^{12}}{12} & \text{otherwise}\end{cases}$$

input
```
integrate(x**11*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**12/12 + b*x**12*atan(c*x**3)/12 - b*x**9/(36*c) + b*x**3/(
12*c**3) - b*atan(c*x**3)/(12*c**4), Ne(c, 0)), (a*x**12/12, True))
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^{11}(a+b\arctan(cx^3))\,dx=\frac{1}{12}\,ax^{12}+\frac{1}{36}\left(3\,x^{12}\arctan(cx^3)-c\left(\frac{c^2x^9-3\,x^3}{c^4}+\frac{3\,\arctan(cx^3)}{c^5}\right)\right)b$$

input
```
integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/12*a*x^12 + 1/36*(3*x^12*arctan(c*x^3) - c*((c^2*x^9 - 3*x^3)/c^4 + 3*ar
ctan(c*x^3)/c^5))*b
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int x^{11}\left(a+b\arctan\left(cx^{3}\right)\right)\,dx=\frac{3\,acx^{12}+\left(3\,cx^{12}\arctan\left(cx^{3}\right)-\frac{3\,\arctan\left(cx^{3}\right)}{c^{3}}-\frac{c^{9}x^{9}-3\,c^{7}x^{3}}{c^{9}}\right)b}{36\,c}$$

input
```
integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/36*(3*a*c*x^12 + (3*c*x^12*arctan(c*x^3) - 3*arctan(c*x^3)/c^3 - (c^9*x^
9 - 3*c^7*x^3)/c^9)*b)/c
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int x^{11}\left(a+b\arctan\left(cx^{3}\right)\right)\,dx=\frac{a\,x^{12}}{12}+\frac{b\,x^{3}}{12\,c^{3}}-\frac{b\,x^{9}}{36\,c}-\frac{b\,\mathrm{atan}\left(c\,x^{3}\right)}{12\,c^{4}}+\frac{b\,x^{12}\,\mathrm{atan}\left(c\,x^{3}\right)}{12}$$

input
```
int(x^11*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^12)/12 + (b*x^3)/(12*c^3) - (b*x^9)/(36*c) - (b*atan(c*x^3))/(12*c^4)
 + (b*x^12*atan(c*x^3))/12
```

3.97 $\int x^8(a+b\arctan(cx^3))\,dx$

3.97.1 Optimal result . . . 630
3.97.2 Mathematica [A] (verified) . . . 630
3.97.3 Rubi [A] (verified) . . . 631
3.97.4 Maple [A] (verified) . . . 632
3.97.5 Fricas [A] (verification not implemented) . . . 632
3.97.6 Sympy [B] (verification not implemented) . . . 633
3.97.7 Maxima [A] (verification not implemented) . . . 633
3.97.8 Giac [A] (verification not implemented) . . . 634
3.97.9 Mupad [B] (verification not implemented) . . . 634

3.97.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)\,dx=-\frac{bx^6}{18c}+\frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)+\frac{b\log\left(1+c^2x^6\right)}{18c^3}$$

output
```
-1/18*b*x^6/c+1/9*x^9*(a+b*arctan(c*x^3))+1/18*b*ln(c^2*x^6+1)/c^3
```

3.97.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)\,dx=-\frac{bx^6}{18c}+\frac{ax^9}{9}+\frac{1}{9}bx^9\arctan\left(cx^3\right)+\frac{b\log\left(1+c^2x^6\right)}{18c^3}$$

input
```
Integrate[x^8*(a + b*ArcTan[c*x^3]),x]
```

output
```
-1/18*(b*x^6)/c + (a*x^9)/9 + (b*x^9*ArcTan[c*x^3])/9 + (b*Log[1 + c^2*x^6
])/(18*c^3)
```

3.97.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+b \arctan \left(c x^3\right)\right) \, dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{9} x^9 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{3} b c \int \frac{x^{11}}{c^2 x^6+1}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{9} x^9 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{18} b c \int \frac{x^6}{c^2 x^6+1}dx^6$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{9} x^9 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{18} b c \int \left(\frac{1}{c^2}-\frac{1}{c^2 \left(c^2 x^6+1\right)}\right)dx^6$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{9} x^9 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{18} b c \left(\frac{x^6}{c^2}-\frac{\log \left(c^2 x^6+1\right)}{c^4}\right)$$

input
```
Int[x^8*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^9*(a + b*ArcTan[c*x^3]))/9 - (b*c*(x^6/c^2 - Log[1 + c^2*x^6]/c^4))/18
```

3.97.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.97.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{x^9 a}{9}+\frac{b\,x^9\arctan\left(c\,x^3\right)}{9}-\frac{b\,x^6}{18c}+\frac{b\ln\left(c^2x^6+1\right)}{18c^3}$	45
parts	$\frac{x^9 a}{9}+\frac{b\,x^9\arctan\left(c\,x^3\right)}{9}-\frac{b\,x^6}{18c}+\frac{b\ln\left(c^2x^6+1\right)}{18c^3}$	45
parallelrisch	$\frac{2x^9\arctan\left(c\,x^3\right)b\,c^3+2a\,c^3x^9-c^2b\,x^6+b\ln\left(c^2x^6+1\right)}{18c^3}$	52
risch	$-\frac{i x^9 b\ln\left(i c\,x^3+1\right)}{18}+\frac{i x^9 b\ln\left(-i c\,x^3+1\right)}{18}+\frac{x^9 a}{9}-\frac{b\,x^6}{18c}+\frac{b\ln\left(-c^2x^6-1\right)}{18c^3}$	68

input
```
int(x^8*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/9*x^9*a+1/9*b*x^9*arctan(c*x^3)-1/18*b*x^6/c+1/18*b*ln(c^2*x^6+1)/c^3
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{2\,bc^3x^9\arctan\left(cx^3\right)+2\,ac^3x^9-bc^2x^6+b\log\left(c^2x^6+1\right)}{18\,c^3}$$

input
```
integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/18*(2*b*c^3*x^9*arctan(c*x^3) + 2*a*c^3*x^9 - b*c^2*x^6 + b*log(c^2*x^6
+ 1))/c^3
```

3.97.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(39) = 78$.

Time = 73.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.49

$$\int x^8(a+b\arctan(cx^3))\,dx = \begin{cases} \frac{ax^9}{9}+\frac{bx^9\operatorname{atan}(cx^3)}{9}-\frac{bx^6}{18c}-\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}(cx^3)}{9c^2}+\frac{b\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{9c^3}+\frac{b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{9c^3} & \text{for } c\neq 0 \\ \frac{ax^9}{9} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**9/9 + b*x**9*atan(c*x**3)/9 - b*x**6/(18*c) - b*sqrt(-1/c*
*2)*atan(c*x**3)/(9*c**2) + b*log(x - (-1/c**2)**(1/6))/(9*c**3) + b*log(4
*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(9*c**3), Ne(c, 0)), (a
*x**9/9, True))
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int x^8(a+b\arctan(cx^3))\,dx = \frac{1}{9}ax^9+\frac{1}{18}\left(2x^9\arctan(cx^3)-\left(\frac{x^6}{c^2}-\frac{\log(c^2x^6+1)}{c^4}\right)c\right)b$$

input
```
integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/9*a*x^9 + 1/18*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*c
)*b
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int x^8 \left(a + b \arctan \left(cx^3\right)\right) \, dx = \frac{2\, acx^9 + \left(2\, cx^9 \arctan \left(cx^3\right) - x^6 + \frac{\log\left(c^2 x^6 + 1\right)}{c^2}\right) b}{18\, c}$$

input
```
integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/18*(2*a*c*x^9 + (2*c*x^9*arctan(c*x^3) - x^6 + log(c^2*x^6 + 1)/c^2)*b)/
c
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int x^8 \left(a + b \arctan \left(cx^3\right)\right) \, dx = \frac{a\, x^9}{9} + \frac{b\, \ln \left(c^2\, x^6 + 1\right)}{18\, c^3} - \frac{b\, x^6}{18\, c} + \frac{b\, x^9\, \mathrm{atan}(c\, x^3)}{9}$$

input
```
int(x^8*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^9)/9 + (b*log(c^2*x^6 + 1))/(18*c^3) - (b*x^6)/(18*c) + (b*x^9*atan(c
*x^3))/9
```

3.98 $\int x^5(a+b\arctan(cx^3))\,dx$

3.98.1 Optimal result 635
3.98.2 Mathematica [A] (verified) 635
3.98.3 Rubi [A] (verified) 636
3.98.4 Maple [A] (verified) 637
3.98.5 Fricas [A] (verification not implemented) 638
3.98.6 Sympy [A] (verification not implemented) 638
3.98.7 Maxima [A] (verification not implemented) 638
3.98.8 Giac [A] (verification not implemented) 639
3.98.9 Mupad [B] (verification not implemented) 639

3.98.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int x^5(a+b\arctan(cx^3))\,dx=-\frac{bx^3}{6c}+\frac{b\arctan(cx^3)}{6c^2}+\frac{1}{6}x^6(a+b\arctan(cx^3))$$

output
```
-1/6*b*x^3/c+1/6*b*arctan(c*x^3)/c^2+1/6*x^6*(a+b*arctan(c*x^3))
```

3.98.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^5(a+b\arctan(cx^3))\,dx=-\frac{bx^3}{6c}+\frac{ax^6}{6}+\frac{b\arctan(cx^3)}{6c^2}+\frac{1}{6}bx^6\arctan(cx^3)$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x^3]),x]
```

output
```
-1/6*(b*x^3)/c + (a*x^6)/6 + (b*ArcTan[c*x^3])/(6*c^2) + (b*x^6*ArcTan[c*x
^3])/6
```

3.98.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 807, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b \arctan \left(c x^3\right)\right) \, dx$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{6} x^6 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{2} b c \int \frac{x^8}{c^2 x^6+1}dx$$

$$\Big\downarrow \; 807$$

$$\frac{1}{6} x^6 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{6} b c \int \frac{x^6}{c^2 x^6+1}dx^3$$

$$\Big\downarrow \; 262$$

$$\frac{1}{6} x^6 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{6} b c \left(\frac{x^3}{c^2}-\frac{\int \frac{1}{c^2 x^6+1}dx^3}{c^2}\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{6} x^6 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{6} b c \left(\frac{x^3}{c^2}-\frac{\arctan \left(c x^3\right)}{c^3}\right)$$

input
```
Int[x^5*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^6*(a + b*ArcTan[c*x^3]))/6 - (b*c*(x^3/c^2 - ArcTan[c*x^3]/c^3))/6
```

3.98.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.98.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{a\,x^{6}}{6}+\frac{b\,x^{6}\arctan\left(c\,x^{3}\right)}{6}-\frac{b\,x^{3}}{6c}+\frac{b\arctan\left(c\,x^{3}\right)}{6c^{2}}$	41
parts	$\frac{a\,x^{6}}{6}+\frac{b\,x^{6}\arctan\left(c\,x^{3}\right)}{6}-\frac{b\,x^{3}}{6c}+\frac{b\arctan\left(c\,x^{3}\right)}{6c^{2}}$	41
parallelrisch	$\frac{\arctan\left(c\,x^{3}\right)b\,c^{2}x^{6}+a\,c^{2}x^{6}-bc\,x^{3}+b\arctan\left(c\,x^{3}\right)}{6c^{2}}$	44
risch	$-\frac{ix^{6}b\ln\left(ic\,x^{3}+1\right)}{12}+\frac{ix^{6}b\ln\left(-ic\,x^{3}+1\right)}{12}+\frac{a\,x^{6}}{6}-\frac{b\,x^{3}}{6c}+\frac{b\arctan\left(c\,x^{3}\right)}{6c^{2}}+\frac{b^{2}}{24a\,c^{2}}$	74

input
```
int(x^5*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x^6+1/6*b*x^6*arctan(c*x^3)-1/6*b*x^3/c+1/6*b*arctan(c*x^3)/c^2
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int x^5 (a + b \arctan (cx^3)) \, dx = \frac{ac^2x^6 - bcx^3 + (bc^2x^6 + b) \arctan (cx^3)}{6 \, c^2}$$

input
```
integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/6*(a*c^2*x^6 - b*c*x^3 + (b*c^2*x^6 + b)*arctan(c*x^3))/c^2
```

3.98.6 Sympy [A] (verification not implemented)

Time = 36.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^5 (a + b \arctan (cx^3)) \, dx = \begin{cases} \frac{ax^6}{6} + \frac{bx^6 \operatorname{atan}(cx^3)}{6} - \frac{bx^3}{6c} + \frac{b \operatorname{atan}(cx^3)}{6c^2} & \text{for } c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**6/6 + b*x**6*atan(c*x**3)/6 - b*x**3/(6*c) + b*atan(c*x**3
)/(6*c**2), Ne(c, 0)), (a*x**6/6, True))
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^5 (a + b \arctan (cx^3)) \, dx = \frac{1}{6} ax^6 + \frac{1}{6} \left(x^6 \arctan (cx^3) - c \left(\frac{x^3}{c^2} - \frac{\arctan (cx^3)}{c^3} \right) \right) b$$

input
```
integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/6*(x^6*arctan(c*x^3) - c*(x^3/c^2 - arctan(c*x^3)/c^3))*b
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^5(a+b\arctan(cx^3))\,dx=\frac{acx^6+\frac{(c^2x^6\arctan(cx^3)-cx^3+\arctan(cx^3))b}{c}}{6\,c}$$

input
```
integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/6*(a*c*x^6 + (c^2*x^6*arctan(c*x^3) - c*x^3 + arctan(c*x^3))*b/c)/c
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^5(a+b\arctan(cx^3))\,dx=\frac{a\,x^6}{6}-\frac{b\,x^3}{6\,c}+\frac{b\,\mathrm{atan}(c\,x^3)}{6\,c^2}+\frac{b\,x^6\,\mathrm{atan}(c\,x^3)}{6}$$

input
```
int(x^5*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^6)/6 - (b*x^3)/(6*c) + (b*atan(c*x^3))/(6*c^2) + (b*x^6*atan(c*x^3))/
6
```

3.99 $\int x^2(a+b\arctan(cx^3))\,dx$

3.99.1 Optimal result 640
3.99.2 Mathematica [A] (verified) 640
3.99.3 Rubi [A] (verified) 641
3.99.4 Maple [A] (verified) 642
3.99.5 Fricas [A] (verification not implemented) 642
3.99.6 Sympy [B] (verification not implemented) 643
3.99.7 Maxima [A] (verification not implemented) 643
3.99.8 Giac [A] (verification not implemented) 643
3.99.9 Mupad [B] (verification not implemented) 644

3.99.1 Optimal result

Integrand size = 14, antiderivative size = 36

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)-\frac{b\log\left(1+c^2x^6\right)}{6c}$$

output
```
1/3*x^3*(a+b*arctan(c*x^3))-1/6*b*ln(c^2*x^6+1)/c
```

3.99.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{ax^3}{3}+\frac{1}{3}bx^3\arctan\left(cx^3\right)-\frac{b\log\left(1+c^2x^6\right)}{6c}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x^3]),x]
```

output
```
(a*x^3)/3 + (b*x^3*ArcTan[c*x^3])/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

3.99.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5361, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \arctan \left(c x^3\right)\right) \, dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3} x^3 \left(a+b \arctan \left(c x^3\right)\right)-b c \int \frac{x^5}{c^2 x^6+1}dx$$

$$\Big\downarrow \text{ 792}$$

$$\frac{1}{3} x^3 \left(a+b \arctan \left(c x^3\right)\right)-\frac{b \log \left(c^2 x^6+1\right)}{6 c}$$

input
```
Int[x^2*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^3*(a + b*ArcTan[c*x^3]))/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

3.99.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.99.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
parts	$\frac{x^3 a}{3}+\frac{b \arctan \left(c \,x^{3}\right) x^{3}}{3}-\frac{b \ln \left(c^{2} x^{6}+1\right)}{6 c}$	36
derivativedivides	$\frac{a c \,x^{3}+b \left(c \,x^{3} \arctan \left(c \,x^{3}\right)-\frac{\ln \left(c^{2} x^{6}+1\right)}{2}\right)}{3 c}$	39
default	$\frac{a c \,x^{3}+b \left(c \,x^{3} \arctan \left(c \,x^{3}\right)-\frac{\ln \left(c^{2} x^{6}+1\right)}{2}\right)}{3 c}$	39
parallelrisch	$-\frac{-2 x^{3} \arctan \left(c \,x^{3}\right) b c -2 a c \,x^{3}+b \ln \left(c^{2} x^{6}+1\right)}{6 c}$	39
risch	$-\frac{i x^{3} b \ln \left(i c \,x^{3}+1\right)}{6}+\frac{i b \,x^{3} \ln \left(-i c \,x^{3}+1\right)}{6}+\frac{x^{3} a}{3}-\frac{b \ln \left(-c^{2} x^{6}-1\right)}{6 c}$	59

input
```
int(x^2*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*a+1/3*b*arctan(c*x^3)*x^3-1/6*b*ln(c^2*x^6+1)/c
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int x^2 \left(a+b \arctan \left(c x^3\right)\right) \, dx=\frac{2 \, b c x^{3} \arctan \left(c x^{3}\right) + 2 \, a c x^{3} - b \log \left(c^{2} x^{6} + 1\right)}{6 \, c}$$

input
```
integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/6*(2*b*c*x^3*arctan(c*x^3) + 2*a*c*x^3 - b*log(c^2*x^6 + 1))/c
```

3.99.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(29) = 58.

Time = 20.58 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.83

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx = \begin{cases} \frac{ax^3}{3}+\frac{bx^3\operatorname{atan}\left(cx^3\right)}{3}+\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^3\right)}{3}-\frac{b\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{3c}-\frac{b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{3c} & \text{for } c\neq 0 \\ \frac{ax^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**3/3 + b*x**3*atan(c*x**3)/3 + b*sqrt(-1/c**2)*atan(c*x**3)
/3 - b*log(x - (-1/c**2)**(1/6))/(3*c) - b*log(4*x**2 + 4*x*(-1/c**2)**(1/
6) + 4*(-1/c**2)**(1/3))/(3*c), Ne(c, 0)), (a*x**3/3, True))
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx = \frac{1}{3}\,ax^3+\frac{\left(2\,cx^3\arctan\left(cx^3\right)-\log\left(c^2x^6+1\right)\right)b}{6\,c}$$

input
```
integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b/c
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx = \frac{2\,acx^3+\left(2\,cx^3\arctan\left(cx^3\right)-\log\left(c^2x^6+1\right)\right)b}{6\,c}$$

input
```
integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/6*(2*a*c*x^3 + (2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b)/c
```

3.99.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int x^2 \left(a + b \arctan\left(cx^3\right)\right) \, dx = \frac{a\,x^3}{3} - \frac{b\,\ln\left(c^2\,x^6 + 1\right)}{6\,c} + \frac{b\,x^3\,\mathrm{atan}(c\,x^3)}{3}$$

input
```
int(x^2*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^3)/3 - (b*log(c^2*x^6 + 1))/(6*c) + (b*x^3*atan(c*x^3))/3
```

3.100 $\int \frac{a+b\arctan(cx^3)}{x}\,dx$

3.100.1 Optimal result 645
3.100.2 Mathematica [A] (verified) 645
3.100.3 Rubi [A] (verified) 646
3.100.4 Maple [C] (verified) 647
3.100.5 Fricas [F] 647
3.100.6 Sympy [F] 648
3.100.7 Maxima [F] 648
3.100.8 Giac [F] 648
3.100.9 Mupad [B] (verification not implemented) 649

3.100.1 Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan(cx^3)}{x}\,dx = a\log(x)+\frac{1}{6}ib\,\text{PolyLog}\left(2,-icx^3\right)-\frac{1}{6}ib\,\text{PolyLog}\left(2,icx^3\right)$$

output
```
a*ln(x)+1/6*I*b*polylog(2,-I*c*x^3)-1/6*I*b*polylog(2,I*c*x^3)
```

3.100.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx^3)}{x}\,dx = a\log(x)+\frac{1}{6}ib\,\text{PolyLog}\left(2,-icx^3\right)-\frac{1}{6}ib\,\text{PolyLog}\left(2,icx^3\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x,x]
```

output
```
a*Log[x] + (I/6)*b*PolyLog[2, (-I)*c*x^3] - (I/6)*b*PolyLog[2, I*c*x^3]
```

3.100.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{1}{3}\int \frac{a+b\arctan\left(cx^3\right)}{x^3}dx^3$$

$$\Big\downarrow \text{5355}$$

$$\frac{1}{3}\left(\frac{1}{2}ib\int \frac{\log\left(1-icx^3\right)}{x^3}dx^3-\frac{1}{2}ib\int \frac{\log\left(icx^3+1\right)}{x^3}dx^3+a\log\left(x^3\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{3}\left(a\log\left(x^3\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,-icx^3\right)-\frac{1}{2}ib\,\text{PolyLog}\left(2,icx^3\right)\right)$$

input
```
Int[(a + b*ArcTan[c*x^3])/x,x]
```

output
```
(a*Log[x^3] + (I/2)*b*PolyLog[2, (-I)*c*x^3] - (I/2)*b*PolyLog[2, I*c*x^3]
)/3
```

3.100.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

3.100.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

method	result
default	$a\ln(x)+b\ln(x)\arctan(c\,x^3)-\frac{b\left(\sum_{\_R1=\mathrm{RootOf}(c^2\_Z^6+1)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^3}\right)}{2c}$
parts	$a\ln(x)+b\ln(x)\arctan(c\,x^3)-\frac{b\left(\sum_{\_R1=\mathrm{RootOf}(c^2\_Z^6+1)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^3}\right)}{2c}$
risch	$-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}(c\_Z^3+\mathrm{RootOf}(\_Z^2+1,\mathrm{index}=1))}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)b}{2}+\frac{i\ln(x)\ln(-icx^3+1)b}{2}+a$

input
```
int((a+b*arctan(c*x^3))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*ln(x)*arctan(c*x^3)-1/2*b/c*sum(1/_R1^3*(ln(x)*ln((_R1-x)/_R1)+d
ilog((_R1-x)/_R1)),_R1=RootOf(_Z^6*c^2+1))
```

3.100.5 Fricas [F]

$$\int\frac{a+b\arctan(cx^3)}{x}\,dx=\int\frac{b\arctan(cx^3)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))/x,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x^3) + a)/x, x)
```

3.100.6 Sympy [F]

$$\int \frac{a + b\arctan(cx^3)}{x}\,dx = \int \frac{a + b\,\mathrm{atan}(cx^3)}{x}\,dx$$

input
```
integrate((a+b*atan(c*x**3))/x,x)
```

output
```
Integral((a + b*atan(c*x**3))/x, x)
```

3.100.7 Maxima [F]

$$\int \frac{a + b\arctan(cx^3)}{x}\,dx = \int \frac{b\arctan(cx^3) + a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))/x,x, algorithm="maxima")
```

output
```
b*integrate(arctan(c*x^3)/x, x) + a*log(x)
```

3.100.8 Giac [F]

$$\int \frac{a + b\arctan(cx^3)}{x}\,dx = \int \frac{b\arctan(cx^3) + a}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)/x, x)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arctan(cx^3)}{x}\,dx = a\,\ln(x) - \frac{b\left(\mathrm{Li}_2(1-c\,x^3\,1\mathrm{i}) - \mathrm{Li}_2(1\mathrm{i}\,c\,x^3+1)\right)\,1\mathrm{i}}{6}$$

input
```
int((a + b*atan(c*x^3))/x,x)
```

output
```
a*log(x) - (b*(dilog(1 - c*x^3*1i) - dilog(c*x^3*1i + 1))*1i)/6
```

3.101 $\int \frac{a+b\arctan(cx^3)}{x^4}\,dx$

3.101.1 Optimal result 650
3.101.2 Mathematica [A] (verified) 650
3.101.3 Rubi [A] (verified) 651
3.101.4 Maple [A] (verified) 652
3.101.5 Fricas [A] (verification not implemented) 653
3.101.6 Sympy [B] (verification not implemented) 653
3.101.7 Maxima [A] (verification not implemented) 654
3.101.8 Giac [A] (verification not implemented) 654
3.101.9 Mupad [B] (verification not implemented) 654

3.101.1 Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = -\frac{a+b\arctan(cx^3)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1+c^2x^6\right)$$

output
```
1/3*(-a-b*arctan(c*x^3))/x^3+b*c*ln(x)-1/6*b*c*ln(c^2*x^6+1)
```

3.101.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{b\arctan(cx^3)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1+c^2x^6\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*ArcTan[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 + c^2*x^6
])/6
```

3.101.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5361, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^4}dx$$

$$\Big\downarrow \text{5361}$$

$$bc\int \frac{1}{x\left(c^2x^6+1\right)}dx-\frac{a+b\arctan\left(cx^3\right)}{3x^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{6}bc\int \frac{1}{x^6\left(c^2x^6+1\right)}dx^6-\frac{a+b\arctan\left(cx^3\right)}{3x^3}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{6}bc\left(\int \frac{1}{x^6}dx^6-c^2\int \frac{1}{c^2x^6+1}dx^6\right)-\frac{a+b\arctan\left(cx^3\right)}{3x^3}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{6}bc\left(\log\left(x^6\right)-c^2\int \frac{1}{c^2x^6+1}dx^6\right)-\frac{a+b\arctan\left(cx^3\right)}{3x^3}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{6}bc\left(\log\left(x^6\right)-\log\left(c^2x^6+1\right)\right)-\frac{a+b\arctan\left(cx^3\right)}{3x^3}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^4,x]
```

output
```
-1/3*(a + b*ArcTan[c*x^3])/x^3 + (b*c*(Log[x^6] - Log[1 + c^2*x^6]))/6
```

3.101.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.101.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{a}{3x^3}+b\left(-\frac{\arctan(c\,x^3)}{3x^3}+c\left(\ln(x)-\frac{\ln(c^2x^6+1)}{6}\right)\right)$	39
parts	$-\frac{a}{3x^3}+b\left(-\frac{\arctan(c\,x^3)}{3x^3}+c\left(\ln(x)-\frac{\ln(c^2x^6+1)}{6}\right)\right)$	39
parallelrisch	$\frac{6bc\ln(x)x^3-bc\ln(c^2x^6+1)x^3-2b\arctan(c\,x^3)-2a}{6x^3}$	45
risch	$\frac{ib\ln(ic\,x^3+1)}{6x^3}-\frac{-6bc\ln(x)x^3+bc\ln(-c^2x^6-1)x^3+ib\ln(-ic\,x^3+1)+2a}{6x^3}$	68

input
```
int((a+b*arctan(c*x^3))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*(-1/3/x^3*arctan(c*x^3)+c*(ln(x)-1/6*ln(c^2*x^6+1)))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = -\frac{bcx^3\log(c^2x^6+1)-6\,bcx^3\log(x)+2\,b\arctan(cx^3)+2\,a}{6\,x^3}$$

input
```
integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(b*c*x^3*log(c^2*x^6 + 1) - 6*b*c*x^3*log(x) + 2*b*arctan(c*x^3) + 2*
a)/x^3
```

3.101.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(39) = 78.

Time = 39.53 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.82

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = \begin{cases} -\frac{a}{3x^3}+bc\log(x)-\frac{bc\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{3}-\frac{bc\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{3}-\frac{b\,\mathrm{atan}(cx^3)}{3\sqrt{-\frac{1}{c^2}}}-\frac{b\,\mathrm{atan}(cx^3)}{3x^3} & \text{for } c \neq 0 \\ -\frac{a}{3x^3} & \text{otherwis} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3) + b*c*log(x) - b*c*log(x - (-1/c**2)**(1/6))/3 - b*
c*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/3 - b*atan(c*x**
3)/(3*sqrt(-1/c**2)) - b*atan(c*x**3)/(3*x**3), Ne(c, 0)), (-a/(3*x**3), T
rue))
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{a + b\arctan(cx^3)}{x^4}\,dx = -\frac{1}{6}\left(c\left(\log(c^2x^6+1) - \log(x^6)\right) + \frac{2\arctan(cx^3)}{x^3}\right)b - \frac{a}{3x^3}$$

input
```
integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="maxima")
```

output
```
-1/6*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*b - 1/3*a/x^3
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \frac{a + b\arctan(cx^3)}{x^4}\,dx = -\frac{bc^3x^3\log(c^2x^6+1) - 2\,bc^3x^3\log(cx^3) + 2\,bc^2\arctan(cx^3) + 2\,ac^2}{6\,c^2x^3}$$

input
```
integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="giac")
```

output
```
-1/6*(b*c^3*x^3*log(c^2*x^6 + 1) - 2*b*c^3*x^3*log(c*x^3) + 2*b*c^2*arctan
(c*x^3) + 2*a*c^2)/(c^2*x^3)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{a + b\arctan(cx^3)}{x^4}\,dx = b\,c\,\ln(x) - \frac{a}{3\,x^3} - \frac{b\,\mathrm{atan}(c\,x^3)}{3\,x^3} - \frac{b\,c\,\ln(c^2\,x^6+1)}{6}$$

input
```
int((a + b*atan(c*x^3))/x^4,x)
```

output
```
b*c*log(x) - a/(3*x^3) - (b*atan(c*x^3))/(3*x^3) - (b*c*log(c^2*x^6 + 1))/
6
```

3.102 $\int \frac{a+b\arctan(cx^3)}{x^7}\,dx$

3.102.1 Optimal result . . . 655
3.102.2 Mathematica [C] (verified) . . . 655
3.102.3 Rubi [A] (verified) . . . 656
3.102.4 Maple [A] (verified) . . . 657
3.102.5 Fricas [A] (verification not implemented) . . . 658
3.102.6 Sympy [A] (verification not implemented) . . . 658
3.102.7 Maxima [A] (verification not implemented) . . . 658
3.102.8 Giac [C] (verification not implemented) . . . 659
3.102.9 Mupad [B] (verification not implemented) . . . 659

3.102.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{bc}{6x^3}-\frac{1}{6}bc^2\arctan(cx^3)-\frac{a+b\arctan(cx^3)}{6x^6}$$

output
```
-1/6*b*c/x^3-1/6*b*c^2*arctan(c*x^3)+1/6*(-a-b*arctan(c*x^3))/x^6
```

3.102.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{a}{6x^6}-\frac{b\arctan(cx^3)}{6x^6}-\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^6\right)}{6x^3}$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^7,x]
```

output
```
-1/6*a/x^6 - (b*ArcTan[c*x^3])/(6*x^6) - (b*c*Hypergeometric2F1[-1/2, 1, 1
/2, -(c^2*x^6)])/(6*x^3)
```

3.102.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 807, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^7}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}bc\int \frac{1}{x^4\left(c^2x^6+1\right)}dx-\frac{a+b\arctan\left(cx^3\right)}{6x^6}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{6}bc\int \frac{1}{x^6\left(c^2x^6+1\right)}dx^3-\frac{a+b\arctan\left(cx^3\right)}{6x^6}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{6}bc\left(c^2\left(-\int \frac{1}{c^2x^6+1}dx^3\right)-\frac{1}{x^3}\right)-\frac{a+b\arctan\left(cx^3\right)}{6x^6}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}bc\left(-c\arctan\left(cx^3\right)-\frac{1}{x^3}\right)-\frac{a+b\arctan\left(cx^3\right)}{6x^6}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^7,x]
```

output
```
-1/6*(a + b*ArcTan[c*x^3])/x^6 + (b*c*(-x^(-3) - c*ArcTan[c*x^3]))/6
```

3.102.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.102.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a}{6x^6}-\frac{b\arctan\left(c\,x^3\right)}{6x^6}-\frac{b\,c^2\arctan\left(c\,x^3\right)}{6}-\frac{bc}{6x^3}$	39
parts	$-\frac{a}{6x^6}-\frac{b\arctan\left(c\,x^3\right)}{6x^6}-\frac{b\,c^2\arctan\left(c\,x^3\right)}{6}-\frac{bc}{6x^3}$	39
parallelrisch	$-\frac{\arctan\left(c\,x^3\right)b\,c^2x^6-a\,c^2x^6+bc\,x^3+b\arctan\left(c\,x^3\right)+a}{6x^6}$	45
risch	$\frac{ib\ln\left(ic\,x^3+1\right)}{12x^6}-\frac{ib\,c^2\ln\left(c\,x^3+i\right)x^6-ib\,c^2\ln\left(c\,x^3-i\right)x^6+2bc\,x^3+ib\ln\left(-ic\,x^3+1\right)+2a}{12x^6}$	87

input
```
int((a+b*arctan(c*x^3))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6-1/6*b/x^6*arctan(c*x^3)-1/6*b*c^2*arctan(c*x^3)-1/6*b*c/x^3
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{a + b \arctan(cx^3)}{x^7} \, dx = -\frac{bcx^3 + (bc^2x^6 + b) \arctan(cx^3) + a}{6 x^6}$$

input
```
integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="fricas")
```

output
```
-1/6*(b*c*x^3 + (b*c^2*x^6 + b)*arctan(c*x^3) + a)/x^6
```

3.102.6 Sympy [A] (verification not implemented)

Time = 37.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{a + b \arctan(cx^3)}{x^7} \, dx = -\frac{a}{6x^6} - \frac{bc^2 \operatorname{atan}(cx^3)}{6} - \frac{bc}{6x^3} - \frac{b \operatorname{atan}(cx^3)}{6x^6}$$

input
```
integrate((a+b*atan(c*x**3))/x**7,x)
```

output
```
-a/(6*x**6) - b*c**2*atan(c*x**3)/6 - b*c/(6*x**3) - b*atan(c*x**3)/(6*x**
6)
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{a + b \arctan(cx^3)}{x^7} \, dx = -\frac{1}{6} \left(\left(c \arctan(cx^3) + \frac{1}{x^3} \right) c + \frac{\arctan(cx^3)}{x^6} \right) b - \frac{a}{6 x^6}$$

input
```
integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="maxima")
```

output
```
-1/6*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*b - 1/6*a/x^6
```

3.102.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = \frac{i\,bc^5x^6\log(i\,cx^3+1) - i\,bc^5x^6\log(-i\,cx^3+1) - 2\,bc^4x^3 - 2\,bc^3\arctan(cx^3) - 2\,ac^3}{12\,c^3x^6}$$

input
```
integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="giac")
```

output
```
1/12*(I*b*c^5*x^6*log(I*c*x^3 + 1) - I*b*c^5*x^6*log(-I*c*x^3 + 1) - 2*b*c
^4*x^3 - 2*b*c^3*arctan(c*x^3) - 2*a*c^3)/(c^3*x^6)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{\frac{b\,c\,x^3}{3}+\frac{a}{3}}{2\,x^6} - \frac{b\,c^2\,\mathrm{atan}(c\,x^3)}{6} - \frac{b\,\mathrm{atan}(c\,x^3)}{6\,x^6}$$

input
```
int((a + b*atan(c*x^3))/x^7,x)
```

output
```
- (a/3 + (b*c*x^3)/3)/(2*x^6) - (b*c^2*atan(c*x^3))/6 - (b*atan(c*x^3))/(6
*x^6)
```

3.103 $\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx$

3.103.1 Optimal result 660
3.103.2 Mathematica [A] (verified) 660
3.103.3 Rubi [A] (verified) 661
3.103.4 Maple [A] (verified) 662
3.103.5 Fricas [A] (verification not implemented) 663
3.103.6 Sympy [B] (verification not implemented) 663
3.103.7 Maxima [A] (verification not implemented) 664
3.103.8 Giac [A] (verification not implemented) 664
3.103.9 Mupad [B] (verification not implemented) 664

3.103.1 Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx=-\frac{bc}{18x^6}-\frac{a+b\arctan(cx^3)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log\left(1+c^2x^6\right)$$

output
```
-1/18*b*c/x^6+1/9*(-a-b*arctan(c*x^3))/x^9-1/3*b*c^3*ln(x)+1/18*b*c^3*ln(c
^2*x^6+1)
```

3.103.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx=-\frac{a}{9x^9}-\frac{bc}{18x^6}-\frac{b\arctan(cx^3)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log\left(1+c^2x^6\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^10,x]
```

output
```
-1/9*a/x^9 - (b*c)/(18*x^6) - (b*ArcTan[c*x^3])/(9*x^9) - (b*c^3*Log[x])/3
 + (b*c^3*Log[1 + c^2*x^6])/18
```

3.103.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^{10}}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}bc\int \frac{1}{x^7\left(c^2x^6+1\right)}dx-\frac{a+b\arctan\left(cx^3\right)}{9x^9}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{18}bc\int \frac{1}{x^{12}\left(c^2x^6+1\right)}dx^6-\frac{a+b\arctan\left(cx^3\right)}{9x^9}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{18}bc\int \left(\frac{c^4}{c^2x^6+1}-\frac{c^2}{x^6}+\frac{1}{x^{12}}\right)dx^6-\frac{a+b\arctan\left(cx^3\right)}{9x^9}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{18}bc\left(c^2\left(-\log\left(x^6\right)\right)+c^2\log\left(c^2x^6+1\right)-\frac{1}{x^6}\right)-\frac{a+b\arctan\left(cx^3\right)}{9x^9}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^10,x]
```

output
```
-1/9*(a + b*ArcTan[c*x^3])/x^9 + (b*c*(-x^(-6) - c^2*Log[x^6] + c^2*Log[1
+ c^2*x^6]))/18
```

3.103.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.103.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a}{9x^9}+b\left(-\frac{\arctan\left(c\,x^3\right)}{9x^9}+\frac{c\left(-\frac{1}{6x^6}-c^2\ln\left(x\right)+\frac{c^2\ln\left(c^2x^6+1\right)}{6}\right)}{3}\right)$	53
parts	$-\frac{a}{9x^9}+b\left(-\frac{\arctan\left(c\,x^3\right)}{9x^9}+\frac{c\left(-\frac{1}{6x^6}-c^2\ln\left(x\right)+\frac{c^2\ln\left(c^2x^6+1\right)}{6}\right)}{3}\right)$	53
parallelrisch	$-\frac{6b\,c^3\ln\left(x\right)x^9-b\,c^3\ln\left(c^2x^6+1\right)x^9-b\,c^3x^9+bc\,x^3+2b\arctan\left(c\,x^3\right)+2a}{18x^9}$	64
risch	$\frac{ib\ln\left(ic\,x^3+1\right)}{18x^9}-\frac{6b\,c^3\ln\left(x\right)x^9-b\,c^3\ln\left(c^2x^6+1\right)x^9+bc\,x^3+ib\ln\left(-ic\,x^3+1\right)+2a}{18x^9}$	78

input
```
int((a+b*arctan(c*x^3))/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a/x^9+b*(-1/9/x^9*arctan(c*x^3)+1/3*c*(-1/6/x^6-c^2*ln(x)+1/6*c^2*ln(
c^2*x^6+1)))
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx = \frac{bc^3x^9\log(c^2x^6+1)-6\,bc^3x^9\log(x)-bcx^3-2\,b\arctan(cx^3)-2\,a}{18\,x^9}$$

input
```
integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="fricas")
```

output
```
1/18*(b*c^3*x^9*log(c^2*x^6 + 1) - 6*b*c^3*x^9*log(x) - b*c*x^3 - 2*b*arct
an(c*x^3) - 2*a)/x^9
```

3.103.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(53) = 106.

Time = 135.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.35

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx = \begin{cases} -\frac{a}{9x^9}-\frac{bc^4\sqrt{-\frac{1}{c^2}}\operatorname{atan}(cx^3)}{9}-\frac{bc^3\log(x)}{3}+\frac{bc^3\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{9}+\frac{bc^3\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{9}-\frac{bc}{18x^6}-\frac{b\operatorname{atan}(c\ldots}{9x^9} \\ -\frac{a}{9x^9} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))/x**10,x)
```

output
```
Piecewise((-a/(9*x**9) - b*c**4*sqrt(-1/c**2)*atan(c*x**3)/9 - b*c**3*log(
x)/3 + b*c**3*log(x - (-1/c**2)**(1/6))/9 + b*c**3*log(4*x**2 + 4*x*(-1/c*
*2)**(1/6) + 4*(-1/c**2)**(1/3))/9 - b*c/(18*x**6) - b*atan(c*x**3)/(9*x**
9), Ne(c, 0)), (-a/(9*x**9), True))
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx = \frac{1}{18}\left(\left(c^2\log\left(c^2x^6+1\right)-c^2\log\left(x^6\right)-\frac{1}{x^6}\right)c-\frac{2\arctan(cx^3)}{x^9}\right)b - \frac{a}{9x^9}$$

input
```
integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="maxima")
```

output
```
1/18*((c^2*log(c^2*x^6 + 1) - c^2*log(x^6) - 1/x^6)*c - 2*arctan(c*x^3)/x^
9)*b - 1/9*a/x^9
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx = \frac{bc^7x^9\log\left(c^2x^6+1\right)-2\,bc^7x^9\log\left(cx^3\right)-bc^5x^3-2\,bc^4\arctan(cx^3)-2\,ac^4}{18\,c^4x^9}$$

input
```
integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="giac")
```

output
```
1/18*(b*c^7*x^9*log(c^2*x^6 + 1) - 2*b*c^7*x^9*log(c*x^3) - b*c^5*x^3 - 2*
b*c^4*arctan(c*x^3) - 2*a*c^4)/(c^4*x^9)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx = \frac{b\,c^3\,\ln\left(c^2\,x^6+1\right)}{18}-\frac{a}{9\,x^9}-\frac{b\,c^3\,\ln(x)}{3}-\frac{b\,\mathrm{atan}(c\,x^3)}{9\,x^9}-\frac{b\,c}{18\,x^6}$$

input
```
int((a + b*atan(c*x^3))/x^10,x)
```

output
```
(b*c^3*log(c^2*x^6 + 1))/18 - a/(9*x^9) - (b*c^3*log(x))/3 - (b*atan(c*x^3
))/(9*x^9) - (b*c)/(18*x^6)
```

3.104 $\int x^3(a + b\arctan(cx^3))\,dx$

3.104.1 Optimal result 665
3.104.2 Mathematica [A] (verified) 666
3.104.3 Rubi [A] (verified) 666
3.104.4 Maple [A] (verified) 671
3.104.5 Fricas [B] (verification not implemented) 672
3.104.6 Sympy [A] (verification not implemented) 672
3.104.7 Maxima [A] (verification not implemented) 673
3.104.8 Giac [A] (verification not implemented) 673
3.104.9 Mupad [B] (verification not implemented) 674

3.104.1 Optimal result

Integrand size = 14, antiderivative size = 174

$$\begin{aligned}\int x^3(a+b\arctan(cx^3))\,dx = &-\frac{3bx}{4c}+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{4c^{4/3}}\\&+\frac{1}{4}x^4(a+b\arctan(cx^3))-\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{8c^{4/3}}\\&+\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{8c^{4/3}}-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}\\&+\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}\end{aligned}$$

output
```
-3/4*b*x/c+1/4*b*arctan(c^(1/3)*x)/c^(4/3)+1/4*x^4*(a+b*arctan(c*x^3))+1/8
*b*arctan(2*c^(1/3)*x-3^(1/2))/c^(4/3)+1/8*b*arctan(2*c^(1/3)*x+3^(1/2))/c
^(4/3)-1/16*b*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)+1/16*b*l
n(1+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)
```

3.104.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.03

$$\int x^3(a+b\arctan(cx^3))\,dx=-\frac{3bx}{4c}+\frac{ax^4}{4}+\frac{b\arctan(\sqrt[3]{c}x)}{4c^{4/3}}+\frac{1}{4}bx^4\arctan(cx^3)-\frac{b\arctan(\sqrt{3}-2\sqrt[3]{c}x)}{8c^{4/3}}+\frac{b\arctan(\sqrt{3}+2\sqrt[3]{c}x)}{8c^{4/3}}-\frac{\sqrt{3}b\log(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2)}{16c^{4/3}}+\frac{\sqrt{3}b\log(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2)}{16c^{4/3}}$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x^3]),x]
```

output
```
(-3*b*x)/(4*c) + (a*x^4)/4 + (b*ArcTan[c^(1/3)*x])/(4*c^(4/3)) + (b*x^4*Ar
cTan[c*x^3])/4 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(8*c^(4/3)) + (b*ArcTan
[Sqrt[3] + 2*c^(1/3)*x])/(8*c^(4/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*
x + c^(2/3)*x^2])/(16*c^(4/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^
(2/3)*x^2])/(16*c^(4/3))
```

3.104.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5361, 843, 753, 27, 216, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\arctan(cx^3))\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4(a+b\arctan(cx^3))-\frac{3}{4}bc\int\frac{x^6}{c^2x^6+1}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{4}x^4(a+b\arctan(cx^3))-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^6+1}dx}{c^2}\right)$$

$$\Big\downarrow \ 753$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{3}\int\frac{1}{c^{2/3}x^2+1}dx+\frac{1}{3}\int\frac{2-\sqrt{3}\sqrt[3]{c}x}{2\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}dx+\frac{1}{3}\int\frac{\sqrt{3}\sqrt[3]{c}x+2}{2\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{c^2}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{3}\int\frac{1}{c^{2/3}x^2+1}dx+\frac{1}{6}\int\frac{2-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{6}\int\frac{\sqrt{3}\sqrt[3]{c}x+2}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{c^2}\right)$$

$$\Big\downarrow \ 216$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\int\frac{2-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{6}\int\frac{\sqrt{3}\sqrt[3]{c}x+2}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\arctan\left(\sqrt[3]{c}x\right)}{3\sqrt[3]{c}}}{c^2}\right)$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\int-\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\cdots\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\cdots}}{2\sqrt[3]{c}}\cdots\right)}{c^2}\right)$$

$$\Big\downarrow \ 25$$

3.104. $\int x^3(a+b\arctan(cx^3))\,dx$

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right.}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x\ldots}}{2\sqrt[3]{c}}\right.}{c^2}\right.$$

$\downarrow$ 27

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}\ldots}{c^{2/3}x^2+\ldots}\right.}{c^2}\right.$$

$\downarrow$ 1082

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\int\frac{1}{-\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\int\ldots}{\ldots}\right.}{c^2}\right.$$

$\downarrow$ 217

$$\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{4}bc\left(\frac{x}{c^2}-\frac{\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\arctan\left(\sqrt{3}\left(\ldots\right.\right.}{\sqrt[3]{\ldots}}\right.}{c^2}\right.$$

$\downarrow$ 1103

3.104. $\int x^3(a+b\arctan\left(cx^3\right))\,dx$

$$\frac{1}{4}x^4(a + b\arctan(cx^3)) - \frac{3}{4}bc\left(\frac{x}{c^2} - \frac{\frac{1}{6}\left(-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}} - \frac{\sqrt{3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}\right) + \frac{1}{6}\left(\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right)}{\sqrt[3]{c}} + \frac{\sqrt{3}\log\left(c^{2/3}x^2+\sqrt{3}\right.}{2\sqrt[3]{c}}\right.}{c^2}\right.$$

input
```
Int[x^3*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^4*(a + b*ArcTan[c*x^3]))/4 - (3*b*c*(x/c^2 - (ArcTan[c^(1/3)*x]/(3*c^(1
/3)) + (-(ArcTan[Sqrt[3]*(1 - (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3)) - (Sqrt[3]*
Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/6 + (ArcTan[Sqrt[3]
*(1 + (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3) + (Sqrt[3]*Log[1 + Sqrt[3]*c^(1/3)*x
 + c^(2/3)*x^2])/(2*c^(1/3)))/6)/c^2))/4
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.104. $\int x^3(a+b\arctan(cx^3))\,dx$

3.104.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

method	result
default	$\frac{a\,x^{4}}{4}+b\left(\frac{x^{4}\arctan\left(c\,x^{3}\right)}{4}-\frac{3c\left(\frac{x}{c^{2}}-\frac{\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}\ln\left(x^{2}+\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{12}+\frac{\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6}-\frac{\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}\ln\left(x \cdots\right.}{c^{2}}\right)}{4}\right)$
parts	$\frac{a\,x^{4}}{4}+b\left(\frac{x^{4}\arctan\left(c\,x^{3}\right)}{4}-\frac{3c\left(\frac{x}{c^{2}}-\frac{\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}\ln\left(x^{2}+\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{12}+\frac{\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6}-\frac{\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}\ln\left(x \cdots\right.}{c^{2}}\right)}{4}\right)$

input
```
int(x^3*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+b*(1/4*x^4*arctan(c*x^3)-3/4*c*(1/c^2*x-(1/12*3^(1/2)*(1/c^2)^(1
/6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6*(1/c^2)^(1/6)*arctan
(2*x/(1/c^2)^(1/6)+3^(1/2))-1/12*3^(1/2)*(1/c^2)^(1/6)*ln(x^2-3^(1/2)*(1/c
^2)^(1/6)*x+(1/c^2)^(1/3))+1/6*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1
/2))+1/3*(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6)))/c^2))
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(126) = 252$.

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.55

$$\int x^3 (a + b \arctan (cx^3)) \, dx$$
$$= \frac{4\,bcx^4 \arctan (cx^3) + 4\,acx^4 + \left(\sqrt{-3}c + c\right)\left(-\frac{b^6}{c^8}\right)^{\frac{1}{6}} \log \left(bx + \frac{1}{2}\left(\sqrt{-3}c + c\right)\left(-\frac{b^6}{c^8}\right)^{\frac{1}{6}}\right) - \left(\sqrt{-3}c + c\right)\left(-\frac{b}{c}\right.}{}$$

input
```
integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/16*(4*b*c*x^4*arctan(c*x^3) + 4*a*c*x^4 + (sqrt(-3)*c + c)*(-b^6/c^8)^(1
/6)*log(b*x + 1/2*(sqrt(-3)*c + c)*(-b^6/c^8)^(1/6)) - (sqrt(-3)*c + c)*(-
b^6/c^8)^(1/6)*log(b*x - 1/2*(sqrt(-3)*c + c)*(-b^6/c^8)^(1/6)) + (sqrt(-3
)*c - c)*(-b^6/c^8)^(1/6)*log(b*x + 1/2*(sqrt(-3)*c - c)*(-b^6/c^8)^(1/6))
 - (sqrt(-3)*c - c)*(-b^6/c^8)^(1/6)*log(b*x - 1/2*(sqrt(-3)*c - c)*(-b^6/
c^8)^(1/6)) + 2*c*(-b^6/c^8)^(1/6)*log(b*x + c*(-b^6/c^8)^(1/6)) - 2*c*(-b
^6/c^8)^(1/6)*log(b*x - c*(-b^6/c^8)^(1/6)) - 12*b*x)/c
```

3.104.6 Sympy [A] (verification not implemented)

Time = 24.30 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.47

$$\int x^3 (a + b \arctan (cx^3)) \, dx$$
$$= \begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{atan}(cx^3)}{4} - \frac{3bx}{4c} - \frac{3b\sqrt[6]{-\frac{1}{c^2}} \log \left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{16c} + \frac{3b\sqrt[6]{-\frac{1}{c^2}} \log \left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{16c} + \\ \frac{ax^4}{4} \end{cases}$$

input
```
integrate(x**3*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**4/4 + b*x**4*atan(c*x**3)/4 - 3*b*x/(4*c) - 3*b*(-1/c**2)*
*(1/6)*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(16*c) + 3*
b*(-1/c**2)**(1/6)*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))
/(16*c) + sqrt(3)*b*(-1/c**2)**(1/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6))
 - sqrt(3)/3)/(8*c) + sqrt(3)*b*(-1/c**2)**(1/6)*atan(2*sqrt(3)*x/(3*(-1/c
**2)**(1/6)) + sqrt(3)/3)/(8*c) + b*atan(c*x**3)/(4*c**2*(-1/c**2)**(1/3))
, Ne(c, 0)), (a*x**4/4, True))
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.85

$$\int x^3(a+b\arctan(cx^3))\,dx = \frac{1}{4}ax^4 + \frac{1}{16}\left(4x^4\arctan(cx^3) + c\left(\frac{\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}} - \frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}} + \frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{1}{3}}} + \frac{2\arctan\left(\frac{2c^{\frac{2}{3}}x+\sqrt{}}{c^{\frac{1}{3}}}\right.}{c^{\frac{1}{3}}}}{c^2}\right.\right.$$

input
```
integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/16*(4*x^4*arctan(c*x^3) + c*((sqrt(3)*log(c^(2/3)*x^2 + sqrt
(3)*c^(1/3)*x + 1)/c^(1/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x +
 1)/c^(1/3) + 4*arctan(c^(1/3)*x)/c^(1/3) + 2*arctan((2*c^(2/3)*x + sqrt(3
)*c^(1/3))/c^(1/3))/c^(1/3) + 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(
1/3))/c^(1/3))/c^2 - 12*x/c^2))*b
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.96

$$\int x^3(a+b\arctan(cx^3))\,dx = \frac{1}{16}bc^7\left(\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^8|c|^{\frac{1}{3}}} - \frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^8|c|^{\frac{1}{3}}} + \frac{2\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^8|c|^{\frac{1}{3}}} + \frac{2\,\mathrm{arct}}{}\right.$$
$$+ \frac{bcx^4\arctan(cx^3) + acx^4 - 3bx}{4c}$$

input
```
integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/16*b*c^7*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/(c^
8*abs(c)^(1/3)) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3
))/(c^8*abs(c)^(1/3)) + 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3)
)/(c^8*abs(c)^(1/3)) + 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))
/(c^8*abs(c)^(1/3)) + 4*arctan(x*abs(c)^(1/3))/(c^8*abs(c)^(1/3))) + 1/4*(
b*c*x^4*arctan(c*x^3) + a*c*x^4 - 3*b*x)/c
```

3.104.9 Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.66

$$\int x^3 \left(a+b \arctan \left(c x^3\right)\right) \, dx$$
$$= \frac{a\,x^4}{4} - \frac{b\left(\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)-\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\right)}{8\,c^{4/3}}$$
$$+ \frac{b\,x^4\,\mathrm{atan}\left(c\,x^3\right)}{4} - \frac{3\,b\,x}{4\,c} - \frac{\sqrt{3}\,b\,\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)\right)\,1{}\mathrm{i}}{8\,c^{4/3}}$$

input
```
int(x^3*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^4)/4 - (b*(atan((-1)^(2/3)*c^(1/3)*x) - atan((c^(1/3)*x*(3^(1/2)*1i +
 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/(8*c^(4/3))
+ (b*x^4*atan(c*x^3))/4 - (3*b*x)/(4*c) - (3^(1/2)*b*(atan((c^(1/3)*x*(3^(
1/2)*1i + 1))/2) + atan((-1)^(2/3)*c^(1/3)*x))*1i)/(8*c^(4/3))
```

3.105 $\int (a + b \arctan(cx^3)) \, dx$

3.105.1 Optimal result 675
3.105.2 Mathematica [A] (verified) 675
3.105.3 Rubi [A] (verified) 676
3.105.4 Maple [A] (verified) 677
3.105.5 Fricas [A] (verification not implemented) 678
3.105.6 Sympy [A] (verification not implemented) 678
3.105.7 Maxima [A] (verification not implemented) 679
3.105.8 Giac [A] (verification not implemented) 680
3.105.9 Mupad [B] (verification not implemented) 680

3.105.1 Optimal result

Integrand size = 10, antiderivative size = 101

$$\int (a + b \arctan(cx^3)) \, dx = ax + bx \arctan(cx^3) + \frac{\sqrt{3} b \arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + \frac{b \log(1 + c^{2/3}x^2)}{2\sqrt[3]{c}} - \frac{b \log(1 - c^{2/3}x^2 + c^{4/3}x^4)}{4\sqrt[3]{c}}$$

output
```
a*x+b*x*arctan(c*x^3)+1/2*b*ln(1+c^(2/3)*x^2)/c^(1/3)-1/4*b*ln(1-c^(2/3)*x
^2+c^(4/3)*x^4)/c^(1/3)+1/2*b*arctan(1/3*(1-2*c^(2/3)*x^2)*3^(1/2))*3^(1/2
)/c^(1/3)
```

3.105.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.30

$$\int (a + b \arctan(cx^3)) \, dx = ax + bx \arctan(cx^3) - \frac{b\left(-2\sqrt{3}\arctan\left(\sqrt{3} - 2\sqrt[3]{c}x\right) - 2\sqrt{3}\arctan\left(\sqrt{3} + 2\sqrt[3]{c}x\right) - 2\log\left(1 + c^{2/3}x^2\right) + \log\left(1 - \sqrt{3}\sqrt[3]{c}x + c^{2}\right.}{4\sqrt[3]{c}}$$

input
```
Integrate[a + b*ArcTan[c*x^3],x]
```

output
```
a*x + b*x*ArcTan[c*x^3] - (b*(-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*c^(1/3)*x] - 2
*Sqrt[3]*ArcTan[Sqrt[3] + 2*c^(1/3)*x] - 2*Log[1 + c^(2/3)*x^2] + Log[1 -
Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2] + Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2
]))/(4*c^(1/3))
```

3.105.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b \arctan \left(c x^3\right)\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$a x+\frac{\sqrt{3} b \arctan \left(\frac{1-2 c^{2/3} x^2}{\sqrt{3}}\right)}{2 \sqrt[3]{c}}+b x \arctan \left(c x^3\right)+\frac{b \log \left(c^{2/3} x^2+1\right)}{2 \sqrt[3]{c}}-\frac{b \log \left(c^{4/3} x^4-c^{2/3} x^2+1\right)}{4 \sqrt[3]{c}}$$

input
```
Int[a + b*ArcTan[c*x^3],x]
```

output
```
a*x + b*x*ArcTan[c*x^3] + (Sqrt[3]*b*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/
(2*c^(1/3)) + (b*Log[1 + c^(2/3)*x^2])/(2*c^(1/3)) - (b*Log[1 - c^(2/3)*x^
2 + c^(4/3)*x^4])/(4*c^(1/3))
```

3.105.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.105.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

method	result	siz
default	$ax+bx\arctan\left(c\,x^{3}\right)+\frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}$	98
parts	$ax+bx\arctan\left(c\,x^{3}\right)+\frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}$	98

input
```
int(a+b*arctan(c*x^3),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arctan(c*x^3)+1/2*b/c/(1/c^2)^(1/3)*ln(x^2+(1/c^2)^(1/3))-1/4*b/c/
(1/c^2)^(1/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))-1/2*b*3^(1/2)/c/(1/c
^2)^(1/3)*arctan(1/3*3^(1/2)*(2*x^2/(1/c^2)^(1/3)-1))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.32

$$\int \left(a + b\arctan\left(cx^3\right)\right) dx = \left[\frac{4\,bcx\arctan\left(cx^3\right) + \sqrt{3}bc\sqrt{-\frac{1}{c^{\frac{2}{3}}}}\log\left(\frac{2\,c^2x^6 - 3\,c^{\frac{2}{3}}x^2 - \sqrt{3}\left(2\,c^{\frac{5}{3}}x^4 + cx^2 - c^{\frac{1}{3}}\right)\sqrt{-\frac{1}{c^{\frac{2}{3}}}} - 1}{c^2x^6 + 1}\right) + 4\,acx - bc^{\frac{2}{3}}\log\left(c^2x^4 - \right.}{4\,c}\right.$$

input
```
integrate(a+b*arctan(c*x^3),x, algorithm="fricas")
```

output
```
[1/4*(4*b*c*x*arctan(c*x^3) + sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c^2*x^6
- 3*c^(2/3)*x^2 - sqrt(3)*(2*c^(5/3)*x^4 + c*x^2 - c^(1/3))*sqrt(-1/c^(2/3
)) - 1)/(c^2*x^6 + 1)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 - c^(4/3)*x^2 + c
^(2/3)) + 2*b*c^(2/3)*log(c*x^2 + c^(1/3)))/c, 1/4*(4*b*c*x*arctan(c*x^3)
+ 2*sqrt(3)*b*c^(2/3)*arctan(-1/3*sqrt(3)*(2*c*x^2 - c^(1/3))/c^(1/3)) + 4
*a*c*x - b*c^(2/3)*log(c^2*x^4 - c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(2/3)*log(
c*x^2 + c^(1/3)))/c]
```

3.105.6 Sympy [A] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 755, normalized size of antiderivative = 7.48

$$\int \left(a + b\arctan\left(cx^3\right)\right) dx = ax + b\left(\begin{cases} 0 \\ -\infty ix \\ \infty ix \\ -\frac{4c^4x^6\left(-\frac{1}{c^2}\right)^{\frac{5}{3}}\log\left(x - \sqrt[6]{-\frac{1}{c^2}}\right)}{4cx^6 + \frac{4}{c}} + \frac{3c^4x^6\left(-\frac{1}{c^2}\right)^{\frac{5}{3}}\log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{4cx^6 + \frac{4}{c}} - \frac{c^4x^6\left(-\frac{1}{c^2}\right)^{\frac{5}{3}}\log\left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}}\right.}{4cx^6 + \frac{4}{c}} \end{cases}\right.$$

input
```
integrate(a+b*atan(c*x**3),x)
```

output
```
a*x + b*Piecewise((0, Eq(c, 0)), (-oo*I*x, Eq(c, -I/x**3)), (oo*I*x, Eq(c,
 I/x**3)), (-4*c**4*x**6*(-1/c**2)**(5/3)*log(x - (-1/c**2)**(1/6))/(4*c*x
**6 + 4/c) + 3*c**4*x**6*(-1/c**2)**(5/3)*log(4*x**2 - 4*x*(-1/c**2)**(1/6
) + 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - c**4*x**6*(-1/c**2)**(5/3)*log(
4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - 2*s
qrt(3)*c**4*x**6*(-1/c**2)**(5/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) -
sqrt(3)/3)/(4*c*x**6 + 4/c) + 2*sqrt(3)*c**4*x**6*(-1/c**2)**(5/3)*atan(2*
sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*x**6 + 4/c) - 4*c**4*x**6
*(-1/c**2)**(5/3)*log(2)/(4*c*x**6 + 4/c) - 4*c**3*x**6*(-1/c**2)**(7/6)*a
tan(c*x**3)/(4*c*x**6 + 4/c) - 4*c**2*(-1/c**2)**(5/3)*log(x - (-1/c**2)**
(1/6))/(4*c*x**6 + 4/c) + 3*c**2*(-1/c**2)**(5/3)*log(4*x**2 - 4*x*(-1/c**
2)**(1/6) + 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - c**2*(-1/c**2)**(5/3)*l
og(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) -
2*sqrt(3)*c**2*(-1/c**2)**(5/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sq
rt(3)/3)/(4*c*x**6 + 4/c) + 2*sqrt(3)*c**2*(-1/c**2)**(5/3)*atan(2*sqrt(3)
*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*x**6 + 4/c) - 4*c**2*(-1/c**2)**
(5/3)*log(2)/(4*c*x**6 + 4/c) + 4*c*x**7*atan(c*x**3)/(4*c*x**6 + 4/c) - 4
*c*(-1/c**2)**(7/6)*atan(c*x**3)/(4*c*x**6 + 4/c) + 4*x*atan(c*x**3)/(4*c*
*2*x**6 + 4), True))
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \left(a+b\arctan\left(cx^3\right)\right)\,dx = -\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{4}{3}}}-\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}\right)-4x\arctan\left(cx^3\right)\right)b + ax$$

input
```
integrate(a+b*arctan(c*x^3),x, algorithm="maxima")
```

output
```
-1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c
^(4/3) + log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1)/c^(4/3) - 2*log((c^(2/3)*x^2 +
 1)/c^(2/3))/c^(4/3)) - 4*x*arctan(c*x^3))*b + a*x
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \left(a+b\arctan\left(cx^3\right)\right)\,dx = -\frac{1}{4}\left(c\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}+\frac{|c|^{\frac{2}{3}}\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}-\frac{2\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{4}{3}}}\right)\right. + ax$$

input
```
integrate(a+b*arctan(c*x^3),x, algorithm="giac")
```

output
```
-1/4*(c*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3)
)*abs(c)^(2/3))/c^2 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(
4/3))/c^2 - 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3)) - 4*x*arctan(c*x^3))
*b + a*x
```

3.105.9 Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int \left(a+b\arctan\left(cx^3\right)\right)\,dx = a\,x+b\,x\,\mathrm{atan}\left(c\,x^3\right)+\frac{b\,\ln\left(c^{2/3}\,x^2+1\right)}{2\,c^{1/3}} - \frac{\ln\left(2-4\,c^{2/3}\,x^2+\sqrt{3}\,2\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}} - \frac{\ln\left(4\,c^{2/3}\,x^2-2+\sqrt{3}\,2\mathrm{i}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}}$$

input
```
int(a + b*atan(c*x^3),x)
```

output
```
a*x + b*x*atan(c*x^3) + (b*log(c^(2/3)*x^2 + 1))/(2*c^(1/3)) - (log(3^(1/2
)*2i - 4*c^(2/3)*x^2 + 2)*(b - 3^(1/2)*b*1i))/(4*c^(1/3)) - (log(3^(1/2)*2
i + 4*c^(2/3)*x^2 - 2)*(b + 3^(1/2)*b*1i))/(4*c^(1/3))
```

3.106 $\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$

3.106.1 Optimal result 681
3.106.2 Mathematica [A] (verified) 681
3.106.3 Rubi [A] (verified) 682
3.106.4 Maple [A] (verified) 686
3.106.5 Fricas [B] (verification not implemented) 687
3.106.6 Sympy [A] (verification not implemented) 687
3.106.7 Maxima [A] (verification not implemented) 688
3.106.8 Giac [A] (verification not implemented) 688
3.106.9 Mupad [B] (verification not implemented) 689

3.106.1 Optimal result

Integrand size = 14, antiderivative size = 165

$$\begin{aligned}&\int \frac{a+b\arctan(cx^3)}{x^3}\,dx\\&=\frac{1}{2}bc^{2/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{a+b\arctan(cx^3)}{2x^2}-\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)\\&\quad+\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)-\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)\end{aligned}$$

output
```
1/2*b*c^(2/3)*arctan(c^(1/3)*x)+1/2*(-a-b*arctan(c*x^3))/x^2+1/4*b*c^(2/3)
*arctan(2*c^(1/3)*x-3^(1/2))+1/4*b*c^(2/3)*arctan(2*c^(1/3)*x+3^(1/2))-1/8
*b*c^(2/3)*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)+1/8*b*c^(2/3)*ln(1+
c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.03

$$\begin{aligned}&\int \frac{a+b\arctan(cx^3)}{x^3}\,dx\\&=-\frac{a}{2x^2}+\frac{1}{2}bc^{2/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{b\arctan(cx^3)}{2x^2}-\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)\\&\quad+\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)-\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)\end{aligned}$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^3,x]
```

output
```
-1/2*a/x^2 + (b*c^(2/3)*ArcTan[c^(1/3)*x])/2 - (b*ArcTan[c*x^3])/(2*x^2) -
 (b*c^(2/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/4 + (b*c^(2/3)*ArcTan[Sqrt[3] +
 2*c^(1/3)*x])/4 - (Sqrt[3]*b*c^(2/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*
x^2])/8 + (Sqrt[3]*b*c^(2/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8
```

3.106.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5361, 753, 27, 216, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^3}dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{3}{2}bc\int \frac{1}{c^2x^6+1}dx-\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow \text{ 753}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\int \frac{1}{c^{2/3}x^2+1}dx+\frac{1}{3}\int \frac{2-\sqrt{3}\sqrt[3]{c}x}{2\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}dx+\frac{1}{3}\int \frac{\sqrt{3}\sqrt[3]{c}x+2}{2\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}dx\right)-\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\int \frac{1}{c^{2/3}x^2+1}dx+\frac{1}{6}\int \frac{2-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{6}\int \frac{\sqrt{3}\sqrt[3]{c}x+2}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx\right)-\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{3}{2}bc\left(\frac{1}{6}\int \frac{2-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{6}\int \frac{\sqrt{3}\sqrt[3]{c}x+2}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\arctan\left(\sqrt[3]{c}x\right)}{3\sqrt[3]{c}}\right)-\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow 1142$$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\int -\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\int\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow 25$$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\int \frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\int \frac{}{c^{2}}\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow 27$$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{1}{2}\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow 1082$$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\int \frac{1}{-\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+}\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow 217$$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

3.106. $\int \frac{a+b\arctan\left(cx^3\right)}{x^3}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}-\frac{\sqrt{3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right)}{\sqrt[3]{c}}+\frac{\sqrt{3}\ldots}{}\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{2x^2}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c*x^3])/x^2 + (3*b*c*(ArcTan[c^(1/3)*x]/(3*c^(1/3)) + (
-(ArcTan[Sqrt[3]*(1 - (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3)) - (Sqrt[3]*Log[1 -
Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/6 + (ArcTan[Sqrt[3]*(1 + (2
*c^(1/3)*x)/Sqrt[3])]/c^(1/3) + (Sqrt[3]*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/
3)*x^2])/(2*c^(1/3)))/6))/2
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.106. $\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$

3.106.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

method	result
default	$-\frac{a}{2x^2}+b\left(-\frac{\arctan(cx^3)}{2x^2}+\frac{3c\left(\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6}-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2-\cdots\right.}{\cdots}\right.}{2}\right.$
parts	$-\frac{a}{2x^2}+b\left(-\frac{\arctan(cx^3)}{2x^2}+\frac{3c\left(\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6}-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2-\cdots\right.}{\cdots}\right.}{2}\right.$

input
```
int((a+b*arctan(c*x^3))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*(-1/2/x^2*arctan(c*x^3)+3/2*c*(1/12*3^(1/2)*(1/c^2)^(1/6)*ln(
x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6*(1/c^2)^(1/6)*arctan(2*x/(1
/c^2)^(1/6)+3^(1/2))-1/12*3^(1/2)*(1/c^2)^(1/6)*ln(x^2-3^(1/2)*(1/c^2)^(1/
6)*x+(1/c^2)^(1/3))+1/6*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))+1/
3*(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))))
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(119) = 238$.

Time = 0.25 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.64

$$\int \frac{a + b\arctan(cx^3)}{x^3}\,dx$$
$$= \frac{2\left(-b^6c^4\right)^{\frac{1}{6}}x^2\log\left(bcx + \left(-b^6c^4\right)^{\frac{1}{6}}\right) - 2\left(-b^6c^4\right)^{\frac{1}{6}}x^2\log\left(bcx - \left(-b^6c^4\right)^{\frac{1}{6}}\right) + \left(-b^6c^4\right)^{\frac{1}{6}}\left(\sqrt{-3}x^2 + x^2\right)\log\left(\right.}{}$$

input
```
integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="fricas")
```

output
```
1/8*(2*(-b^6*c^4)^(1/6)*x^2*log(b*c*x + (-b^6*c^4)^(1/6)) - 2*(-b^6*c^4)^(
1/6)*x^2*log(b*c*x - (-b^6*c^4)^(1/6)) + (-b^6*c^4)^(1/6)*(sqrt(-3)*x^2 +
x^2)*log(2*b*c*x + (-b^6*c^4)^(1/6)*(sqrt(-3) + 1)) - (-b^6*c^4)^(1/6)*(sq
rt(-3)*x^2 + x^2)*log(2*b*c*x - (-b^6*c^4)^(1/6)*(sqrt(-3) + 1)) + (-b^6*c
^4)^(1/6)*(sqrt(-3)*x^2 - x^2)*log(2*b*c*x + (-b^6*c^4)^(1/6)*(sqrt(-3) -
1)) - (-b^6*c^4)^(1/6)*(sqrt(-3)*x^2 - x^2)*log(2*b*c*x - (-b^6*c^4)^(1/6)
*(sqrt(-3) - 1)) - 4*b*arctan(c*x^3) - 4*a)/x^2
```

3.106.6 Sympy [A] (verification not implemented)

Time = 29.93 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.48

$$\int \frac{a + b\arctan(cx^3)}{x^3}\,dx$$
$$= \begin{cases} -\frac{a}{2x^2} + \frac{b\,\mathrm{atan}\,(cx^3)}{2\sqrt[3]{-\frac{1}{c^2}}} - \frac{b\,\mathrm{atan}\,(cx^3)}{2x^2} + \frac{3b\log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}} - \frac{3b\log\left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}} - \frac{\sqrt{3}b\,\mathrm{atan}}{4} \\ -\frac{a}{2x^2} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))/x**3,x)
```

output
```
Piecewise((-a/(2*x**2) + b*atan(c*x**3)/(2*(-1/c**2)**(1/3)) - b*atan(c*x*
*3)/(2*x**2) + 3*b*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))
/(8*c*(-1/c**2)**(5/6)) - 3*b*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c*
*2)**(1/3))/(8*c*(-1/c**2)**(5/6)) - sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**
2)**(1/6)) - sqrt(3)/3)/(4*c*(-1/c**2)**(5/6)) - sqrt(3)*b*atan(2*sqrt(3)*
x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*(-1/c**2)**(5/6)), Ne(c, 0)), (-a
/(2*x**2), True))
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$$
$$=\frac{1}{8}\left(\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}+\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{1}{3}}}+\frac{2\arctan\left(\frac{2c^{\frac{2}{3}}x+}{c^{\frac{1}{3}}}\right.}{c^{\frac{1}{3}}}\right.\right.$$
$$-\frac{a}{2x^2}$$

input
```
integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="maxima")
```

output
```
1/8*((sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(1/3) - sqrt(3)*l
og(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(1/3) + 4*arctan(c^(1/3)*x)/c^(1
/3) + 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(1/3) + 2*arctan
((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(1/3))*c - 4*arctan(c*x^3)/x^2
)*b - 1/2*a/x^2
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$$
$$=\frac{1}{8}\left(\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\arctan\left(\right.}{}\right.$$
$$-\frac{b\arctan(cx^3)+a}{2x^2}$$

input
```
integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="giac")
```

output
```
1/8*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/
3) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/
3) + 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 2*
arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 4*arctan(
x*abs(c)^(1/3))/abs(c)^(1/3))*b*c - 1/2*(b*arctan(c*x^3) + a)/x^2
```

3.106.9 Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.65

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx = -\frac{a}{2x^2} - \frac{b\,c^{2/3}\left(\frac{\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)}{2} - \frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)}{2} + \mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{2} - \frac{b\,\mathrm{atan}(c\,x^3)}{2\,x^2} - \frac{\sqrt{3}\,b\,c^{2/3}\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right) + \mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)\right)\,1\mathrm{i}}{4}$$

input
```
int((a + b*atan(c*x^3))/x^3,x)
```

output
```
- a/(2*x^2) - (b*c^(2/3)*(atan((-1)^(2/3)*c^(1/3)*x)/2 - atan((c^(1/3)*x*(
3^(1/2)*1i + 1))/2)/2 + atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/
2 - (b*atan(c*x^3))/(2*x^2) - (3^(1/2)*b*c^(2/3)*(atan((c^(1/3)*x*(3^(1/2)
*1i + 1))/2) + atan((-1)^(2/3)*c^(1/3)*x))*1i)/4
```

3.107 $\int \frac{a+b\arctan(cx^3)}{x^6}\,dx$

3.107.1 Optimal result 690
3.107.2 Mathematica [A] (verified) 690
3.107.3 Rubi [A] (verified) 691
3.107.4 Maple [A] (verified) 694
3.107.5 Fricas [A] (verification not implemented) 695
3.107.6 Sympy [B] (verification not implemented) 696
3.107.7 Maxima [A] (verification not implemented) 697
3.107.8 Giac [A] (verification not implemented) 697
3.107.9 Mupad [B] (verification not implemented) 698

3.107.1 Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{3bc}{10x^2}-\frac{a+b\arctan(cx^3)}{5x^5}+\frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)$$

output
```
-3/10*b*c/x^2+1/5*(-a-b*arctan(c*x^3))/x^5+1/10*b*c^(5/3)*ln(1+c^(2/3)*x^2
)-1/20*b*c^(5/3)*ln(1-c^(2/3)*x^2+c^(4/3)*x^4)+1/10*b*c^(5/3)*arctan(1/3*(
1-2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.59

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{a}{5x^5}-\frac{3bc}{10x^2}-\frac{b\arctan(cx^3)}{5x^5}+\frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)+\frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^6,x]
```

output
```
-1/5*a/x^5 - (3*b*c)/(10*x^2) - (b*ArcTan[c*x^3])/(5*x^5) + (Sqrt[3]*b*c^(
5/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/10 + (Sqrt[3]*b*c^(5/3)*ArcTan[Sqrt[3]
 + 2*c^(1/3)*x])/10 + (b*c^(5/3)*Log[1 + c^(2/3)*x^2])/10 - (b*c^(5/3)*Log
[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/20 - (b*c^(5/3)*Log[1 + Sqrt[3]*c^(
1/3)*x + c^(2/3)*x^2])/20
```

3.107.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {5361, 807, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^6}dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{3}{5}bc\int \frac{1}{x^3\left(c^2x^6+1\right)}dx-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{3}{10}bc\int \frac{1}{x^4\left(c^2x^6+1\right)}dx^2-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{3}{10}bc\left(c^2\left(-\int \frac{x^2}{c^2x^6+1}dx^2\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\int \frac{c^{2/3}x^2+1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{3c^{2/3}}-\frac{\int \frac{1}{c^{2/3}x^2+1}dx^2}{3c^{2/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\int \frac{c^{2/3}x^2+1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{3c^{2/3}}-\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\frac{3}{2}\int \frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{\int -\frac{c^{2/3}\left(1-2c^{2/3}x^2\right)}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{2c^{2/3}}}{3c^{2/3}}-\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$\downarrow$ 25

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\frac{3}{2}\int \frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{\int \frac{c^{2/3}\left(1-2c^{2/3}x^2\right)}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{2c^{2/3}}}{3c^{2/3}}-\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$\downarrow$ 27

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\frac{3}{2}\int \frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{1}{2}\int \frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{3c^{2/3}}-\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$\downarrow$ 1082

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\frac{3\int \frac{1}{-x^4-3}d\left(1-2c^{2/3}x^2\right)}{c^{2/3}}-\frac{1}{2}\int \frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{3c^{2/3}}-\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$\downarrow$ 217

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{-\frac{1}{2}\int \frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{\sqrt{3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{c^{2/3}}}{3c^{2/3}}-\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

$\downarrow$ 1103

$$\frac{3}{10}bc\left(-\left(c^2\left(\frac{\frac{\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{2c^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{c^{2/3}}}{3c^{2/3}}-\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{4/3}}\right)\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan\left(cx^3\right)}{5x^5}$$

3.107. $\int \frac{a+b\arctan\left(cx^3\right)}{x^6}\,dx$

input
```
Int[(a + b*ArcTan[c*x^3])/x^6,x]
```

output
```
-1/5*(a + b*ArcTan[c*x^3])/x^5 + (3*b*c*(-x^(-2) - c^2*(-1/3*Log[1 + c^(2/
3)*x^2]/c^(4/3) + (-((Sqrt[3]*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/c^(2/3)
) + Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4]/(2*c^(2/3)))/(3*c^(2/3)))))/10
```

3.107.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.107.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a}{5x^5}-\frac{b\arctan(c\,x^3)}{5x^5}-\frac{3bc}{10x^2}+\frac{bc\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right.}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}$
parts	$-\frac{a}{5x^5}-\frac{b\arctan(c\,x^3)}{5x^5}-\frac{3bc}{10x^2}+\frac{bc\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right.}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}$
risch	$\frac{ib\ln(ic\,x^3+1)}{10x^5}-\frac{ib\ln(-ic\,x^3+1)}{10x^5}-\frac{3bc}{10x^2}-\frac{bc\ln\left(x+\left(\frac{i}{c}\right)^{\frac{1}{3}}\right)}{10\left(\frac{i}{c}\right)^{\frac{2}{3}}}+\frac{bc\ln\left(x^2-\left(\frac{i}{c}\right)^{\frac{1}{3}}x+\left(\frac{i}{c}\right)^{\frac{2}{3}}\right)}{20\left(\frac{i}{c}\right)^{\frac{2}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{i}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right.}{10\left(\frac{i}{c}\right)^{\frac{2}{3}}}$

input
```
int((a+b*arctan(c*x^3))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5-1/5*b/x^5*arctan(c*x^3)-3/10*b*c/x^2+1/10*b*c/(1/c^2)^(1/3)*ln(
x^2+(1/c^2)^(1/3))-1/20*b*c/(1/c^2)^(1/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)
^(2/3))-1/10*b*c*3^(1/2)/(1/c^2)^(1/3)*arctan(1/3*3^(1/2)*(2*x^2/(1/c^2)^(
1/3)-1))
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx =$$

$$-\frac{2\sqrt{3}b(c^2)^{\frac{1}{3}}cx^5\arctan\left(\frac{2}{3}\sqrt{3}(c^2)^{\frac{1}{3}}x^2-\frac{1}{3}\sqrt{3}\right)+b(c^2)^{\frac{1}{3}}cx^5\log\left(c^2x^4-(c^2)^{\frac{2}{3}}x^2+(c^2)^{\frac{1}{3}}\right)-2\,b(c^2)^{\frac{1}{3}}cx^5\,\mathrm{lo}}{20\,x^5}$$

input
```
integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="fricas")
```

output
```
-1/20*(2*sqrt(3)*b*(c^2)^(1/3)*c*x^5*arctan(2/3*sqrt(3)*(c^2)^(1/3)*x^2 -
1/3*sqrt(3)) + b*(c^2)^(1/3)*c*x^5*log(c^2*x^4 - (c^2)^(2/3)*x^2 + (c^2)^(
1/3)) - 2*b*(c^2)^(1/3)*c*x^5*log(c^2*x^2 + (c^2)^(2/3)) + 6*b*c*x^3 + 4*b
*arctan(c*x^3) + 4*a)/x^5
```

3.107.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(112) = 224$.

Time = 61.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.49

$$\int \frac{a + b\arctan(cx^3)}{x^6}\, dx$$
$$= \begin{cases} -\frac{a}{5x^5} + \frac{bc^2\sqrt[6]{-\frac{1}{c^2}}\operatorname{atan}(cx^3)}{5} - \frac{bc\log\left(x - \sqrt[6]{-\frac{1}{c^2}}\right)}{5\sqrt[3]{-\frac{1}{c^2}}} + \frac{3bc\log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{20\sqrt[3]{-\frac{1}{c^2}}} - \frac{bc\log\left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{\right.}}{20\sqrt[3]{-\frac{1}{c^2}}} \\ -\frac{a}{5x^5} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))/x**6,x)
```

output
```
Piecewise((-a/(5*x**5) + b*c**2*(-1/c**2)**(1/6)*atan(c*x**3)/5 - b*c*log(
x - (-1/c**2)**(1/6))/(5*(-1/c**2)**(1/3)) + 3*b*c*log(4*x**2 - 4*x*(-1/c*
*2)**(1/6) + 4*(-1/c**2)**(1/3))/(20*(-1/c**2)**(1/3)) - b*c*log(4*x**2 +
4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(20*(-1/c**2)**(1/3)) - sqrt(3)
*b*c*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/(10*(-1/c**2)**(1/
3)) + sqrt(3)*b*c*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(10*(
-1/c**2)**(1/3)) - 3*b*c/(10*x**2) - b*atan(c*x**3)/(5*x**5), Ne(c, 0)), (
-a/(5*x**5), True))
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{1}{20}\left(\left(\left(2\sqrt{3}c^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)+c^{\frac{2}{3}}\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)-2\,c^{\frac{2}{3}}\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)+\frac{6}{x^2}\right) - \frac{a}{5\,x^5}$$

input
```
integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="maxima")
```

output
```
-1/20*((2*sqrt(3)*c^(2/3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(
2/3)) + c^(2/3)*log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1) - 2*c^(2/3)*log((c^(2/3
)*x^2 + 1)/c^(2/3)) + 6/x^2)*c + 4*arctan(c*x^3)/x^5)*b - 1/5*a/x^5
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{1}{20}\,bc^3\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}+\frac{|c|^{\frac{2}{3}}\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}-\frac{2\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{4}{3}}}\right) - \frac{3\,bcx^3+2\,b\arctan(cx^3)+2\,a}{10\,x^5}$$

input
```
integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="giac")
```

output
```
-1/20*b*c^3*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(
2/3))*abs(c)^(2/3))/c^2 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(
c)^(4/3))/c^2 - 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3)) - 1/10*(3*b*c*x^
3 + 2*b*arctan(c*x^3) + 2*a)/x^5
```

3.107.9 Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = \frac{b\,c^{5/3}\,\ln\left(c^{2/3}\,x^2+1\right)}{10} - \frac{\frac{3\,b\,c\,x^3}{2}+a}{5\,x^5} - \frac{b\,\mathrm{atan}(c\,x^3)}{5\,x^5} - \frac{b\,c^{5/3}\,\ln\left(\sqrt{3}\,c^{2/3}\,x^2 - c^{2/3}\,x^2\,1\mathrm{i} + 2\mathrm{i}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{20} + \frac{b\,c^{5/3}\,\ln\left(-c^{2/3}\,x^2\,1\mathrm{i} - \sqrt{3}\,c^{2/3}\,x^2 + 2\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{20}$$

input
```
int((a + b*atan(c*x^3))/x^6,x)
```

output
```
(b*c^(5/3)*log(c^(2/3)*x^2 + 1))/10 - (a + (3*b*c*x^3)/2)/(5*x^5) - (b*ata
n(c*x^3))/(5*x^5) - (b*c^(5/3)*log(3^(1/2)*c^(2/3)*x^2 - c^(2/3)*x^2*1i +
2i)*(3^(1/2)*1i + 1))/20 + (b*c^(5/3)*log(2i - 3^(1/2)*c^(2/3)*x^2 - c^(2/
3)*x^2*1i)*(3^(1/2)*1i - 1))/20
```

3.108 $\int x^7(a+b\arctan(cx^3))\,dx$

3.108.1 Optimal result . . . 699
3.108.2 Mathematica [A] (verified) . . . 700
3.108.3 Rubi [A] (verified) . . . 700
3.108.4 Maple [A] (verified) . . . 704
3.108.5 Fricas [B] (verification not implemented) . . . 706
3.108.6 Sympy [A] (verification not implemented) . . . 706
3.108.7 Maxima [A] (verification not implemented) . . . 707
3.108.8 Giac [A] (verification not implemented) . . . 708
3.108.9 Mupad [B] (verification not implemented) . . . 708

3.108.1 Optimal result

Integrand size = 14, antiderivative size = 176

$$\begin{aligned}\int x^7(a+b\arctan(cx^3))\,dx=&-\frac{3bx^5}{40c}+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{8c^{8/3}}\\&+\frac{1}{8}x^8(a+b\arctan(cx^3))-\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{16c^{8/3}}\\&+\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{16c^{8/3}}+\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}\\&-\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}\end{aligned}$$

output
```
-3/40*b*x^5/c+1/8*b*arctan(c^(1/3)*x)/c^(8/3)+1/8*x^8*(a+b*arctan(c*x^3))+
1/16*b*arctan(2*c^(1/3)*x-3^(1/2))/c^(8/3)+1/16*b*arctan(2*c^(1/3)*x+3^(1/
2))/c^(8/3)+1/32*b*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3)-1/3
2*b*ln(1+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3)
```

3.108.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03

$$\int x^7\left(a+b\arctan\left(cx^3\right)\right)\,dx=-\frac{3bx^5}{40c}+\frac{ax^8}{8}+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}bx^8\arctan\left(cx^3\right)-\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{16c^{8/3}}+\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{16c^{8/3}}+\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}-\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}$$

input
```
Integrate[x^7*(a + b*ArcTan[c*x^3]),x]
```

output
```
(-3*b*x^5)/(40*c) + (a*x^8)/8 + (b*ArcTan[c^(1/3)*x])/(8*c^(8/3)) + (b*x^8
*ArcTan[c*x^3])/8 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(16*c^(8/3)) + (b*Ar
cTan[Sqrt[3] + 2*c^(1/3)*x])/(16*c^(8/3)) + (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(
1/3)*x + c^(2/3)*x^2])/(32*c^(8/3)) - (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x
 + c^(2/3)*x^2])/(32*c^(8/3))
```

3.108.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5361, 843, 824, 27, 216, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\int\frac{x^{10}}{c^2x^6+1}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{\int\frac{x^4}{c^2x^6+1}dx}{c^2}\right)$$

$\Big\downarrow$ 824

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{\frac{\int\frac{1}{c^{2/3}x^2+1}dx}{3c^{4/3}}+\frac{\int-\frac{1-\sqrt{3}\sqrt[3]{c}x}{2\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}+\frac{\int-\frac{\sqrt{3}\sqrt[3]{c}x+1}{2\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}}{c^2}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{\frac{\int\frac{1}{c^{2/3}x^2+1}dx}{3c^{4/3}}-\frac{\int\frac{1-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int\frac{\sqrt{3}\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}}{c^2}\right)$$

$\Big\downarrow$ 216

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{\int\frac{1-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int\frac{\sqrt{3}\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\arctan\left(\sqrt[3]{c}x\right)}{3c^{5/3}}}{c^2}\right)$$

$\Big\downarrow$ 1142

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{-\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\int-\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\arctan\left(\sqrt[3]{c}x\right)}{3c^{5/3}}}{c^2}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\arctan\left(\sqrt[3]{c}x\right)}{3c^{5/3}}}{c^2}\right)$$

3.108. $\int x^7\left(a+b\arctan\left(cx^3\right)\right)\,dx$

$\downarrow$ 27

$$\frac{1}{8}x^8(a+b\arctan(cx^3))-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\text{arcta}}{}}{c^2}\right.$$

$\downarrow$ 1082

$$\frac{1}{8}x^8(a+b\arctan(cx^3))-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\int\frac{1}{-\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\int\frac{1}{-\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)^2-\frac{1}{3}}d\left(\right.}{\sqrt{3}\sqrt[3]{c}}}{6c^{4/3}}}{c^2}\right.$$

$\downarrow$ 217

$$\frac{1}{8}x^8(a+b\arctan(cx^3))-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctan}}{3}}{c^2}\right.$$

$\downarrow$ 1103

$$\frac{1}{8}x^8(a+b\arctan(cx^3))-\frac{3}{8}bc\left(\frac{x^5}{5c^2}-\frac{-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}-\frac{\sqrt{3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}-\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}+\frac{\arctan(}{3c^{5/}}}{c^2}\right.$$

input
```
Int[x^7*(a + b*ArcTan[c*x^3]),x]
```

3.108. $\int x^7(a+b\arctan(cx^3))\,dx$

output
```
(x^8*(a + b*ArcTan[c*x^3]))/8 - (3*b*c*(x^5/(5*c^2) - (ArcTan[c^(1/3)*x]/(
3*c^(5/3)) - (ArcTan[Sqrt[3]*(1 - (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3) - (Sqrt[
3]*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/(6*c^(4/3)) - (-
(ArcTan[Sqrt[3]*(1 + (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3)) + (Sqrt[3]*Log[1 + S
qrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/(6*c^(4/3)))/c^2))/8
```

3.108.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.108.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95

method	result
default	$\frac{x^8 a}{8}+b\left(\frac{x^8\arctan(c\,x^3)}{8}-\frac{3c\left(\frac{x^5}{5c^2}-\frac{-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\right.}{12}}{c^2}\right.}{8}\right.$
parts	$\frac{x^8 a}{8}+b\left(\frac{x^8\arctan(c\,x^3)}{8}-\frac{3c\left(\frac{x^5}{5c^2}-\frac{-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\right.}{12}}{c^2}\right.}{8}\right.$

input
```
int(x^7*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^8*a+b*(1/8*x^8*arctan(c*x^3)-3/8*c*(1/5/c^2*x^5-(-1/12*3^(1/2)*(1/c^
2)^(5/6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/
6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))+1/12*3^(1/2)*(1/c^2)^(5/6)*ln(x^2-3^(
1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x/(1/c^
2)^(1/6)-3^(1/2))+1/3/c^2/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6)))/c^2))
```

3.108. $\int x^7(a+b\arctan(cx^3))\,dx$

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(128) = 256$.

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.73

$$\int x^7\left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$= \frac{20\,bcx^8\arctan\left(cx^3\right)+20\,acx^8-12\,bx^5+10\,c\left(-\frac{b^6}{c^{16}}\right)^{\frac{1}{6}}\log\left(c^{13}\left(-\frac{b^6}{c^{16}}\right)^{\frac{5}{6}}+b^5x\right)-10\,c\left(-\frac{b^6}{c^{16}}\right)^{\frac{1}{6}}\log\left(-c^{13}\right.}{}$$

input
```
integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/160*(20*b*c*x^8*arctan(c*x^3) + 20*a*c*x^8 - 12*b*x^5 + 10*c*(-b^6/c^16)
^(1/6)*log(c^13*(-b^6/c^16)^(5/6) + b^5*x) - 10*c*(-b^6/c^16)^(1/6)*log(-c
^13*(-b^6/c^16)^(5/6) + b^5*x) - 5*(sqrt(-3)*c - c)*(-b^6/c^16)^(1/6)*log(
b^5*x + 1/2*(sqrt(-3)*c^13 + c^13)*(-b^6/c^16)^(5/6)) + 5*(sqrt(-3)*c - c)
*(-b^6/c^16)^(1/6)*log(b^5*x - 1/2*(sqrt(-3)*c^13 + c^13)*(-b^6/c^16)^(5/6
)) - 5*(sqrt(-3)*c + c)*(-b^6/c^16)^(1/6)*log(b^5*x + 1/2*(sqrt(-3)*c^13 -
 c^13)*(-b^6/c^16)^(5/6)) + 5*(sqrt(-3)*c + c)*(-b^6/c^16)^(1/6)*log(b^5*x
 - 1/2*(sqrt(-3)*c^13 - c^13)*(-b^6/c^16)^(5/6)))/c
```

3.108.6 Sympy [A] (verification not implemented)

Time = 63.81 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.50

$$\int x^7\left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$= \begin{cases} \frac{ax^8}{8}+\frac{bx^8\operatorname{atan}\left(cx^3\right)}{8}-\frac{3bx^5}{40c}+\frac{3b\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{32c^3\sqrt[6]{-\frac{1}{c^2}}}-\frac{3b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{32c^3\sqrt[6]{-\frac{1}{c^2}}}+\frac{\sqrt{3}b\operatorname{atan}\left(\frac{2\sqrt{3}\cdots}{3\sqrt[6]{-\cdots}}\right)}{16c^3\sqrt[6]{-\cdots}} \\ \frac{ax^8}{8} \end{cases}$$

input
```
integrate(x**7*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**8/8 + b*x**8*atan(c*x**3)/8 - 3*b*x**5/(40*c) + 3*b*log(4*
x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(32*c**3*(-1/c**2)**(1/6
)) - 3*b*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(32*c**3*
(-1/c**2)**(1/6)) + sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt
(3)/3)/(16*c**3*(-1/c**2)**(1/6)) + sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2
)**(1/6)) + sqrt(3)/3)/(16*c**3*(-1/c**2)**(1/6)) - b*atan(c*x**3)/(8*c**4
*(-1/c**2)**(2/3)), Ne(c, 0)), (a*x**8/8, True))
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86

$$\int x^7\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{8}ax^8$$
$$+\frac{1}{160}\left(20\,x^8\arctan\left(cx^3\right)-\left(\frac{12\,x^5}{c^2}+\frac{5\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}}-\cdots\right.}{c^2}\right.\right.$$

input
```
integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/8*a*x^8 + 1/160*(20*x^8*arctan(c*x^3) - (12*x^5/c^2 + 5*(sqrt(3)*log(c^(
2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt
(3)*c^(1/3)*x + 1)/c^(5/3) - 4*arctan(c^(1/3)*x)/c^(5/3) - 2*arctan((2*c^(
2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3) - 2*arctan((2*c^(2/3)*x - sqrt(
3)*c^(1/3))/c^(1/3))/c^(5/3))/c^2)*c)*b
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97

$$\int x^7 \left(a + b \arctan\left(cx^3\right)\right) dx =$$

$$-\frac{1}{32} bc^{15} \left(\frac{\sqrt{3}|c|^{\frac{1}{3}} \log\left(x^2 + \frac{\sqrt{3}x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{18}} - \frac{\sqrt{3}\log\left(x^2 - \frac{\sqrt{3}x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{16}|c|^{\frac{5}{3}}} - \frac{2\arctan\left(\left(2x + \frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^{16}|c|^{\frac{5}{3}}} - \right.$$

$$+ \frac{5\,bcx^8 \arctan\left(cx^3\right) + 5\,acx^8 - 3\,bx^5}{40\,c}$$

input
```
integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
-1/32*b*c^15*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/ab
s(c)^(2/3))/c^18 - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/
3))/(c^16*abs(c)^(5/3)) - 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/
3))/(c^16*abs(c)^(5/3)) - 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/
3))/(c^16*abs(c)^(5/3)) - 4*arctan(x*abs(c)^(1/3))/(c^16*abs(c)^(5/3))) +
1/40*(5*b*c*x^8*arctan(c*x^3) + 5*a*c*x^8 - 3*b*x^5)/c
```

3.108.9 Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.69

$$\int x^7 \left(a + b \arctan\left(cx^3\right)\right) dx = \frac{a\,x^8}{8} - \frac{3\,b\,x^5}{40\,c}$$

$$- \frac{b\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right) + \mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right) + 2\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{16\,c^{8/3}}$$

$$+ \frac{b\,x^8\,\mathrm{atan}(c\,x^3)}{8} + \frac{\sqrt{3}\,b\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right) - \mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)\,1\mathrm{i}}{16\,c^{8/3}}$$

input
```
int(x^7*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^8)/8 - (3*b*x^5)/(40*c) - (b*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)
^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1
/2)*1i + 1))/2)))/(16*c^(8/3)) + (b*x^8*atan(c*x^3))/8 + (3^(1/2)*b*(atan(
(-1)^(2/3)*c^(1/3)*x) - atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1
i)/(16*c^(8/3))
```

3.108. $\int x^7(a + b\arctan(cx^3))\,dx$

3.109 $\int x^4(a+b\arctan(cx^3))\,dx$

3.109.1 Optimal result . . . 710
3.109.2 Mathematica [A] (verified) . . . 710
3.109.3 Rubi [A] (verified) . . . 711
3.109.4 Maple [A] (verified) . . . 715
3.109.5 Fricas [A] (verification not implemented) . . . 715
3.109.6 Sympy [B] (verification not implemented) . . . 716
3.109.7 Maxima [A] (verification not implemented) . . . 717
3.109.8 Giac [A] (verification not implemented) . . . 717
3.109.9 Mupad [B] (verification not implemented) . . . 718

3.109.1 Optimal result

Integrand size = 14, antiderivative size = 117

$$\int x^4(a+b\arctan(cx^3))\,dx=-\frac{3bx^2}{10c}+\frac{1}{5}x^5(a+b\arctan(cx^3))-\frac{\sqrt{3}b\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{b\log(1+c^{2/3}x^2)}{10c^{5/3}}-\frac{b\log(1-c^{2/3}x^2+c^{4/3}x^4)}{20c^{5/3}}$$

output
```
-3/10*b*x^2/c+1/5*x^5*(a+b*arctan(c*x^3))+1/10*b*ln(1+c^(2/3)*x^2)/c^(5/3)
-1/20*b*ln(1-c^(2/3)*x^2+c^(4/3)*x^4)/c^(5/3)-1/10*b*arctan(1/3*(1-2*c^(2/
3)*x^2)*3^(1/2))*3^(1/2)/c^(5/3)
```

3.109.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.58

$$\int x^4(a+b\arctan(cx^3))\,dx=-\frac{3bx^2}{10c}+\frac{ax^5}{5}+\frac{1}{5}bx^5\arctan(cx^3)-\frac{\sqrt{3}b\arctan(\sqrt{3}-2\sqrt[3]{c}x)}{10c^{5/3}}-\frac{\sqrt{3}b\arctan(\sqrt{3}+2\sqrt[3]{c}x)}{10c^{5/3}}+\frac{b\log(1+c^{2/3}x^2)}{10c^{5/3}}-\frac{b\log(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2)}{20c^{5/3}}-\frac{b\log(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2)}{20c^{5/3}}$$

input
```
Integrate[x^4*(a + b*ArcTan[c*x^3]),x]
```

output
```
(-3*b*x^2)/(10*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x^3])/5 - (Sqrt[3]*b*ArcTa
n[Sqrt[3] - 2*c^(1/3)*x])/(10*c^(5/3)) - (Sqrt[3]*b*ArcTan[Sqrt[3] + 2*c^(
1/3)*x])/(10*c^(5/3)) + (b*Log[1 + c^(2/3)*x^2])/(10*c^(5/3)) - (b*Log[1 -
 Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(20*c^(5/3)) - (b*Log[1 + Sqrt[3]*c^(1/
3)*x + c^(2/3)*x^2])/(20*c^(5/3))
```

3.109.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {5361, 807, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\arctan(cx^3))\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{5}x^5(a+b\arctan(cx^3))-\frac{3}{5}bc\int\frac{x^7}{c^2x^6+1}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{5}x^5(a+b\arctan(cx^3))-\frac{3}{10}bc\int\frac{x^6}{c^2x^6+1}dx^2$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{5}x^5(a+b\arctan(cx^3))-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\int\frac{1}{c^2x^6+1}dx^2}{c^2}\right)$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{5}x^5(a+b\arctan(cx^3))-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\int\frac{1}{c^{2/3}x^2+1}dx^2+\frac{1}{3}\int\frac{2-c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{c^2}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{5}x^5(a+b\arctan(cx^3))-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\int\frac{2-c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{\log(c^{2/3}x^2+1)}{3c^{2/3}}}{c^2}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{\int-\frac{c^{2/3}\left(1-2c^{2/3}x^2\right)}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{2c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{\int\frac{c^{2/3}\left(1-2c^{2/3}x^2\right)}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{2c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{1}{2}\int\frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\left(\frac{3\int\frac{1}{-x^4-3}d\left(1-2c^{2/3}x^2\right)}{c^{2/3}}+\frac{1}{2}\int\frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\left(\frac{1}{2}\int\frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{\sqrt{3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

$$\Big\downarrow \; 1103$$

3.109. $\int x^4(a+b\arctan(cx^3))\,dx$

$$\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{x^2}{c^2}-\frac{\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{c^{2/3}}-\frac{\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{2c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}}{c^2}\right)$$

input
```
Int[x^4*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^5*(a + b*ArcTan[c*x^3]))/5 - (3*b*c*(x^2/c^2 - (Log[1 + c^(2/3)*x^2]/(3
*c^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/c^(2/3)) - Lo
g[1 - c^(2/3)*x^2 + c^(4/3)*x^4]/(2*c^(2/3)))/3)/c^2))/10
```

3.109.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.109. $\int x^4(a+b\arctan(cx^3))\,dx$

3.109.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

method	result
default	$\frac{a x^5}{5}+\frac{b x^5 \arctan(c x^3)}{5}-\frac{3b x^2}{10c}+\frac{b \ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b \ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}} x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$
parts	$\frac{a x^5}{5}+\frac{b x^5 \arctan(c x^3)}{5}-\frac{3b x^2}{10c}+\frac{b \ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b \ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}} x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$
risch	$-\frac{i x^5 b \ln(i c x^3+1)}{10}+\frac{i b x^5 \ln(-i c x^3+1)}{10}-\frac{3b x^2}{10c}-\frac{i b \ln\left(x+\left(\frac{i}{c}\right)^{\frac{1}{3}}\right)}{10c^2\left(\frac{i}{c}\right)^{\frac{1}{3}}}+\frac{i b \ln\left(x^2-\left(\frac{i}{c}\right)^{\frac{1}{3}} x+\left(\frac{i}{c}\right)^{\frac{2}{3}}\right)}{20c^2\left(\frac{i}{c}\right)^{\frac{1}{3}}}+\frac{i b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{i}{c}\right)^{\frac{1}{3}}}\right.}{3}\right.}{10c^2\left(\frac{i}{c}\right)^{\frac{1}{3}}}$

input
```
int(x^4*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+1/5*b*x^5*arctan(c*x^3)-3/10*b*x^2/c+1/10*b/c^3/(1/c^2)^(2/3)*ln
(x^2+(1/c^2)^(1/3))-1/20*b/c^3/(1/c^2)^(2/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c
^2)^(2/3))+1/10*b/c^3/(1/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*x^2/(1/c
^2)^(1/3)-1))
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$=\frac{4\,bc^3x^5\arctan\left(cx^3\right)+4\,ac^3x^5-6\,bc^2x^2+2\,\sqrt{3}b(c^2)^{\frac{1}{6}}c\arctan\left(\frac{\sqrt{3}\left(2\,(c^2)^{\frac{2}{3}}x^2-(c^2)^{\frac{1}{3}}\right)(c^2)^{\frac{1}{6}}}{3\,c}\right)-b(c^2)^{\frac{2}{3}}\log\left(c^2x\right.}{20\,c^3}$$

input
```
integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/20*(4*b*c^3*x^5*arctan(c*x^3) + 4*a*c^3*x^5 - 6*b*c^2*x^2 + 2*sqrt(3)*b*
(c^2)^(1/6)*c*arctan(1/3*sqrt(3)*(2*(c^2)^(2/3)*x^2 - (c^2)^(1/3))*(c^2)^(
1/6)/c) - b*(c^2)^(2/3)*log(c^2*x^4 - (c^2)^(2/3)*x^2 + (c^2)^(1/3)) + 2*b
*(c^2)^(2/3)*log(c^2*x^2 + (c^2)^(2/3)))/c^3
```

3.109.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(109) = 218$.

Time = 31.58 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.50

$$\int x^4(a + b\arctan(cx^3))\,dx$$
$$= \begin{cases} \frac{ax^5}{5} - \frac{bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(x - \sqrt[6]{-\frac{1}{c^2}}\right)}{5} + \frac{3bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{20} - \frac{bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{\right.}}{20} \\ \frac{ax^5}{5} \end{cases}$$

input
```
integrate(x**4*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**5/5 - b*c**3*(-1/c**2)**(7/3)*log(x - (-1/c**2)**(1/6))/5
+ 3*b*c**3*(-1/c**2)**(7/3)*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2
)**(1/3))/20 - b*c**3*(-1/c**2)**(7/3)*log(4*x**2 + 4*x*(-1/c**2)**(1/6) +
 4*(-1/c**2)**(1/3))/20 + sqrt(3)*b*c**3*(-1/c**2)**(7/3)*atan(2*sqrt(3)*x
/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/10 - sqrt(3)*b*c**3*(-1/c**2)**(7/3)*at
an(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/10 - b*c**2*(-1/c**2)**(1
1/6)*atan(c*x**3)/5 + b*x**5*atan(c*x**3)/5 - 3*b*x**2/(10*c), Ne(c, 0)),
(a*x**5/5, True))
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{5}\,ax^5+\frac{1}{20}\left(4\,x^5\arctan\left(cx^3\right)-c\left(\frac{6\,x^2}{c^2}-\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)}{c^{\frac{8}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{8}{3}}}-\frac{2\,\log\left(\frac{c^{\frac{2}{3}}}{\cdots}\right.}{c^{\frac{8}{3}}}\right.\right.$$

input
```
integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/20*(4*x^5*arctan(c*x^3) - c*(6*x^2/c^2 - 2*sqrt(3)*arctan(1/
3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^(8/3) + log(c^(4/3)*x^4 - c
^(2/3)*x^2 + 1)/c^(8/3) - 2*log((c^(2/3)*x^2 + 1)/c^(2/3))/c^(8/3)))*b
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{20}\,bc^9\left(\frac{2\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{10}|c|^{\frac{2}{3}}}-\frac{\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}+\frac{2\,\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}\right)+\frac{2\,bcx^5\arctan\left(cx^3\right)+2\,acx^5-3\,bx^2}{10\,c}$$

input
```
integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/20*b*c^9*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(
2/3))/(c^10*abs(c)^(2/3)) - log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/(
c^10*abs(c)^(2/3)) + 2*log(x^2 + 1/abs(c)^(2/3))/(c^10*abs(c)^(2/3))) + 1/
10*(2*b*c*x^5*arctan(c*x^3) + 2*a*c*x^5 - 3*b*x^2)/c
```

3.109.9 Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x^4(a+b\arctan(cx^3))\,dx=\frac{a\,x^5}{5}+\frac{b\,\ln\left(c^{2/3}\,x^2+1\right)}{10\,c^{5/3}}-\frac{3\,b\,x^2}{10\,c}-\frac{\ln\left(1-2\,c^{2/3}\,x^2+\sqrt{3}\,1\mathrm{i}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}}-\frac{\ln\left(2\,c^{2/3}\,x^2-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}}+\frac{b\,x^5\,\mathrm{atan}(c\,x^3)}{5}$$

input
```
int(x^4*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^5)/5 + (b*log(c^(2/3)*x^2 + 1))/(10*c^(5/3)) - (3*b*x^2)/(10*c) - (lo
g(3^(1/2)*1i - 2*c^(2/3)*x^2 + 1)*(b + 3^(1/2)*b*1i))/(20*c^(5/3)) - (log(
3^(1/2)*1i + 2*c^(2/3)*x^2 - 1)*(b - 3^(1/2)*b*1i))/(20*c^(5/3)) + (b*x^5*
atan(c*x^3))/5
```

3.110 $\int x(a+b\arctan(cx^3))\,dx$

3.110.1 Optimal result 719
3.110.2 Mathematica [A] (verified) 720
3.110.3 Rubi [A] (verified) 720
3.110.4 Maple [A] (verified) 724
3.110.5 Fricas [B] (verification not implemented) 725
3.110.6 Sympy [A] (verification not implemented) 726
3.110.7 Maxima [A] (verification not implemented) 727
3.110.8 Giac [A] (verification not implemented) 727
3.110.9 Mupad [B] (verification not implemented) 728

3.110.1 Optimal result

Integrand size = 12, antiderivative size = 165

$$\int x(a+b\arctan(cx^3))\,dx = -\frac{b\arctan(\sqrt[3]{c}x)}{2c^{2/3}} + \frac{1}{2}x^2(a+b\arctan(cx^3)) + \frac{b\arctan(\sqrt{3}-2\sqrt[3]{c}x)}{4c^{2/3}} - \frac{b\arctan(\sqrt{3}+2\sqrt[3]{c}x)}{4c^{2/3}} - \frac{\sqrt{3}b\log(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2)}{8c^{2/3}} + \frac{\sqrt{3}b\log(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2)}{8c^{2/3}}$$

output
```
-1/2*b*arctan(c^(1/3)*x)/c^(2/3)+1/2*x^2*(a+b*arctan(c*x^3))-1/4*b*arctan(
2*c^(1/3)*x-3^(1/2))/c^(2/3)-1/4*b*arctan(2*c^(1/3)*x+3^(1/2))/c^(2/3)-1/8
*b*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)+1/8*b*ln(1+c^(2/3)*
x^2+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)
```

3.110.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.03

$$\int x(a+b\arctan(cx^3))\,dx=\frac{ax^2}{2}-\frac{b\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}bx^2\arctan(cx^3)+\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}+\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}$$

input
```
Integrate[x*(a + b*ArcTan[c*x^3]),x]
```

output
```
(a*x^2)/2 - (b*ArcTan[c^(1/3)*x])/(2*c^(2/3)) + (b*x^2*ArcTan[c*x^3])/2 +
(b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(4*c^(2/3)) - (b*ArcTan[Sqrt[3] + 2*c^(1
/3)*x])/(4*c^(2/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])
/(8*c^(2/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(
2/3))
```

3.110.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5361, 824, 27, 216, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx^3))\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2(a+b\arctan(cx^3))-\frac{3}{2}bc\int\frac{x^4}{c^2x^6+1}dx$$

$$\Big\downarrow \text{824}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{2}bc\left(\frac{\int\frac{1}{c^{2/3}x^2+1}dx}{3c^{4/3}}+\frac{\int-\frac{1-\sqrt{3}\sqrt[3]{c}x}{2\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}+\frac{\int-\frac{\sqrt{3}\sqrt[3]{c}x+1}{2\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{2}bc\left(\frac{\int\frac{1}{c^{2/3}x^2+1}dx}{3c^{4/3}}-\frac{\int\frac{1-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int\frac{\sqrt{3}\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{2}bc\left(-\frac{\int\frac{1-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int\frac{\sqrt{3}\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\arctan\left(\sqrt[3]{c}x\right)}{3c^{5/3}}\right)$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{2}bc\left(-\frac{-\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\int-\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\text{a}\ldots\right.$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{2}bc\left(-\frac{\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{\sqrt{3}\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\text{arcta}\ldots}{\ldots}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{3}{2}bc\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{1}{2}\int\frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{1}{2}\int\frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}\ldots\right.$$

3.110. $\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx$

$\downarrow$ 1082

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-$$
$$\frac{3}{2}bc\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\int\frac{1}{-\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\int\frac{1}{-\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)^2-\frac{1}{3}}\ldots}{\sqrt{3}\sqrt[3]{\ldots}}}{6c^{4/3}}\right.$$

$\downarrow$ 217

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-$$
$$\frac{3}{2}bc\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}+\ldots\right.$$

$\downarrow$ 1103

$$\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-$$
$$\frac{3}{2}bc\left(-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}-\frac{\sqrt{3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}-\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arc}\ldots}{}\right.$$

input
```
Int[x*(a + b*ArcTan[c*x^3]),x]
```

output
```
(x^2*(a + b*ArcTan[c*x^3]))/2 - (3*b*c*(ArcTan[c^(1/3)*x]/(3*c^(5/3)) - (A
rcTan[Sqrt[3]*(1 - (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3) - (Sqrt[3]*Log[1 - Sqrt
[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/(6*c^(4/3)) - (-(ArcTan[Sqrt[3]
*(1 + (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3)) + (Sqrt[3]*Log[1 + Sqrt[3]*c^(1/3)*
x + c^(2/3)*x^2])/(2*c^(1/3)))/(6*c^(4/3))))/2
```

3.110. $\int x(a+b\arctan(cx^3))\,dx$

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.110.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.93

method	result
default	$\frac{a x^2}{2}+b\left(\frac{x^2\arctan(c x^3)}{2}-\frac{3c\left(-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\cdots\right)\right)}{12}\cdots\right)}{2}\cdots\right)$
parts	$\frac{a x^2}{2}+b\left(\frac{x^2\arctan(c x^3)}{2}-\frac{3c\left(-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\cdots\right)\right)}{12}\cdots\right)}{2}\cdots\right)$

input
```
int(x*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+b*(1/2*x^2*arctan(c*x^3)-3/2*c*(-1/12*3^(1/2)*(1/c^2)^(5/6)*ln(x
^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x
/(1/c^2)^(1/6)+3^(1/2))+1/12*3^(1/2)*(1/c^2)^(5/6)*ln(x^2-3^(1/2)*(1/c^2)^
(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1
/2))+1/3/c^2/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))))
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(119) = 238$.

Time = 0.25 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.68

$$\int x(a+b\arctan(cx^3))\,dx = \frac{1}{2}bx^2\arctan(cx^3) + \frac{1}{2}ax^2 + \frac{1}{8}\left(-\frac{b^6}{c^4}\right)^{\frac{1}{6}}(\sqrt{-3}-1)\log\left(b^5x + \frac{1}{2}(\sqrt{-3}c^3+c^3)\left(-\frac{b^6}{c^4}\right)^{\frac{5}{6}}\right) - \frac{1}{8}\left(-\frac{b^6}{c^4}\right)^{\frac{1}{6}}(\sqrt{-3}-1)\log\left(b^5x - \frac{1}{2}(\sqrt{-3}c^3+c^3)\left(-\frac{b^6}{c^4}\right)^{\frac{5}{6}}\right) + \frac{1}{8}\left(-\frac{b^6}{c^4}\right)^{\frac{1}{6}}(\sqrt{-3}+1)\log\left(b^5x + \frac{1}{2}(\sqrt{-3}c^3-c^3)\left(-\frac{b^6}{c^4}\right)^{\frac{5}{6}}\right) - \frac{1}{8}\left(-\frac{b^6}{c^4}\right)^{\frac{1}{6}}(\sqrt{-3}+1)\log\left(b^5x - \frac{1}{2}(\sqrt{-3}c^3-c^3)\left(-\frac{b^6}{c^4}\right)^{\frac{5}{6}}\right) - \frac{1}{4}\left(-\frac{b^6}{c^4}\right)^{\frac{1}{6}}\log\left(b^5x + \left(-\frac{b^6}{c^4}\right)^{\frac{5}{6}}c^3\right) + \frac{1}{4}\left(-\frac{b^6}{c^4}\right)^{\frac{1}{6}}\log\left(b^5x - \left(-\frac{b^6}{c^4}\right)^{\frac{5}{6}}c^3\right)$$

input
```
integrate(x*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
1/2*b*x^2*arctan(c*x^3) + 1/2*a*x^2 + 1/8*(-b^6/c^4)^(1/6)*(sqrt(-3) - 1)*
log(b^5*x + 1/2*(sqrt(-3)*c^3 + c^3)*(-b^6/c^4)^(5/6)) - 1/8*(-b^6/c^4)^(1
/6)*(sqrt(-3) - 1)*log(b^5*x - 1/2*(sqrt(-3)*c^3 + c^3)*(-b^6/c^4)^(5/6))
+ 1/8*(-b^6/c^4)^(1/6)*(sqrt(-3) + 1)*log(b^5*x + 1/2*(sqrt(-3)*c^3 - c^3)
*(-b^6/c^4)^(5/6)) - 1/8*(-b^6/c^4)^(1/6)*(sqrt(-3) + 1)*log(b^5*x - 1/2*(
sqrt(-3)*c^3 - c^3)*(-b^6/c^4)^(5/6)) - 1/4*(-b^6/c^4)^(1/6)*log(b^5*x + (
-b^6/c^4)^(5/6)*c^3) + 1/4*(-b^6/c^4)^(1/6)*log(b^5*x - (-b^6/c^4)^(5/6)*c
^3)
```

3.110.6 Sympy [A] (verification not implemented)

Time = 18.14 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.49

$$\int x(a+b\arctan(cx^3))\,dx$$
$$= \begin{cases} \frac{ax^2}{2} + \frac{bx^2\operatorname{atan}(cx^3)}{2} - \frac{3b\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\sqrt[6]{-\frac{1}{c^2}}} + \frac{3b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\sqrt[6]{-\frac{1}{c^2}}} - \frac{\sqrt{3}b\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[6]{-\frac{1}{c^2}}}-\frac{\sqrt{3}}{3}\right)}{4c\sqrt[6]{-\frac{1}{c^2}}} \\ \frac{ax^2}{2} \end{cases}$$

input
```
integrate(x*(a+b*atan(c*x**3)),x)
```

output
```
Piecewise((a*x**2/2 + b*x**2*atan(c*x**3)/2 - 3*b*log(4*x**2 - 4*x*(-1/c**
2)**(1/6) + 4*(-1/c**2)**(1/3))/(8*c*(-1/c**2)**(1/6)) + 3*b*log(4*x**2 +
4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(8*c*(-1/c**2)**(1/6)) - sqrt(3
)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/(4*c*(-1/c**2)**(1/
6)) - sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*(-
1/c**2)**(1/6)) + b*atan(c*x**3)/(2*c**2*(-1/c**2)**(2/3)), Ne(c, 0)), (a*
x**2/2, True))
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{2}\,ax^2$$
$$+\frac{1}{8}\left(4\,x^2\arctan\left(cx^3\right)+c\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{4\,\arctan\left(c^{\frac{1}{3}}\right.}{c^{\frac{5}{3}}}\right.\right.$$

input
```
integrate(x*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/8*(4*x^2*arctan(c*x^3) + c*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3
)*c^(1/3)*x + 1)/c^(5/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1
)/c^(5/3) - 4*arctan(c^(1/3)*x)/c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*
c^(1/3))/c^(1/3))/c^(5/3) - 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/
3))/c^(5/3)))*b
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.95

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$=\frac{1}{8}\,bc^5\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6}-\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6}-\frac{2\,\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^4|c|^{\frac{5}{3}}}-\right.$$
$$+\frac{1}{2}\,bx^2\arctan\left(cx^3\right)+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
1/8*b*c^5*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c
)^(2/3))/c^6 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/a
bs(c)^(2/3))/c^6 - 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^
4*abs(c)^(5/3)) - 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^4
*abs(c)^(5/3)) - 4*arctan(x*abs(c)^(1/3))/(c^4*abs(c)^(5/3))) + 1/2*b*x^2*
arctan(c*x^3) + 1/2*a*x^2
```

3.110.9 Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int x(a+b\arctan(cx^3))\,dx = \frac{a\,x^2}{2} + \frac{b\left(\operatorname{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)+\operatorname{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+2\operatorname{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{4\,c^{2/3}} + \frac{b\,x^2\operatorname{atan}(c\,x^3)}{2} - \frac{\sqrt{3}\,b\left(\operatorname{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)-\operatorname{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)\,1\mathrm{i}}{4\,c^{2/3}}$$

input
```
int(x*(a + b*atan(c*x^3)),x)
```

output
```
(a*x^2)/2 + (b*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^(2/3)*c^(1/3)*x*(3
^(1/2)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/(
4*c^(2/3)) + (b*x^2*atan(c*x^3))/2 - (3^(1/2)*b*(atan((-1)^(2/3)*c^(1/3)*x
) - atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1i)/(4*c^(2/3))
```

3.111 $\int \frac{a+b\arctan(cx^3)}{x^2} dx$

3.111.1 Optimal result 729
3.111.2 Mathematica [A] (verified) 729
3.111.3 Rubi [A] (verified) 730
3.111.4 Maple [A] (verified) 733
3.111.5 Fricas [A] (verification not implemented) 734
3.111.6 Sympy [B] (verification not implemented) 734
3.111.7 Maxima [A] (verification not implemented) 735
3.111.8 Giac [A] (verification not implemented) 735
3.111.9 Mupad [B] (verification not implemented) 736

3.111.1 Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{a+b\arctan(cx^3)}{x^2} dx = -\frac{a+b\arctan(cx^3)}{x} - \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right) + \frac{1}{2}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2\right) - \frac{1}{4}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)$$

output
```
(-a-b*arctan(c*x^3))/x+1/2*b*c^(1/3)*ln(1+c^(2/3)*x^2)-1/4*b*c^(1/3)*ln(1-
c^(2/3)*x^2+c^(4/3)*x^4)-1/2*b*c^(1/3)*arctan(1/3*(1-2*c^(2/3)*x^2)*3^(1/2
))*3^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.63

$$\int \frac{a+b\arctan(cx^3)}{x^2} dx = -\frac{a}{x} - \frac{b\arctan(cx^3)}{x} - \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right) - \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right) + \frac{1}{2}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2\right) - \frac{1}{4}b\sqrt[3]{c}\log\left(1-\sqrt{3}\sqrt[3]{c}x + c^{2/3}x^2\right) - \frac{1}{4}b\sqrt[3]{c}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^2,x]
```

output
```
-(a/x) - (b*ArcTan[c*x^3])/x - (Sqrt[3]*b*c^(1/3)*ArcTan[Sqrt[3] - 2*c^(1/
3)*x])/2 - (Sqrt[3]*b*c^(1/3)*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/2 + (b*c^(1/3
)*Log[1 + c^(2/3)*x^2])/2 - (b*c^(1/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)
*x^2])/4 - (b*c^(1/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/4
```

3.111.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5361, 807, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx^3)}{x^2}dx$$

$$\Big\downarrow \text{5361}$$

$$3bc\int \frac{x}{c^2x^6+1}dx-\frac{a+b\arctan(cx^3)}{x}$$

$$\Big\downarrow \text{807}$$

$$\frac{3}{2}bc\int \frac{1}{c^2x^6+1}dx^2-\frac{a+b\arctan(cx^3)}{x}$$

$$\Big\downarrow \text{750}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\int \frac{1}{c^{2/3}x^2+1}dx^2+\frac{1}{3}\int \frac{2-c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2\right)-\frac{a+b\arctan(cx^3)}{x}$$

$$\Big\downarrow \text{16}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\int \frac{2-c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan(cx^3)}{x}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{\int -\frac{c^{2/3}(1-2c^{2/3}x^2)}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{2c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan(cx^3)}{x}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{\int \frac{c^{2/3}\left(1-2c^{2/3}x^2\right)}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2}{2c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2+\frac{1}{2}\int \frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 1082$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{3\int \frac{1}{-x^4-3}d\left(1-2c^{2/3}x^2\right)}{c^{2/3}}+\frac{1}{2}\int \frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 217$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2c^{2/3}x^2}{c^{4/3}x^4-c^{2/3}x^2+1}dx^2-\frac{\sqrt{3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 1103$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{c^{2/3}}-\frac{\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{2c^{2/3}}\right)+\frac{\log\left(c^{2/3}x^2+1\right)}{3c^{2/3}}\right)-\frac{a+b\arctan\left(cx^3\right)}{x}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^2,x]
```

output
```
-((a + b*ArcTan[c*x^3])/x) + (3*b*c*(Log[1 + c^(2/3)*x^2]/(3*c^(2/3)) + (-
((Sqrt[3]*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/c^(2/3)) - Log[1 - c^(2/3)*
x^2 + c^(4/3)*x^4]/(2*c^(2/3)))/3))/2
```

3.111. $\int \frac{a+b\arctan\left(cx^3\right)}{x^2}\,dx$

3.111.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.111. $\int \frac{a+b\arctan(cx^3)}{x^2}\,dx$

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.111.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

method	result
default	$-\frac{a}{x}-\frac{b\arctan\left(c\,x^{3}\right)}{x}+\frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}$
parts	$-\frac{a}{x}-\frac{b\arctan\left(c\,x^{3}\right)}{x}+\frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}$
risch	$\frac{ib\ln\left(ic\,x^{3}+1\right)}{2x}-\frac{ib\ln\left(-ic\,x^{3}+1\right)}{2x}-\frac{ib\ln\left(x+\left(\frac{i}{c}\right)^{\frac{1}{3}}\right)}{2\left(\frac{i}{c}\right)^{\frac{1}{3}}}+\frac{ib\ln\left(x^{2}-\left(\frac{i}{c}\right)^{\frac{1}{3}}x+\left(\frac{i}{c}\right)^{\frac{2}{3}}\right)}{4\left(\frac{i}{c}\right)^{\frac{1}{3}}}+\frac{ib\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{i}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2\left(\frac{i}{c}\right)^{\frac{1}{3}}}-\frac{a}{x}-$

input
```
int((a+b*arctan(c*x^3))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x-b/x*arctan(c*x^3)+1/2*b/c/(1/c^2)^(2/3)*ln(x^2+(1/c^2)^(1/3))-1/4*b/c
/(1/c^2)^(2/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b/c/(1/c^2)^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*x^2/(1/c^2)^(1/3)-1))
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{a + b\arctan(cx^3)}{x^2}\,dx$$
$$= \frac{2\sqrt{3}bc^{\frac{1}{3}}x\arctan\left(\frac{2}{3}\sqrt{3}c^{\frac{2}{3}}x^2 - \frac{1}{3}\sqrt{3}\right) - bc^{\frac{1}{3}}x\log\left(c^2x^4 - c^{\frac{4}{3}}x^2 + c^{\frac{2}{3}}\right) + 2\,bc^{\frac{1}{3}}x\log\left(cx^2 + c^{\frac{1}{3}}\right) - 4\,b\arctan}{4\,x}$$

input
```
integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*b*c^(1/3)*x*arctan(2/3*sqrt(3)*c^(2/3)*x^2 - 1/3*sqrt(3)) -
 b*c^(1/3)*x*log(c^2*x^4 - c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(1/3)*x*log(c*x^
2 + c^(1/3)) - 4*b*arctan(c*x^3) - 4*a)/x
```

3.111.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(97) = 194.

Time = 25.23 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.52

$$\int \frac{a + b\arctan(cx^3)}{x^2}\,dx$$
$$= \begin{cases} -\frac{a}{x} + bc^2\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}\operatorname{atan}(cx^3) - bc\sqrt[3]{-\frac{1}{c^2}}\log\left(x - \sqrt[6]{-\frac{1}{c^2}}\right) + \frac{3bc\sqrt[3]{-\frac{1}{c^2}}\log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{4} - \frac{bc\sqrt[3]{-}}{} \\ -\frac{a}{x} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))/x**2,x)
```

output
```
Piecewise((-a/x + b*c**2*(-1/c**2)**(5/6)*atan(c*x**3) - b*c*(-1/c**2)**(1
/3)*log(x - (-1/c**2)**(1/6)) + 3*b*c*(-1/c**2)**(1/3)*log(4*x**2 - 4*x*(-
1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/4 - b*c*(-1/c**2)**(1/3)*log(4*x**2 +
 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/4 + sqrt(3)*b*c*(-1/c**2)**(1/
3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/2 - sqrt(3)*b*c*(-1/
c**2)**(1/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/2 - b*atan
(c*x**3)/x, Ne(c, 0)), (-a/x, True))
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx$$
$$=\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}-\frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{2}{3}}}+\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}\right)-\frac{4\arctan(cx^3)}{x}\right)b$$
$$-\frac{a}{x}$$

input
```
integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="maxima")
```

output
```
1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^
(2/3) - log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1)/c^(2/3) + 2*log((c^(2/3)*x^2 +
1)/c^(2/3))/c^(2/3)) - 4*arctan(c*x^3)/x)*b - a/x
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx$$
$$=\frac{1}{4}bc\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{|c|^{\frac{2}{3}}}-\frac{\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{|c|^{\frac{2}{3}}}+\frac{2\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{2}{3}}}\right)$$
$$-\frac{b\arctan(cx^3)+a}{x}$$

input
```
integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="giac")
```

output
```
1/4*b*c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(2/3
))/abs(c)^(2/3) - log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/abs(c)^(2/3
) + 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(2/3)) - (b*arctan(c*x^3) + a)/x
```

3.111.9 Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx = \frac{b\,c^{1/3}\,\ln\left(c^{2/3}\,x^2+1\right)}{2} - \frac{a}{x} - \frac{b\,\mathrm{atan}(c\,x^3)}{x} - \frac{b\,c^{1/3}\,\ln\left(-\sqrt{3}-c^{2/3}\,x^2\,2\mathrm{i}+1\mathrm{i}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4} + \frac{b\,c^{1/3}\,\ln\left(-\sqrt{3}+c^{2/3}\,x^2\,2\mathrm{i}-\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}$$

input
```
int((a + b*atan(c*x^3))/x^2,x)
```

output
```
(b*c^(1/3)*log(c^(2/3)*x^2 + 1))/2 - a/x - (b*atan(c*x^3))/x - (b*c^(1/3)*
log(1i - c^(2/3)*x^2*2i - 3^(1/2))*(3^(1/2)*1i + 1))/4 + (b*c^(1/3)*log(c^
(2/3)*x^2*2i - 3^(1/2) - 1i)*(3^(1/2)*1i - 1))/4
```

3.112 $\int \frac{a+b\arctan(cx^3)}{x^5}\,dx$

3.112.1 Optimal result 737
3.112.2 Mathematica [A] (verified) 737
3.112.3 Rubi [A] (verified) 738
3.112.4 Maple [A] (verified) 742
3.112.5 Fricas [B] (verification not implemented) 743
3.112.6 Sympy [A] (verification not implemented) 744
3.112.7 Maxima [A] (verification not implemented) 744
3.112.8 Giac [A] (verification not implemented) 745
3.112.9 Mupad [B] (verification not implemented) 746

3.112.1 Optimal result

Integrand size = 14, antiderivative size = 174

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx$$
$$= -\frac{3bc}{4x} - \frac{1}{4}bc^{4/3}\arctan\left(\sqrt[3]{c}x\right) - \frac{a+b\arctan(cx^3)}{4x^4} + \frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)$$
$$-\frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right) - \frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right) + \frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right.$$

output
```
-3/4*b*c/x-1/4*b*c^(4/3)*arctan(c^(1/3)*x)+1/4*(-a-b*arctan(c*x^3))/x^4-1/
8*b*c^(4/3)*arctan(2*c^(1/3)*x-3^(1/2))-1/8*b*c^(4/3)*arctan(2*c^(1/3)*x+3
^(1/2))-1/16*b*c^(4/3)*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)+1/16*b*
c^(4/3)*ln(1+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)
```

3.112.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx$$
$$= -\frac{a}{4x^4} - \frac{3bc}{4x} - \frac{1}{4}bc^{4/3}\arctan\left(\sqrt[3]{c}x\right) - \frac{b\arctan(cx^3)}{4x^4} + \frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)$$
$$-\frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right) - \frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right) + \frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right.$$

input
```
Integrate[(a + b*ArcTan[c*x^3])/x^5,x]
```

output
```
-1/4*a/x^4 - (3*b*c)/(4*x) - (b*c^(4/3)*ArcTan[c^(1/3)*x])/4 - (b*ArcTan[c
*x^3])/(4*x^4) + (b*c^(4/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/8 - (b*c^(4/3)*
ArcTan[Sqrt[3] + 2*c^(1/3)*x])/8 - (Sqrt[3]*b*c^(4/3)*Log[1 - Sqrt[3]*c^(1
/3)*x + c^(2/3)*x^2])/16 + (Sqrt[3]*b*c^(4/3)*Log[1 + Sqrt[3]*c^(1/3)*x +
c^(2/3)*x^2])/16
```

3.112.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5361, 847, 824, 27, 216, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^5}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{4}bc\int \frac{1}{x^2\left(c^2x^6+1\right)}dx-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{4}bc\left(c^2\left(-\int \frac{x^4}{c^2x^6+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \text{824}$$

$$\frac{3}{4}bc\left(-\left(c^2\left(\frac{\int \frac{1}{c^{2/3}x^2+1}dx}{3c^{4/3}}+\frac{\int -\frac{1-\sqrt{3}\sqrt[3]{c}x}{2\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}+\frac{\int -\frac{\sqrt{3}\sqrt[3]{c}x+1}{2\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}bc\left(-\left(c^2\left(\frac{\int \frac{1}{c^{2/3}x^2+1}dx}{3c^{4/3}}-\frac{\int \frac{1-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int \frac{\sqrt{3}\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \; 216$$

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{\int \frac{1-\sqrt{3}\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int \frac{\sqrt{3}\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\arctan\left(\sqrt[3]{c}x\right)}{3c^{5/3}}\right)\right)-\frac{1}{x}\right)-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \; 1142$$

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{-\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\int -\frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}}{6c^{4/3}}\right.\right.\right.$$

$$-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{\frac{\sqrt{3}\int \frac{\sqrt[3]{c}\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{\sqrt{3}\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}-\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}\right.\right.\right.$$

$$-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{1}{2}\int \frac{1}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{1}{2}\int \frac{1}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{}}}{6c^{4/3}}\right.\right.\right.$$

$$-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

3.112. $\int \frac{a+b\arctan\left(cx^3\right)}{x^5}\,dx$

$\Big\downarrow$ 1082

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\int \frac{1}{-\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\int \frac{1}{-\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right.}}{}}{6c^{4/3}}\right.\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$\Big\downarrow$ 217

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1}dx+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[3]{c}x+\sqrt{3}}{c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1}dx-\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right.\right.}{\sqrt[3]{c}}}{6c^{4/3}}\right.\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

$\Big\downarrow$ 1103

$$\frac{3}{4}bc\left(-\left(c^2\left(-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)\right)}{\sqrt[3]{c}}-\frac{\sqrt{3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}-\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[3]{c}x}{\sqrt{3}}+1\right)\right.}{\sqrt[3]{c}}}{6c^{4/3}}\right.\right.\right.$$

$$\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$

input
```
Int[(a + b*ArcTan[c*x^3])/x^5,x]
```

output
```
-1/4*(a + b*ArcTan[c*x^3])/x^4 + (3*b*c*(-x^(-1) - c^2*(ArcTan[c^(1/3)*x]/
(3*c^(5/3)) - (ArcTan[Sqrt[3]*(1 - (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3) - (Sqrt
[3]*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/(6*c^(4/3)) - (
-(ArcTan[Sqrt[3]*(1 + (2*c^(1/3)*x)/Sqrt[3])]/c^(1/3)) + (Sqrt[3]*Log[1 +
Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(2*c^(1/3)))/(6*c^(4/3)))))/4
```

3.112. $\int \frac{a+b\arctan\left(cx^3\right)}{x^5}\,dx$

3.112.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.112.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.94

method	result
default	$-\frac{a}{4x^4}+b\left(-\frac{\arctan(cx^3)}{4x^4}+\frac{3c\left(-\frac{1}{x}-\left(-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2 \cdots\right)}{\cdots}\cdots\right)\right)}{4}\cdots\right)$
parts	$-\frac{a}{4x^4}+b\left(-\frac{\arctan(cx^3)}{4x^4}+\frac{3c\left(-\frac{1}{x}-\left(-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2 \cdots\right)}{\cdots}\cdots\right)\right)}{4}\cdots\right)$

input
```
int((a+b*arctan(c*x^3))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/x^4+b*(-1/4/x^4*arctan(c*x^3)+3/4*c*(-1/x-(-1/12*3^(1/2)*(1/c^2)^(5
/6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*ar
ctan(2*x/(1/c^2)^(1/6)+3^(1/2))+1/12*3^(1/2)*(1/c^2)^(5/6)*ln(x^2-3^(1/2)*
(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1
/6)-3^(1/2))+1/3/c^2/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6)))*c^2))
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(126) = 252$.

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.73

$$\int \frac{a + b\arctan(cx^3)}{x^5}\,dx =$$
$$-\frac{2\left(-b^6c^8\right)^{\frac{1}{6}}x^4\log\left(b^5c^7x + \left(-b^6c^8\right)^{\frac{5}{6}}\right) - 2\left(-b^6c^8\right)^{\frac{1}{6}}x^4\log\left(b^5c^7x - \left(-b^6c^8\right)^{\frac{5}{6}}\right) + 12\,bcx^3 - \left(-b^6c^8\right)^{\frac{1}{6}}\left(\sqrt{\ }\right.}{}$$

input
```
integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="fricas")
```

output
```
-1/16*(2*(-b^6*c^8)^(1/6)*x^4*log(b^5*c^7*x + (-b^6*c^8)^(5/6)) - 2*(-b^6*
c^8)^(1/6)*x^4*log(b^5*c^7*x - (-b^6*c^8)^(5/6)) + 12*b*c*x^3 - (-b^6*c^8)
^(1/6)*(sqrt(-3)*x^4 - x^4)*log(2*b^5*c^7*x + (-b^6*c^8)^(5/6)*(sqrt(-3) +
 1)) + (-b^6*c^8)^(1/6)*(sqrt(-3)*x^4 - x^4)*log(2*b^5*c^7*x - (-b^6*c^8)^
(5/6)*(sqrt(-3) + 1)) - (-b^6*c^8)^(1/6)*(sqrt(-3)*x^4 + x^4)*log(2*b^5*c^
7*x + (-b^6*c^8)^(5/6)*(sqrt(-3) - 1)) + (-b^6*c^8)^(1/6)*(sqrt(-3)*x^4 +
x^4)*log(2*b^5*c^7*x - (-b^6*c^8)^(5/6)*(sqrt(-3) - 1)) + 4*b*arctan(c*x^3
) + 4*a)/x^4
```

3.112.6 Sympy [A] (verification not implemented)

Time = 49.05 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.52

$$\int \frac{a + b \arctan (cx^3)}{x^5} \, dx$$
$$= \begin{cases} -\frac{a}{4x^4} + \frac{3bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}} \log \left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{16} - \frac{3bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}} \log \left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{16} + \frac{\sqrt{3}bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}} \operatorname{atan}}{8} \\ -\frac{a}{4x^4} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))/x**5,x)
```

output
```
Piecewise((-a/(4*x**4) + 3*b*c**3*(-1/c**2)**(5/6)*log(4*x**2 - 4*x*(-1/c*
*2)**(1/6) + 4*(-1/c**2)**(1/3))/16 - 3*b*c**3*(-1/c**2)**(5/6)*log(4*x**2
 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/16 + sqrt(3)*b*c**3*(-1/c**2
)**(5/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/8 + sqrt(3)*b*
c**3*(-1/c**2)**(5/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/8
 - b*c**2*(-1/c**2)**(1/3)*atan(c*x**3)/4 - 3*b*c/(4*x) - b*atan(c*x**3)/(
4*x**4), Ne(c, 0)), (-a/(4*x**4), True))
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

$$\int \frac{a + b \arctan (cx^3)}{x^5} \, dx$$
$$= \frac{1}{16} \left(\left(c^2 \left(\frac{\sqrt{3} \log \left(c^{\frac{2}{3}}x^2 + \sqrt{3}c^{\frac{1}{3}}x + 1\right)}{c^{\frac{5}{3}}} - \frac{\sqrt{3} \log \left(c^{\frac{2}{3}}x^2 - \sqrt{3}c^{\frac{1}{3}}x + 1\right)}{c^{\frac{5}{3}}} - \frac{4 \arctan \left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}} - \frac{2 \arctan \left(\right.}{c} \right.\right.\right.$$
$$- \frac{a}{4\,x^4}$$

input
```
integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="maxima")
```

output
```
1/16*((c^2*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - sqr
t(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - 4*arctan(c^(1/3)*x
)/c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3) - 2*
arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3)) - 12/x)*c - 4*arc
tan(c*x^3)/x^4)*b - 1/4*a/x^4
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.93

$$\int \frac{a + b\arctan(cx^3)}{x^5}\,dx$$
$$= \frac{1}{16}bc^3\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2 + \frac{\sqrt{3}x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2} - \frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2 - \frac{\sqrt{3}x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2} - \frac{2\,|c|^{\frac{1}{3}}\arctan\left(\left(2\,x + \frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right.}{c^2}\right.$$
$$- \frac{3\,bcx^3 + b\arctan(cx^3) + a}{4\,x^4}$$

input
```
integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="giac")
```

output
```
1/16*b*c^3*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(
c)^(2/3))/c^2 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/
abs(c)^(2/3))/c^2 - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs
(c)^(1/3))/c^2 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)
^(1/3))/c^2 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^2) - 1/4*(3*b*c*x^3
+ b*arctan(c*x^3) + a)/x^4
```

3.112.9 Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^5}\,dx = -\frac{a}{4\,x^4} + \frac{b\,c^{4/3}\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)+\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{8} - \frac{b\,\mathrm{atan}(c\,x^3)}{4\,x^4} - \frac{3\,b\,c}{4\,x} - \frac{\sqrt{3}\,b\,c^{4/3}\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)-\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)\,1\mathrm{i}}{8}$$

input
```
int((a + b*atan(c*x^3))/x^5,x)
```

output
```
(b*c^(4/3)*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/
2)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/8 - a
/(4*x^4) - (b*atan(c*x^3))/(4*x^4) - (3*b*c)/(4*x) - (3^(1/2)*b*c^(4/3)*(a
tan((-1)^(2/3)*c^(1/3)*x) - atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2
))*1i)/8
```

3.113 $\int x^{11}(a+b\arctan(cx^3))^2\,dx$

3.113.1 Optimal result 747
3.113.2 Mathematica [A] (verified) 747
3.113.3 Rubi [A] (verified) 748
3.113.4 Maple [A] (verified) 751
3.113.5 Fricas [A] (verification not implemented) 751
3.113.6 Sympy [B] (verification not implemented) 752
3.113.7 Maxima [A] (verification not implemented) 752
3.113.8 Giac [A] (verification not implemented) 753
3.113.9 Mupad [B] (verification not implemented) 753

3.113.1 Optimal result

Integrand size = 16, antiderivative size = 124

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{abx^3}{6c^3}+\frac{b^2x^6}{36c^2}+\frac{b^2x^3\arctan(cx^3)}{6c^3}-\frac{bx^9(a+b\arctan(cx^3))}{18c}-\frac{(a+b\arctan(cx^3))^2}{12c^4}+\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{b^2\log(1+c^2x^6)}{9c^4}$$

output
```
1/6*a*b*x^3/c^3+1/36*b^2*x^6/c^2+1/6*b^2*x^3*arctan(c*x^3)/c^3-1/18*b*x^9*
(a+b*arctan(c*x^3))/c-1/12*(a+b*arctan(c*x^3))^2/c^4+1/12*x^12*(a+b*arctan
(c*x^3))^2-1/9*b^2*ln(c^2*x^6+1)/c^4
```

3.113.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\frac{cx^3(6ab+b^2cx^3-2abc^2x^6+3a^2c^3x^9)-2b(bcx^3(-3+c^2x^6)+a(3-3c^4x^{12}))\arctan(cx^3)+3b^2(-1+c^4}{36c^4}$$

input
```
Integrate[x^11*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
(c*x^3*(6*a*b + b^2*c*x^3 - 2*a*b*c^2*x^6 + 3*a^2*c^3*x^9) - 2*b*(b*c*x^3*
(-3 + c^2*x^6) + a*(3 - 3*c^4*x^12))*ArcTan[c*x^3] + 3*b^2*(-1 + c^4*x^12)
*ArcTan[c*x^3]^2 - 4*b^2*Log[1 + c^2*x^6])/(36*c^4)
```

3.113.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5363, 5361, 5451, 5361, 243, 49, 2009, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3}\int x^9\left(a+b\arctan\left(cx^3\right)\right)^2dx^3$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\int\frac{x^{12}\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3\right)$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\int x^6\left(a+b\arctan\left(cx^3\right)\right)dx^3}{c^2}-\frac{\int\frac{x^6\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{3}bc\int\frac{x^9}{c^2x^6+1}dx^3}{c^2}-\frac{\int\frac{x^6\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}bc\int\frac{x^6}{c^2x^6+1}dx^6}{c^2}-\frac{\int\frac{x^6\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}bc\int\left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^6+1)}\right)dx^6}{c^2}-\frac{\int\frac{x^6(a+b\arctan(cx^3)}{c^2x^6+1}}{c^2}\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}bc\left(\frac{x^6}{c^2}-\frac{\log(c^2x^6+1)}{c^4}\right)}{c^2}-\frac{\int\frac{x^6(a+b\arctan(cx^3))}{c^2x^6+1}dx^3}{c^2}\right.\right.$$

$\downarrow$ 5451

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}bc\left(\frac{x^6}{c^2}-\frac{\log(c^2x^6+1)}{c^4}\right)}{c^2}-\frac{\frac{\int(a+b\arctan(cx^3))dx^3}{c^2}-}{c^2}\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}bc\left(\frac{x^6}{c^2}-\frac{\log(c^2x^6+1)}{c^4}\right)}{c^2}-\frac{\frac{ax^3+bx^3\arctan(cx^3)-\frac{b\log(}{}}{c^2}}{}\right.\right.$$

$\downarrow$ 5419

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}bc\left(\frac{x^6}{c^2}-\frac{\log(c^2x^6+1)}{c^4}\right)}{c^2}-\frac{\frac{ax^3+bx^3\arctan(cx^3)-\frac{b\log(}{}}{c^2}}{}\right.\right.$$

input
```
Int[x^11*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
((x^12*(a + b*ArcTan[c*x^3])^2)/4 - (b*c*(((x^9*(a + b*ArcTan[c*x^3]))/3 -
 (b*c*(x^6/c^2 - Log[1 + c^2*x^6]/c^4))/6)/c^2 - (-1/2*(a + b*ArcTan[c*x^3
])^2/(b*c^3) + (a*x^3 + b*x^3*ArcTan[c*x^3] - (b*Log[1 + c^2*x^6])/(2*c))/
c^2)/c^2))/2)/3
```

3.113. $\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$

3.113.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.113. $\int x^{11}(a+b\arctan(cx^3))^2\,dx$

3.113.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

method	result
default	$\frac{a^2x^{12}}{12}+\frac{b^2x^{12}\arctan(cx^3)^2}{12}-\frac{b^2\arctan(cx^3)x^9}{18c}+\frac{b^2x^3\arctan(cx^3)}{6c^3}-\frac{b^2\arctan(cx^3)^2}{12c^4}+\frac{b^2x^6}{36c^2}-\frac{b^2\ln(c^2x^6+1)}{9c^4}$
parts	$\frac{a^2x^{12}}{12}+\frac{b^2x^{12}\arctan(cx^3)^2}{12}-\frac{b^2\arctan(cx^3)x^9}{18c}+\frac{b^2x^3\arctan(cx^3)}{6c^3}-\frac{b^2\arctan(cx^3)^2}{12c^4}+\frac{b^2x^6}{36c^2}-\frac{b^2\ln(c^2x^6+1)}{9c^4}$
parallelrisch	$-\frac{-3b^2\arctan(cx^3)^2x^{12}c^4-6ab\arctan(cx^3)x^{12}c^4-3c^4a^2x^{12}+2b^2\arctan(cx^3)x^9c^3+2ab\,c^3x^9-x^6b^2c^2-6b^2\arctan(cx^3)x^3c}{36c^4}$
risch	$-\frac{b^2(c^4x^{12}-1)\ln(icx^3+1)^2}{48c^4}-\frac{ib(6a\,c^4x^{12}+3ib\,c^4x^{12}\ln(-icx^3+1)-2b\,c^3x^9+6bc\,x^3-3ib\ln(-icx^3+1))\ln(icx^3+1)}{72c^4}+\frac{ia}{}$

input
```
int(x^11*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^2*x^12+1/12*b^2*x^12*arctan(c*x^3)^2-1/18*b^2*arctan(c*x^3)/c*x^9+1
/6*b^2*x^3*arctan(c*x^3)/c^3-1/12*b^2/c^4*arctan(c*x^3)^2+1/36*b^2*x^6/c^2
-1/9*b^2*ln(c^2*x^6+1)/c^4+1/6*a*b*x^12*arctan(c*x^3)-1/18*a*b/c*x^9+1/6*a
*b*x^3/c^3-1/6*a*b/c^4*arctan(c*x^3)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\frac{3\,a^2c^4x^{12}-2\,abc^3x^9+b^2c^2x^6+6\,abcx^3+3\,(b^2c^4x^{12}-b^2)\arctan\left(cx^3\right)^2-4\,b^2\log\left(c^2x^6+1\right)+2\,(3\,abc^4x^{1}}{36\,c^4}$$

input
```
integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

output
```
1/36*(3*a^2*c^4*x^12 - 2*a*b*c^3*x^9 + b^2*c^2*x^6 + 6*a*b*c*x^3 + 3*(b^2*
c^4*x^12 - b^2)*arctan(c*x^3)^2 - 4*b^2*log(c^2*x^6 + 1) + 2*(3*a*b*c^4*x^
12 - b^2*c^3*x^9 + 3*b^2*c*x^3 - 3*a*b)*arctan(c*x^3))/c^4
```

3.113.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(112) = 224$.

Time = 147.67 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.96

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\begin{cases}\frac{a^2x^{12}}{12}+\frac{abx^{12}\operatorname{atan}\left(cx^3\right)}{6}-\frac{abx^9}{18c}+\frac{abx^3}{6c^3}-\frac{ab\operatorname{atan}\left(cx^3\right)}{6c^4}+\frac{b^2x^{12}\operatorname{atan}^2\left(cx^3\right)}{12}-\frac{b^2x^9\operatorname{atan}\left(cx^3\right)}{18c}+\frac{b^2x^6}{36c^2}+\frac{b^2x^3\operatorname{atan}\left(cx^3\right)}{6c^3}+\ldots\\ \frac{a^2x^{12}}{12}\end{cases}$$

input
```
integrate(x**11*(a+b*atan(c*x**3))**2,x)
```

output
```
Piecewise((a**2*x**12/12 + a*b*x**12*atan(c*x**3)/6 - a*b*x**9/(18*c) + a*
b*x**3/(6*c**3) - a*b*atan(c*x**3)/(6*c**4) + b**2*x**12*atan(c*x**3)**2/1
2 - b**2*x**9*atan(c*x**3)/(18*c) + b**2*x**6/(36*c**2) + b**2*x**3*atan(c
*x**3)/(6*c**3) + 2*b**2*sqrt(-1/c**2)*atan(c*x**3)/(9*c**3) - 2*b**2*log(
x - (-1/c**2)**(1/6))/(9*c**4) - 2*b**2*log(4*x**2 + 4*x*(-1/c**2)**(1/6)
+ 4*(-1/c**2)**(1/3))/(9*c**4) - b**2*atan(c*x**3)**2/(12*c**4), Ne(c, 0))
, (a**2*x**12/12, True))
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.36

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{1}{12}\,b^2x^{12}\arctan\left(cx^3\right)^2+\frac{1}{12}\,a^2x^{12}$$
$$+\frac{1}{18}\left(3\,x^{12}\arctan\left(cx^3\right)-c\left(\frac{c^2x^9-3\,x^3}{c^4}+\frac{3\,\arctan\left(cx^3\right)}{c^5}\right)\right)ab$$
$$-\frac{1}{36}\left(2\,c\left(\frac{c^2x^9-3\,x^3}{c^4}+\frac{3\,\arctan\left(cx^3\right)}{c^5}\right)\arctan\left(cx^3\right)-\frac{c^2x^6+3\,\arctan\left(cx^3\right)^2-3\,\log\left(18\,c^7x^6+18\,c^5\right)\ldots}{c^4}\right.$$

input
```
integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/12*b^2*x^12*arctan(c*x^3)^2 + 1/12*a^2*x^12 + 1/18*(3*x^12*arctan(c*x^3)
 - c*((c^2*x^9 - 3*x^3)/c^4 + 3*arctan(c*x^3)/c^5))*a*b - 1/36*(2*c*((c^2*
x^9 - 3*x^3)/c^4 + 3*arctan(c*x^3)/c^5)*arctan(c*x^3) - (c^2*x^6 + 3*arcta
n(c*x^3)^2 - 3*log(18*c^7*x^6 + 18*c^5) - log(c^2*x^6 + 1))/c^4)*b^2
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\frac{3\,a^2cx^{12}+2\left(3\,cx^{12}\arctan\left(cx^3\right)-\frac{3\,\arctan\left(cx^3\right)}{c^3}-\frac{c^9x^9-3\,c^7x^3}{c^9}\right)ab+\left(3\,cx^{12}\arctan\left(cx^3\right)^2-\frac{2\,c^3x^9\arctan\left(cx^3\right)-c^{\cdots}}{}\right.}{36\,c}$$

input
```
integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

output
```
1/36*(3*a^2*c*x^12 + 2*(3*c*x^12*arctan(c*x^3) - 3*arctan(c*x^3)/c^3 - (c^
9*x^9 - 3*c^7*x^3)/c^9)*a*b + (3*c*x^12*arctan(c*x^3)^2 - (2*c^3*x^9*arcta
n(c*x^3) - c^2*x^6 - 6*c*x^3*arctan(c*x^3) + 3*arctan(c*x^3)^2 + 4*log(c^2
*x^6 + 1))/c^3)*b^2)/c
```

3.113.9 Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{a^2\,x^{12}}{12}-\frac{b^2\,\mathrm{atan}(c\,x^3)^2}{12\,c^4}+\frac{b^2\,x^{12}\,\mathrm{atan}(c\,x^3)^2}{12}-\frac{b^2\,\ln\left(c^2\,x^6+1\right)}{9\,c^4}+\frac{b^2\,x^6}{36\,c^2}+\frac{b^2\,x^3\,\mathrm{atan}(c\,x^3)}{6\,c^3}-\frac{b^2\,x^9\,\mathrm{atan}(c\,x^3)}{18\,c}+\frac{a\,b\,x^3}{6\,c^3}-\frac{a\,b\,x^9}{18\,c}-\frac{a\,b\,\mathrm{atan}(c\,x^3)}{6\,c^4}+\frac{a\,b\,x^{12}\,\mathrm{atan}(c\,x^3)}{6}$$

input
```
int(x^11*(a + b*atan(c*x^3))^2,x)
```

output
```
(a^2*x^12)/12 - (b^2*atan(c*x^3)^2)/(12*c^4) + (b^2*x^12*atan(c*x^3)^2)/12
 - (b^2*log(c^2*x^6 + 1))/(9*c^4) + (b^2*x^6)/(36*c^2) + (b^2*x^3*atan(c*x
^3))/(6*c^3) - (b^2*x^9*atan(c*x^3))/(18*c) + (a*b*x^3)/(6*c^3) - (a*b*x^9
)/(18*c) - (a*b*atan(c*x^3))/(6*c^4) + (a*b*x^12*atan(c*x^3))/6
```

3.114 $\int x^8(a+b\arctan(cx^3))^2\,dx$

3.114.1 Optimal result . . . 754
3.114.2 Mathematica [A] (verified) . . . 754
3.114.3 Rubi [A] (verified) . . . 755
3.114.4 Maple [C] (warning: unable to verify) . . . 758
3.114.5 Fricas [F] . . . 759
3.114.6 Sympy [F] . . . 759
3.114.7 Maxima [F] . . . 759
3.114.8 Giac [F] . . . 760
3.114.9 Mupad [F(-1)] . . . 760

3.114.1 Optimal result

Integrand size = 16, antiderivative size = 154

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{b^2x^3}{9c^2}-\frac{b^2\arctan\left(cx^3\right)}{9c^3}-\frac{bx^6(a+b\arctan\left(cx^3\right))}{9c}-\frac{i(a+b\arctan\left(cx^3\right))^2}{9c^3}+\frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2b(a+b\arctan\left(cx^3\right))\log\left(\frac{2}{1+icx^3}\right)}{9c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{9c^3}$$

output
```
1/9*b^2*x^3/c^2-1/9*b^2*arctan(c*x^3)/c^3-1/9*b*x^6*(a+b*arctan(c*x^3))/c-
1/9*I*(a+b*arctan(c*x^3))^2/c^3+1/9*x^9*(a+b*arctan(c*x^3))^2-2/9*b*(a+b*a
rctan(c*x^3))*ln(2/(1+I*c*x^3))/c^3-1/9*I*b^2*polylog(2,1-2/(1+I*c*x^3))/c
^3
```

3.114.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\frac{b^2cx^3-abc^2x^6+a^2c^3x^9+b^2(i+c^3x^9)\arctan\left(cx^3\right)^2-b\arctan\left(cx^3\right)\left(b+bc^2x^6-2ac^3x^9+2b\log\left(1+e\right.\right.}{9c^3}$$

input
```
Integrate[x^8*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
(b^2*c*x^3 - a*b*c^2*x^6 + a^2*c^3*x^9 + b^2*(I + c^3*x^9)*ArcTan[c*x^3]^2
 - b*ArcTan[c*x^3]*(b + b*c^2*x^6 - 2*a*c^3*x^9 + 2*b*Log[1 + E^((2*I)*Arc
Tan[c*x^3])]) + a*b*Log[1 + c^2*x^6] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c
*x^3])])/(9*c^3)
```

3.114.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5363, 5361, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+b \arctan \left(c x^3\right)\right)^2 \, dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3} \int x^6 \left(a+b \arctan \left(c x^3\right)\right)^2dx^3$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3} \left(\frac{1}{3} x^9 \left(a+b \arctan \left(c x^3\right)\right)^2-\frac{2}{3} b c \int \frac{x^9 \left(a+b \arctan \left(c x^3\right)\right)}{c^2 x^6+1}dx^3\right)$$

$$\Big\downarrow \text{5451}$$

$$\frac{1}{3} \left(\frac{1}{3} x^9 \left(a+b \arctan \left(c x^3\right)\right)^2-\frac{2}{3} b c \left(\frac{\int x^3 \left(a+b \arctan \left(c x^3\right)\right)dx^3}{c^2}-\frac{\int \frac{x^3 \left(a+b \arctan \left(c x^3\right)\right)}{c^2 x^6+1}dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3} \left(\frac{1}{3} x^9 \left(a+b \arctan \left(c x^3\right)\right)^2-\frac{2}{3} b c \left(\frac{\frac{1}{2} x^6 \left(a+b \arctan \left(c x^3\right)\right)-\frac{1}{2} b c \int \frac{x^6}{c^2 x^6+1}dx^3}{c^2}-\frac{\int \frac{x^3 \left(a+b \arctan \left(c x^3\right)\right)}{c^2 x^6+1}dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{3}\Bigg(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\Bigg(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{x^3}{c^2}-\frac{\int\frac{1}{c^2x^6+1}dx^3}{c^2}\right)}{c^2}-\frac{\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}$$

$\Big\downarrow$ 216

$$\frac{1}{3}\Bigg(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\Bigg(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)}{c^2}-\frac{\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}\Bigg)$$

$\Big\downarrow$ 5455

$$\frac{1}{3}\Bigg(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\Bigg(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)}{c^2}-\frac{-\frac{\int\frac{a+b\arctan\left(cx^3\right)}{i-cx^3}dx^3}{c}-\frac{i(}{}}{c^2}$$

$\Big\downarrow$ 5379

$$\frac{1}{3}\Bigg(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\Bigg(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)}{c^2}-\frac{-\frac{\log\left(\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right.\right.}{c}}{c}$$

$\Big\downarrow$ 2849

$$\frac{1}{3}\Bigg(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\Bigg(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)}{c^2}-\frac{-\frac{ib\int\frac{\log\left(\frac{2}{icx^3+1}\right)}{1-\frac{2}{icx^3+1}}d\frac{1}{icx^3+1}}{c}+}{}$$

$\Big\downarrow$ 2752

$$\frac{1}{3}\Bigg(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2}{3}bc\Bigg(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)}{c^2}-\frac{-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{2bc^2}-\frac{\log}{}}{}$$

input
```
Int[x^8*(a + b*ArcTan[c*x^3])^2,x]
```

3.114. $\int x^8\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$

output
```
((x^9*(a + b*ArcTan[c*x^3])^2)/3 - (2*b*c*(((x^6*(a + b*ArcTan[c*x^3]))/2
- (b*c*(x^3/c^2 - ArcTan[c*x^3]/c^3))/2)/c^2 - (((-1/2*I)*(a + b*ArcTan[c*
x^3])^2)/(b*c^2) - (((a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/c + ((I/2
)*b*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c)/c)/c^2))/3)/3
```

3.114.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.114.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.69 (sec) , antiderivative size = 11449, normalized size of antiderivative = 74.34

method	result	size
default	Expression too large to display	11449
parts	Expression too large to display	11449

input
```
int(x^8*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.114.5 Fricas [F]

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int\left(b\arctan\left(cx^3\right)+a\right)^2x^8\,dx$$

input
```
integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^8*arctan(c*x^3)^2 + 2*a*b*x^8*arctan(c*x^3) + a^2*x^8, x)
```

3.114.6 Sympy [F]

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int x^8\left(a+b\operatorname{atan}\left(cx^3\right)\right)^2\,dx$$

input
```
integrate(x**8*(a+b*atan(c*x**3))**2,x)
```

output
```
Integral(x**8*(a + b*atan(c*x**3))**2, x)
```

3.114.7 Maxima [F]

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int\left(b\arctan\left(cx^3\right)+a\right)^2x^8\,dx$$

input
```
integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/9*a^2*x^9 + 1/9*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*
c)*a*b + 1/144*(4*x^9*arctan(c*x^3)^2 - x^9*log(c^2*x^6 + 1)^2 + 144*integ
rate(1/48*(4*c^2*x^14*log(c^2*x^6 + 1) - 8*c*x^11*arctan(c*x^3) + 36*(c^2*
x^14 + x^8)*arctan(c*x^3)^2 + 3*(c^2*x^14 + x^8)*log(c^2*x^6 + 1)^2)/(c^2*
x^6 + 1), x))*b^2
```

3.114.8 Giac [F]

$$\int x^8 \left(a + b \arctan \left(c x^3\right)\right)^2 \, dx = \int \left(b \arctan \left(c x^3\right) + a\right)^2 x^8 \, dx$$

input
```
integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2*x^8, x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a + b \arctan \left(c x^3\right)\right)^2 \, dx = \int x^8 \left(a + b \operatorname{atan}\left(c x^3\right)\right)^2 \, dx$$

input
```
int(x^8*(a + b*atan(c*x^3))^2,x)
```

output
```
int(x^8*(a + b*atan(c*x^3))^2, x)
```

3.115 $\int x^5(a+b\arctan(cx^3))^2 \, dx$

3.115.1 Optimal result 761
3.115.2 Mathematica [A] (verified) 761
3.115.3 Rubi [A] (verified) 762
3.115.4 Maple [A] (verified) 763
3.115.5 Fricas [A] (verification not implemented) 764
3.115.6 Sympy [B] (verification not implemented) 764
3.115.7 Maxima [A] (verification not implemented) 765
3.115.8 Giac [A] (verification not implemented) 765
3.115.9 Mupad [B] (verification not implemented) 766

3.115.1 Optimal result

Integrand size = 16, antiderivative size = 90

$$\int x^5(a+b\arctan(cx^3))^2 \, dx = -\frac{abx^3}{3c} - \frac{b^2x^3\arctan(cx^3)}{3c} + \frac{(a+b\arctan(cx^3))^2}{6c^2} + \frac{1}{6}x^6(a+b\arctan(cx^3))^2 + \frac{b^2\log(1+c^2x^6)}{6c^2}$$

output
```
-1/3*a*b*x^3/c-1/3*b^2*x^3*arctan(c*x^3)/c+1/6*(a+b*arctan(c*x^3))^2/c^2+1
/6*x^6*(a+b*arctan(c*x^3))^2+1/6*b^2*ln(c^2*x^6+1)/c^2
```

3.115.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int x^5(a+b\arctan(cx^3))^2 \, dx = \frac{acx^3(-2b+acx^3)+2b(a-bcx^3+ac^2x^6)\arctan(cx^3)+b^2(1+c^2x^6)\arctan(cx^3)^2+b^2\log(1+c^2x^6)}{6c^2}$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
(a*c*x^3*(-2*b + a*c*x^3) + 2*b*(a - b*c*x^3 + a*c^2*x^6)*ArcTan[c*x^3] +
b^2*(1 + c^2*x^6)*ArcTan[c*x^3]^2 + b^2*Log[1 + c^2*x^6])/(6*c^2)
```

3.115.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5363, 5361, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b \arctan \left(c x^3\right)\right)^2 \, dx$$

$$\Big\downarrow \text{ 5363}$$

$$\frac{1}{3} \int x^3 \left(a+b \arctan \left(c x^3\right)\right)^2 dx^3$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 \left(a+b \arctan \left(c x^3\right)\right)^2-b c \int \frac{x^6 \left(a+b \arctan \left(c x^3\right)\right)}{c^2 x^6+1}dx^3\right)$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 \left(a+b \arctan \left(c x^3\right)\right)^2-b c \left(\frac{\int \left(a+b \arctan \left(c x^3\right)\right)dx^3}{c^2}-\frac{\int \frac{a+b \arctan \left(c x^3\right)}{c^2 x^6+1}dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 \left(a+b \arctan \left(c x^3\right)\right)^2-b c \left(\frac{a x^3+b x^3 \arctan \left(c x^3\right)-\frac{b \log \left(c^2 x^6+1\right)}{2 c}}{c^2}-\frac{\int \frac{a+b \arctan \left(c x^3\right)}{c^2 x^6+1}dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 \left(a+b \arctan \left(c x^3\right)\right)^2-b c \left(\frac{a x^3+b x^3 \arctan \left(c x^3\right)-\frac{b \log \left(c^2 x^6+1\right)}{2 c}}{c^2}-\frac{\left(a+b \arctan \left(c x^3\right)\right)^2}{2 b c^3}\right)\right)$$

input
```
Int[x^5*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
((x^6*(a + b*ArcTan[c*x^3])^2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x^3])^2/(b*c^
3) + (a*x^3 + b*x^3*ArcTan[c*x^3] - (b*Log[1 + c^2*x^6])/(2*c))/c^2))/3
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.115.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

method	result
parallelrisch	$\frac{b^{2} \arctan \left(c \,x^{3}\right)^{2} x^{6} c^{2}+2 a b \arctan \left(c \,x^{3}\right) x^{6} c^{2}+a^{2} c^{2} x^{6}-2 b^{2} \arctan \left(c \,x^{3}\right) x^{3} c -2 a b c \,x^{3}+b^{2} \arctan \left(c \,x^{3}\right)^{2}+b^{2} \ln \left(c^{2} x^{6}+1\right)+2 a b \,\mathrm{ar}}{6 c^{2}}$
default	$\frac{x^{6} a^{2}}{6}+\frac{b^{2} x^{6} \arctan \left(c \,x^{3}\right)^{2}}{6}-\frac{b^{2} x^{3} \arctan \left(c \,x^{3}\right)}{3 c}+\frac{b^{2} \arctan \left(c \,x^{3}\right)^{2}}{6 c^{2}}+\frac{b^{2} \ln \left(c^{2} x^{6}+1\right)}{6 c^{2}}+\frac{a b \,x^{6} \arctan \left(c \,x^{3}\right)}{3}-\frac{a b \,x^{3}}{3 c}-$
parts	$\frac{x^{6} a^{2}}{6}+\frac{b^{2} x^{6} \arctan \left(c \,x^{3}\right)^{2}}{6}-\frac{b^{2} x^{3} \arctan \left(c \,x^{3}\right)}{3 c}+\frac{b^{2} \arctan \left(c \,x^{3}\right)^{2}}{6 c^{2}}+\frac{b^{2} \ln \left(c^{2} x^{6}+1\right)}{6 c^{2}}+\frac{a b \,x^{6} \arctan \left(c \,x^{3}\right)}{3}-\frac{a b \,x^{3}}{3 c}-$
risch	$-\frac{b^{2} \left(c^{2} x^{6}+1\right) \ln \left(i c \,x^{3}+1\right)^{2}}{24 c^{2}}-\frac{i b \left(4 a^{2} c^{2} x^{6}+2 i x^{6} b \ln \left(-i c \,x^{3}+1\right) a \,c^{2}-4 a b c \,x^{3}+b^{2}+2 i b \ln \left(-i c \,x^{3}+1\right) a \right) \ln \left(i c \,x^{3}+1\right)}{24 a \,c^{2}}+$

input
```
int(x^5*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(b^2*arctan(c*x^3)^2*x^6*c^2+2*a*b*arctan(c*x^3)*x^6*c^2+a^2*c^2*x^6-2
*b^2*arctan(c*x^3)*x^3*c-2*a*b*c*x^3+b^2*arctan(c*x^3)^2+b^2*ln(c^2*x^6+1)
+2*a*b*arctan(c*x^3))/c^2
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int x^5 \left(a + b \arctan\left(cx^3\right)\right)^2 \, dx$$
$$= \frac{a^2c^2x^6 - 2\,abcx^3 + (b^2c^2x^6 + b^2)\arctan\left(cx^3\right)^2 + b^2\log\left(c^2x^6 + 1\right) + 2\left(abc^2x^6 - b^2cx^3 + ab\right)\arctan\left(cx^3\right)}{6\,c^2}$$

input
```
integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

output
```
1/6*(a^2*c^2*x^6 - 2*a*b*c*x^3 + (b^2*c^2*x^6 + b^2)*arctan(c*x^3)^2 + b^2
*log(c^2*x^6 + 1) + 2*(a*b*c^2*x^6 - b^2*c*x^3 + a*b)*arctan(c*x^3))/c^2
```

3.115.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(78) = 156$.

Time = 50.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.16

$$\int x^5 \left(a + b \arctan\left(cx^3\right)\right)^2 \, dx$$
$$= \begin{cases} \frac{a^2x^6}{6} + \frac{abx^6\operatorname{atan}\left(cx^3\right)}{3} - \frac{abx^3}{3c} + \frac{ab\operatorname{atan}\left(cx^3\right)}{3c^2} + \frac{b^2x^6\operatorname{atan}^2\left(cx^3\right)}{6} - \frac{b^2x^3\operatorname{atan}\left(cx^3\right)}{3c} - \frac{b^2\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^3\right)}{3c} + \frac{b^2\log\left(x - \sqrt[6]{-\cdots}\right.}{3c^2} \\ \frac{a^2x^6}{6} \end{cases}$$

input
```
integrate(x**5*(a+b*atan(c*x**3))**2,x)
```

output
```
Piecewise((a**2*x**6/6 + a*b*x**6*atan(c*x**3)/3 - a*b*x**3/(3*c) + a*b*at
an(c*x**3)/(3*c**2) + b**2*x**6*atan(c*x**3)**2/6 - b**2*x**3*atan(c*x**3)
/(3*c) - b**2*sqrt(-1/c**2)*atan(c*x**3)/(3*c) + b**2*log(x - (-1/c**2)**(
1/6))/(3*c**2) + b**2*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/
3))/(3*c**2) + b**2*atan(c*x**3)**2/(6*c**2), Ne(c, 0)), (a**2*x**6/6, Tru
e))
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.40

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\frac{1}{6}\,b^2x^6\arctan\left(cx^3\right)^2+\frac{1}{6}\,a^2x^6+\frac{1}{3}\left(x^6\arctan\left(cx^3\right)-c\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)\right)ab$$
$$-\frac{1}{6}\left(2\,c\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)\arctan\left(cx^3\right)+\frac{\arctan\left(cx^3\right)^2-\log\left(6\,c^5x^6+6\,c^3\right)}{c^2}\right)b^2$$

input
```
integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6*arctan(c*x^3)^2 + 1/6*a^2*x^6 + 1/3*(x^6*arctan(c*x^3) - c*(x^
3/c^2 - arctan(c*x^3)/c^3))*a*b - 1/6*(2*c*(x^3/c^2 - arctan(c*x^3)/c^3)*a
rctan(c*x^3) + (arctan(c*x^3)^2 - log(6*c^5*x^6 + 6*c^3))/c^2)*b^2
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\frac{a^2cx^6+\frac{2\left(c^2x^6\arctan\left(cx^3\right)-cx^3+\arctan\left(cx^3\right)\right)ab}{c}+\frac{\left(c^2x^6\arctan\left(cx^3\right)^2-2\,cx^3\arctan\left(cx^3\right)+\arctan\left(cx^3\right)^2+\log\left(c^2x^6+1\right)\right)b^2}{c}}{6\,c}$$

input
```
integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

output
```
1/6*(a^2*c*x^6 + 2*(c^2*x^6*arctan(c*x^3) - c*x^3 + arctan(c*x^3))*a*b/c +
 (c^2*x^6*arctan(c*x^3)^2 - 2*c*x^3*arctan(c*x^3) + arctan(c*x^3)^2 + log(
c^2*x^6 + 1))*b^2/c)/c
```

3.115.9 Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.24

$$\int x^5 (a+b\arctan(cx^3))^2\,dx = \frac{a^2x^6}{6}+\frac{b^2\,\mathrm{atan}(c\,x^3)^2}{6\,c^2}+\frac{b^2\,x^6\,\mathrm{atan}(c\,x^3)^2}{6}+\frac{b^2\,\ln(c^2\,x^6+1)}{6\,c^2}-\frac{b^2\,x^3\,\mathrm{atan}(c\,x^3)}{3\,c}-\frac{a\,b\,x^3}{3\,c}+\frac{a\,b\,\mathrm{atan}(c\,x^3)}{3\,c^2}+\frac{a\,b\,x^6\,\mathrm{atan}(c\,x^3)}{3}$$

input
```
int(x^5*(a + b*atan(c*x^3))^2,x)
```

output
```
(a^2*x^6)/6 + (b^2*atan(c*x^3)^2)/(6*c^2) + (b^2*x^6*atan(c*x^3)^2)/6 + (b
^2*log(c^2*x^6 + 1))/(6*c^2) - (b^2*x^3*atan(c*x^3))/(3*c) - (a*b*x^3)/(3*
c) + (a*b*atan(c*x^3))/(3*c^2) + (a*b*x^6*atan(c*x^3))/3
```

3.116 $\int x^2(a+b\arctan(cx^3))^2\,dx$

3.116.1 Optimal result 767
3.116.2 Mathematica [A] (verified) 767
3.116.3 Rubi [A] (verified) 768
3.116.4 Maple [A] (verified) 770
3.116.5 Fricas [F] 771
3.116.6 Sympy [F] 771
3.116.7 Maxima [F] 771
3.116.8 Giac [F] 772
3.116.9 Mupad [F(-1)] 772

3.116.1 Optimal result

Integrand size = 16, antiderivative size = 104

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{i(a+b\arctan(cx^3))^2}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^2+\frac{2b(a+b\arctan(cx^3))\log\left(\frac{2}{1+icx^3}\right)}{3c}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{3c}$$

output
```
1/3*I*(a+b*arctan(c*x^3))^2/c+1/3*x^3*(a+b*arctan(c*x^3))^2+2/3*b*(a+b*arc
tan(c*x^3))*ln(2/(1+I*c*x^3))/c+1/3*I*b^2*polylog(2,1-2/(1+I*c*x^3))/c
```

3.116.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{b^2(-i+cx^3)\arctan\left(cx^3\right)^2+2b\arctan\left(cx^3\right)\left(acx^3+b\log\left(1+e^{2i\arctan(cx^3)}\right)\right)+a(acx^3-b\log(1+c^2x^6}{3c}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
(b^2*(-I + c*x^3)*ArcTan[c*x^3]^2 + 2*b*ArcTan[c*x^3]*(a*c*x^3 + b*Log[1 +
 E^((2*I)*ArcTan[c*x^3])]) + a*(a*c*x^3 - b*Log[1 + c^2*x^6]) - I*b^2*Poly
Log[2, -E^((2*I)*ArcTan[c*x^3])])/(3*c)
```

3.116.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5363, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$

$$\Big\downarrow \ 5363$$

$$\frac{1}{3}\int \left(a+b\arctan\left(cx^3\right)\right)^2dx^3$$

$$\Big\downarrow \ 5345$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\int \frac{x^3\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3\right)$$

$$\Big\downarrow \ 5455$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{\int \frac{a+b\arctan(cx^3)}{i-cx^3}dx^3}{c}-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \ 5379$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx^3}\right)(a+b\arctan(cx^3))}{c}-b\int \frac{\log\left(\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3}{c}-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \ 2849$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{-\frac{ib\int \frac{\log\left(\frac{2}{icx^3+1}\right)}{1-\frac{2}{icx^3+1}}d\frac{1}{icx^3+1}}{c}+\frac{\log\left(\frac{2}{1+icx^3}\right)(a+b\arctan(cx^3))}{c}}{c}-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{2bc^2}\right)\right)$$

$$\downarrow \text{2752}$$

$$\frac{1}{3}\left(x^3(a+b\arctan(cx^3))^2-2bc\left(-\frac{i(a+b\arctan(cx^3))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx^3}\right)(a+b\arctan(cx^3))}{c}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)}{2c}}{c}\right)\right)$$

input
```
Int[x^2*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
(x^3*(a + b*ArcTan[c*x^3])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^3])^2)/(
b*c^2) - (((a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/c + ((I/2)*b*PolyLo
g[2, 1 - 2/(1 + I*c*x^3)])/c)/c))/3
```

3.116.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.116.4 Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.35

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\arctan(cx^3)^2(cx^3+i)+2\arctan(cx^3)\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)-2i\arctan(cx^3)^2-i\operatorname{polylog}\left(2,-\frac{(icx^3+1)^2}{c^2x^6+1}\right)\right.}{3c}$
derivativedivides	$\frac{cx^3a^2-i\arctan(cx^3)^2b^2+\arctan(cx^3)^2b^2cx^3-i\operatorname{polylog}\left(2,-\frac{(icx^3+1)^2}{c^2x^6+1}\right)b^2+2\arctan(cx^3)\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)b^2+2}{3c}$
default	$\frac{cx^3a^2-i\arctan(cx^3)^2b^2+\arctan(cx^3)^2b^2cx^3-i\operatorname{polylog}\left(2,-\frac{(icx^3+1)^2}{c^2x^6+1}\right)b^2+2\arctan(cx^3)\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)b^2+2}{3c}$
risch	$\frac{b^2\ln(icx^3+1)\ln(-icx^3+1)x^3}{6}-\frac{ba\ln(icx^3+1)}{3c}-\frac{i\ln(-icx^3+1)^2b^2}{12c}+\frac{i\ln(-icx^3+1)abx^3}{3}+\frac{ib^2\ln(c^2x^6+1)}{6c}$

input
```
int(x^2*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+1/3*b^2/c*(arctan(c*x^3)^2*(c*x^3+I)+2*arctan(c*x^3)*ln(1+(1+I
*c*x^3)^2/(c^2*x^6+1))-2*I*arctan(c*x^3)^2-I*polylog(2,-(1+I*c*x^3)^2/(c^2
*x^6+1)))+2/3*a*b*arctan(c*x^3)*x^3-1/3/c*a*b*ln(c^2*x^6+1)
```

3.116.5 Fricas [F]

$$\int x^2 \left(a+b \arctan \left(c x^3\right)\right)^2 \, dx=\int \left(b \arctan \left(c x^3\right)+a\right)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctan(c*x^3)^2 + 2*a*b*x^2*arctan(c*x^3) + a^2*x^2, x)
```

3.116.6 Sympy [F]

$$\int x^2 \left(a+b \arctan \left(c x^3\right)\right)^2 \, dx=\int x^2 \left(a+b \operatorname{atan}\left(c x^3\right)\right)^2 \, dx$$

input
```
integrate(x**2*(a+b*atan(c*x**3))**2,x)
```

output
```
Integral(x**2*(a + b*atan(c*x**3))**2, x)
```

3.116.7 Maxima [F]

$$\int x^2 \left(a+b \arctan \left(c x^3\right)\right)^2 \, dx=\int \left(b \arctan \left(c x^3\right)+a\right)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/48*(4*x^3*arctan(c*x^3)^2 - x^3*log(c^2*x^6 + 1)^2 + 576*c
^2*integrate(1/16*x^8*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 48*c^2*integrate
(1/16*x^8*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 192*c^2*integrate(1/16*x^
8*log(c^2*x^6 + 1)/(c^2*x^6 + 1), x) + 4*arctan(c*x^3)^3/c - 384*c*integra
te(1/16*x^5*arctan(c*x^3)/(c^2*x^6 + 1), x) + 48*integrate(1/16*x^2*log(c^
2*x^6 + 1)^2/(c^2*x^6 + 1), x))*b^2 + 1/3*(2*c*x^3*arctan(c*x^3) - log(c^2
*x^6 + 1))*a*b/c
```

3.116.8 Giac [F]

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int\left(b\arctan\left(cx^3\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2*x^2, x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int x^2\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^2\,dx$$

input
```
int(x^2*(a + b*atan(c*x^3))^2,x)
```

output
```
int(x^2*(a + b*atan(c*x^3))^2, x)
```

3.117 $\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx$

3.117.1 Optimal result . . . 773
3.117.2 Mathematica [A] (verified) . . . 774
3.117.3 Rubi [A] (verified) . . . 774
3.117.4 Maple [F] . . . 776
3.117.5 Fricas [F] . . . 777
3.117.6 Sympy [F] . . . 777
3.117.7 Maxima [F] . . . 777
3.117.8 Giac [F] . . . 778
3.117.9 Mupad [F(-1)] . . . 778

3.117.1 Optimal result

Integrand size = 16, antiderivative size = 154

$$\begin{aligned}\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx =& \frac{2}{3}(a+b\arctan(cx^3))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx^3}\right)\\ &-\frac{1}{3}ib(a+b\arctan(cx^3))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)\\ &+\frac{1}{3}ib(a+b\arctan(cx^3))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right)\\ &-\frac{1}{6}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)\\ &+\frac{1}{6}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^3}\right)\end{aligned}$$

output
```
-2/3*(a+b*arctan(c*x^3))^2*arctanh(-1+2/(1+I*c*x^3))-1/3*I*b*(a+b*arctan(c
*x^3))*polylog(2,1-2/(1+I*c*x^3))+1/3*I*b*(a+b*arctan(c*x^3))*polylog(2,-1
+2/(1+I*c*x^3))-1/6*b^2*polylog(3,1-2/(1+I*c*x^3))+1/6*b^2*polylog(3,-1+2/
(1+I*c*x^3))
```

3.117.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx = a^2\log(x)+\frac{1}{3}iab\left(\text{PolyLog}\left(2,-icx^3\right)-\text{PolyLog}\left(2,icx^3\right)\right)+\frac{1}{72}b^2\Big(-i\pi^3+16i\arctan\left(cx^3\right)^3+24\arctan\left(cx^3\right)^2\log\left(1-e^{-2i\arctan(cx^3)}\right)-24\arctan\left(cx^3\right)^2\log\left(1+e^{2i\arctan(cx^3)}\right)+24i\arctan\left(cx^3\right)\text{PolyLog}\left(2,e^{-2i\arctan(cx^3)}\right)+24i\arctan\left(cx^3\right)\text{PolyLog}\left(2,-e^{2i\arctan(cx^3)}\right)+12\,\text{PolyLog}\left(3,e^{-2i\arctan(cx^3)}\right)-12\,\text{PolyLog}\left(3,-e^{2i\arctan(cx^3)}\right)\Big)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^2/x,x]
```

output
```
a^2*Log[x] + (I/3)*a*b*(PolyLog[2, (-I)*c*x^3] - PolyLog[2, I*c*x^3]) + (b
^2*((-I)*Pi^3 + (16*I)*ArcTan[c*x^3]^3 + 24*ArcTan[c*x^3]^2*Log[1 - E^((-2
*I)*ArcTan[c*x^3])] - 24*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan[c*x^3])]
+ (24*I)*ArcTan[c*x^3]*PolyLog[2, E^((-2*I)*ArcTan[c*x^3])] + (24*I)*ArcTa
n[c*x^3]*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])] + 12*PolyLog[3, E^((-2*I)*Ar
cTan[c*x^3])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x^3])]))/72
```

3.117.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5359, 5357, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{1}{3}\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^3}dx^3$$

$$\Big\downarrow \; 5357$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2-4bc\int \frac{\left(a+b\arctan\left(cx^3\right)\right)\text{arctanh}\left(1-\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3\right)$$

$$\Big\downarrow \; 5523$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2-4bc\left(\frac{1}{2}\int \frac{\left(a+b\arctan\left(cx^3\right)\right)\log\left(2-\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3-\frac{1}{2}\int\right.\right.$$

$$\Big\downarrow \; 5529$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)\left(a+b\arctan\left(cx^3\right)\right)}{2c}-\frac{1}{2}\right.\right.\right.$$

$$\Big\downarrow \; 7164$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)\left(a+b\arctan\left(cx^3\right)\right)}{2c}+\frac{b}{}\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^3])^2/x,x]
```

output
```
(2*(a + b*ArcTan[c*x^3])^2*ArcTanh[1 - 2/(1 + I*c*x^3)] - 4*b*c*((((I/2)*(
a + b*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c + (b*PolyLog[3, 1
- 2/(1 + I*c*x^3)])/(4*c))/2 + (((-1/2*I)*(a + b*ArcTan[c*x^3])*PolyLog[2,
 -1 + 2/(1 + I*c*x^3)])/c - (b*PolyLog[3, -1 + 2/(1 + I*c*x^3)])/(4*c))/2)
)/3
```

3.117. $\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x}\,dx$

3.117.3.1 Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.117.4 Maple [F]

$$\int \frac{(a + b \arctan(c x^3))^2}{x} dx$$

input
```
int((a+b*arctan(c*x^3))^2/x,x)
```

output
```
int((a+b*arctan(c*x^3))^2/x,x)
```

3.117.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x, x)
```

3.117.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx = \int \frac{(a+b\operatorname{atan}(cx^3))^2}{x}\,dx$$

input
```
integrate((a+b*atan(c*x**3))**2/x,x)
```

output
```
Integral((a + b*atan(c*x**3))**2/x, x)
```

3.117.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x^3)^2 + b^2*log(c^2*x^6 + 1)
^2 + 32*a*b*arctan(c*x^3))/x, x)
```

3.117.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx=\int \frac{(b\arctan(cx^3)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2/x, x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x^3))^2}{x}\,dx$$

input
```
int((a + b*atan(c*x^3))^2/x,x)
```

output
```
int((a + b*atan(c*x^3))^2/x, x)
```

3.118 $\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx$

3.118.1 Optimal result . . . 779
3.118.2 Mathematica [A] (verified) . . . 779
3.118.3 Rubi [A] (verified) . . . 780
3.118.4 Maple [C] (warning: unable to verify) . . . 782
3.118.5 Fricas [F] . . . 782
3.118.6 Sympy [F] . . . 782
3.118.7 Maxima [F] . . . 783
3.118.8 Giac [F] . . . 783
3.118.9 Mupad [F(-1)] . . . 783

3.118.1 Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx = -\frac{1}{3}ic(a+b\arctan(cx^3))^2 - \frac{(a+b\arctan(cx^3))^2}{3x^3} + \frac{2}{3}bc(a+b\arctan(cx^3))\log\left(2-\frac{2}{1-icx^3}\right) - \frac{1}{3}ib^2c\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)$$

output
```
-1/3*I*c*(a+b*arctan(c*x^3))^2-1/3*(a+b*arctan(c*x^3))^2/x^3+2/3*b*c*(a+b*
arctan(c*x^3))*ln(2-2/(1-I*c*x^3))-1/3*I*b^2*c*polylog(2,-1+2/(1-I*c*x^3))
```

3.118.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx = \frac{b^2(-1-icx^3)\arctan(cx^3)^2 + 2b\arctan(cx^3)\left(-a+bcx^3\log\left(1-e^{2i\arctan(cx^3)}\right)\right) - a(a-2bcx^3\log(cx^3)}{3x^3}$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^2/x^4,x]
```

output
```
(b^2*(-1 - I*c*x^3)*ArcTan[c*x^3]^2 + 2*b*ArcTan[c*x^3]*(-a + b*c*x^3*Log[
1 - E^((2*I)*ArcTan[c*x^3])]) - a*(a - 2*b*c*x^3*Log[c*x^3] + b*c*x^3*Log[
1 + c^2*x^6]) - I*b^2*c*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/(3*x^3)
```

3.118.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5363, 5361, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3}\int \frac{(a+b\arctan(cx^3))^2}{x^6}dx^3$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(2bc\int \frac{a+b\arctan(cx^3)}{x^3(c^2x^6+1)}dx^3-\frac{(a+b\arctan(cx^3))^2}{x^3}\right)$$

$$\Big\downarrow \text{5459}$$

$$\frac{1}{3}\left(-\frac{(a+b\arctan(cx^3))^2}{x^3}+2bc\left(i\int \frac{a+b\arctan(cx^3)}{x^3(cx^3+i)}dx^3-\frac{i(a+b\arctan(cx^3))^2}{2b}\right)\right)$$

$$\Big\downarrow \text{5403}$$

$$\frac{1}{3}\left(-\frac{(a+b\arctan(cx^3))^2}{x^3}+2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx^3}\right)}{c^2x^6+1}dx^3-i\log\left(2-\frac{2}{1-icx^3}\right)(a+b\arctan(cx^3))\right)-\right.\right.$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{3}\left(-\frac{(a+b\arctan(cx^3))^2}{x^3}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx^3}\right)(a+b\arctan(cx^3))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx^3}-1\right)\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^3])^2/x^4,x]
```

output
```
(-((a + b*ArcTan[c*x^3])^2/x^3) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^3])^2
)/b + I*((-I)*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)] - (b*PolyLog[
2, -1 + 2/(1 - I*c*x^3)])/2)))/3
```

3.118.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.118. $\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx$

3.118.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.78 (sec) , antiderivative size = 11455, normalized size of antiderivative = 114.55

method	result	size
default	Expression too large to display	11455
parts	Expression too large to display	11455

input
```
int((a+b*arctan(c*x^3))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.118.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx=\int \frac{(b\arctan(cx^3)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x^4, x)
```

3.118.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx=\int \frac{(a+b\,\mathrm{atan}(cx^3))^2}{x^4}\,dx$$

input
```
integrate((a+b*atan(c*x**3))**2/x**4,x)
```

output
```
Integral((a + b*atan(c*x**3))**2/x**4, x)
```

3.118.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*a*b + 1/48*(4
8*x^3*integrate(-1/16*(4*c^2*x^6*log(c^2*x^6 + 1) - 8*c*x^3*arctan(c*x^3)
- 12*(c^2*x^6 + 1)*arctan(c*x^3)^2 - (c^2*x^6 + 1)*log(c^2*x^6 + 1)^2)/(c^
2*x^10 + x^4), x) - 4*arctan(c*x^3)^2 + log(c^2*x^6 + 1)^2)*b^2/x^3 - 1/3*
a^2/x^3
```

3.118.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2/x^4, x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx = \int \frac{(a+b\operatorname{atan}(c\,x^3))^2}{x^4}\,dx$$

input
```
int((a + b*atan(c*x^3))^2/x^4,x)
```

output
```
int((a + b*atan(c*x^3))^2/x^4, x)
```

3.119 $\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx$

3.119.1 Optimal result . . . 784
3.119.2 Mathematica [A] (verified) . . . 784
3.119.3 Rubi [A] (verified) . . . 785
3.119.4 Maple [A] (verified) . . . 787
3.119.5 Fricas [A] (verification not implemented) . . . 788
3.119.6 Sympy [B] (verification not implemented) . . . 788
3.119.7 Maxima [A] (verification not implemented) . . . 789
3.119.8 Giac [F] . . . 789
3.119.9 Mupad [B] (verification not implemented) . . . 790

3.119.1 Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = -\frac{bc(a+b\arctan(cx^3))}{3x^3}-\frac{1}{6}c^2(a+b\arctan(cx^3))^2 -\frac{(a+b\arctan(cx^3))^2}{6x^6}+b^2c^2\log(x)-\frac{1}{6}b^2c^2\log(1+c^2x^6)$$

output
```
-1/3*b*c*(a+b*arctan(c*x^3))/x^3-1/6*c^2*(a+b*arctan(c*x^3))^2-1/6*(a+b*ar
ctan(c*x^3))^2/x^6+b^2*c^2*ln(x)-1/6*b^2*c^2*ln(c^2*x^6+1)
```

3.119.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = -\frac{a^2+2abcx^3+2b(a+bcx^3+ac^2x^6)\arctan(cx^3)+b^2(1+c^2x^6)\arctan(cx^3)^2-6b^2c^2x^6\log(x)+b^2c^2x^6}{6x^6}$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^2/x^7,x]
```

output
```
-1/6*(a^2 + 2*a*b*c*x^3 + 2*b*(a + b*c*x^3 + a*c^2*x^6)*ArcTan[c*x^3] + b^
2*(1 + c^2*x^6)*ArcTan[c*x^3]^2 - 6*b^2*c^2*x^6*Log[x] + b^2*c^2*x^6*Log[1
 + c^2*x^6])/x^6
```

3.119.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5363, 5361, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3}\int \frac{(a+b\arctan(cx^3))^2}{x^9}dx^3$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(bc\int \frac{a+b\arctan(cx^3)}{x^6(c^2x^6+1)}dx^3-\frac{(a+b\arctan(cx^3))^2}{2x^6}\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{3}\left(bc\left(\int \frac{a+b\arctan(cx^3)}{x^6}dx^3-c^2\int \frac{a+b\arctan(cx^3)}{c^2x^6+1}dx^3\right)-\frac{(a+b\arctan(cx^3))^2}{2x^6}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx^3)}{c^2x^6+1}dx^3\right)+bc\int \frac{1}{x^3(c^2x^6+1)}dx^3-\frac{a+b\arctan(cx^3)}{x^3}\right)-\frac{(a+b\arctan(cx^3))^2}{2x^6}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx^3)}{c^2x^6+1}dx^3\right)+\frac{1}{2}bc\int \frac{1}{x^3(c^2x^6+1)}dx^6-\frac{a+b\arctan(cx^3)}{x^3}\right)-\frac{(a+b\arctan(cx^3))}{2x^6}\right.$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{3}\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx^3)}{c^2x^6+1}dx^3\right)+\frac{1}{2}bc\left(\int \frac{1}{x^3}dx^6-c^2\int \frac{1}{c^2x^6+1}dx^6\right)-\frac{a+b\arctan(cx^3)}{x^3}\right)-(a+b\right.$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{3}\left(bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^3\right)}{c^2x^6+1}dx^3\right)+\frac{1}{2}bc\left(\log\left(x^6\right)-c^2\int \frac{1}{c^2x^6+1}dx^6\right)-\frac{a+b\arctan\left(cx^3\right)}{x^3}\right)-\frac{(a+b\arctan$$

$$\Big\downarrow 16$$

$$\frac{1}{3}\left(bc\left(c^2\left(-\int \frac{a+b\arctan\left(cx^3\right)}{c^2x^6+1}dx^3\right)-\frac{a+b\arctan\left(cx^3\right)}{x^3}+\frac{1}{2}bc\left(\log\left(x^6\right)-\log\left(c^2x^6+1\right)\right)\right)-\frac{(a+b\arctan}{2x^6}\right.$$

$$\Big\downarrow 5419$$

$$\frac{1}{3}\left(bc\left(-\frac{c\left(a+b\arctan\left(cx^3\right)\right)^2}{2b}-\frac{a+b\arctan\left(cx^3\right)}{x^3}+\frac{1}{2}bc\left(\log\left(x^6\right)-\log\left(c^2x^6+1\right)\right)\right)-\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{2x^6}\right.$$

input
```
Int[(a + b*ArcTan[c*x^3])^2/x^7,x]
```

output
```
(-1/2*(a + b*ArcTan[c*x^3])^2/x^6 + b*c*(-((a + b*ArcTan[c*x^3])/x^3) - (c
*(a + b*ArcTan[c*x^3])^2)/(2*b) + (b*c*(Log[x^6] - Log[1 + c^2*x^6]))/2))/
3
```

3.119.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.119.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

method	result
default	$-\frac{a^2}{6x^6}-\frac{b^2\arctan(cx^3)^2}{6x^6}-\frac{b^2\arctan(cx^3)^2c^2}{6}-\frac{b^2c\arctan(cx^3)}{3x^3}+b^2c^2\ln(x)-\frac{b^2c^2\ln(c^2x^6+1)}{6}-\frac{ab\arctan(}{3x^6}$
parts	$-\frac{a^2}{6x^6}-\frac{b^2\arctan(cx^3)^2}{6x^6}-\frac{b^2\arctan(cx^3)^2c^2}{6}-\frac{b^2c\arctan(cx^3)}{3x^3}+b^2c^2\ln(x)-\frac{b^2c^2\ln(c^2x^6+1)}{6}-\frac{ab\arctan(}{3x^6}$
parallelrisch	$\frac{-b^2\arctan(cx^3)^2x^6c^2+6b^2c^2\ln(x)x^6-b^2c^2\ln(c^2x^6+1)x^6-2ab\arctan(cx^3)x^6c^2+a^2c^2x^6-2b^2\arctan(cx^3)x^3c-2abcx^3-b^2}{6x^6}$
risch	$\frac{b^2(c^2x^6+1)\ln(icx^3+1)^2}{24x^6}+\frac{ib(ibc^2x^6\ln(-icx^3+1)+2bcx^3+2a+ib\ln(-icx^3+1))\ln(icx^3+1)}{12x^6}-\frac{4i\ln((-7ibc+ac)x^3+7b}{}$

input
```
int((a+b*arctan(c*x^3))^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2/x^6-1/6*b^2/x^6*arctan(c*x^3)^2-1/6*b^2*arctan(c*x^3)^2*c^2-1/3*b
^2*c*arctan(c*x^3)/x^3+b^2*c^2*ln(x)-1/6*b^2*c^2*ln(c^2*x^6+1)-1/3*a*b/x^6
*arctan(c*x^3)-1/3*a*b*arctan(c*x^3)*c^2-1/3*a*b*c/x^3
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = -\frac{b^2c^2x^6\log(c^2x^6+1) - 6\,b^2c^2x^6\log(x) + 2\,abcx^3 + (b^2c^2x^6+b^2)\arctan(cx^3)^2 + a^2 + 2\,(abc^2x^6+b^2cx^3}{6\,x^6}$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="fricas")
```

output
```
-1/6*(b^2*c^2*x^6*log(c^2*x^6 + 1) - 6*b^2*c^2*x^6*log(x) + 2*a*b*c*x^3 +
(b^2*c^2*x^6 + b^2)*arctan(c*x^3)^2 + a^2 + 2*(a*b*c^2*x^6 + b^2*c*x^3 + a
*b)*arctan(c*x^3))/x^6
```

3.119.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(80) = 160.

Time = 71.42 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.38

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = \begin{cases} -\frac{a^2}{6x^6} - \frac{abc^2\operatorname{atan}(cx^3)}{3} - \frac{abc}{3x^3} - \frac{ab\operatorname{atan}(cx^3)}{3x^6} + \frac{b^2c^3\sqrt{-\frac{1}{c^2}}\operatorname{atan}(cx^3)}{3} + b^2c^2\log(x) - \frac{b^2c^2\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{3} - \frac{b^2c^2\log\Big(}{} \\ -\frac{a^2}{6x^6} \end{cases}$$

input
```
integrate((a+b*atan(c*x**3))**2/x**7,x)
```

output
```
Piecewise((-a**2/(6*x**6) - a*b*c**2*atan(c*x**3)/3 - a*b*c/(3*x**3) - a*b
*atan(c*x**3)/(3*x**6) + b**2*c**3*sqrt(-1/c**2)*atan(c*x**3)/3 + b**2*c**
2*log(x) - b**2*c**2*log(x - (-1/c**2)**(1/6))/3 - b**2*c**2*log(4*x**2 +
4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/3 - b**2*c**2*atan(c*x**3)**2/6
 - b**2*c*atan(c*x**3)/(3*x**3) - b**2*atan(c*x**3)**2/(6*x**6), Ne(c, 0))
, (-a**2/(6*x**6), True))
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = -\frac{1}{3}\left(\left(c\arctan\left(cx^3\right)+\frac{1}{x^3}\right)c+\frac{\arctan\left(cx^3\right)}{x^6}\right)ab + \frac{1}{6}\left(\left(\arctan\left(cx^3\right)^2-\log\left(c^2x^6+1\right)+6\,\log\left(x\right)\right)c^2-2\left(c\arctan\left(cx^3\right)+\frac{1}{x^3}\right)c\arctan\left(cx^3\right)\right)b^2 - \frac{b^2\arctan\left(cx^3\right)^2}{6\,x^6}-\frac{a^2}{6\,x^6}$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="maxima")
```

output
```
-1/3*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*a*b + 1/6*((arctan(
c*x^3)^2 - log(c^2*x^6 + 1) + 6*log(x))*c^2 - 2*(c*arctan(c*x^3) + 1/x^3)*
c*arctan(c*x^3))*b^2 - 1/6*b^2*arctan(c*x^3)^2/x^6 - 1/6*a^2/x^6
```

3.119.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x^7}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2/x^7, x)
```

3.119.9 Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.75

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = b^2c^2\ln(x) - \frac{b^2c^2\operatorname{atan}(c\,x^3)^2}{6} - \frac{b^2\operatorname{atan}(c\,x^3)^2}{6\,x^6} - \frac{b^2c^2\ln(c^2x^6+1)}{6} - \frac{a^2}{6\,x^6} - \frac{b^2c\operatorname{atan}(c\,x^3)}{3\,x^3} - \frac{a\,b\,c}{3\,x^3} - \frac{a\,b\,c^2\operatorname{atan}\left(\frac{a^2c\,x^3}{a^2+49\,b^2}+\frac{49\,b^2c\,x^3}{a^2+49\,b^2}\right)}{3} - \frac{a\,b\operatorname{atan}(c\,x^3)}{3\,x^6}$$

input
```
int((a + b*atan(c*x^3))^2/x^7,x)
```

output
```
b^2*c^2*log(x) - (b^2*c^2*atan(c*x^3)^2)/6 - (b^2*atan(c*x^3)^2)/(6*x^6) -
 (b^2*c^2*log(c^2*x^6 + 1))/6 - a^2/(6*x^6) - (b^2*c*atan(c*x^3))/(3*x^3)
- (a*b*c)/(3*x^3) - (a*b*c^2*atan((a^2*c*x^3)/(a^2 + 49*b^2) + (49*b^2*c*x
^3)/(a^2 + 49*b^2)))/3 - (a*b*atan(c*x^3))/(3*x^6)
```

3.120 $\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx$

3.120.1 Optimal result . . . 791
3.120.2 Mathematica [A] (verified) . . . 792
3.120.3 Rubi [A] (verified) . . . 792
3.120.4 Maple [C] (warning: unable to verify) . . . 795
3.120.5 Fricas [F] . . . 795
3.120.6 Sympy [F] . . . 796
3.120.7 Maxima [F] . . . 796
3.120.8 Giac [F] . . . 796
3.120.9 Mupad [F(-1)] . . . 797

3.120.1 Optimal result

Integrand size = 16, antiderivative size = 154

$$\begin{aligned}\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx =& -\frac{b^2c^2}{9x^3}-\frac{1}{9}b^2c^3\arctan(cx^3)-\frac{bc(a+b\arctan(cx^3))}{9x^6}\\ &+\frac{1}{9}ic^3(a+b\arctan(cx^3))^2-\frac{(a+b\arctan(cx^3))^2}{9x^9}\\ &-\frac{2}{9}bc^3(a+b\arctan(cx^3))\log\left(2-\frac{2}{1-icx^3}\right)\\ &+\frac{1}{9}ib^2c^3\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)\end{aligned}$$

output
```
-1/9*b^2*c^2/x^3-1/9*b^2*c^3*arctan(c*x^3)-1/9*b*c*(a+b*arctan(c*x^3))/x^6
+1/9*I*c^3*(a+b*arctan(c*x^3))^2-1/9*(a+b*arctan(c*x^3))^2/x^9-2/9*b*c^3*(
a+b*arctan(c*x^3))*ln(2-2/(1-I*c*x^3))+1/9*I*b^2*c^3*polylog(2,-1+2/(1-I*c
*x^3))
```

3.120.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx =$$
$$-\frac{a^2+abcx^3+b^2c^2x^6+b^2(1-ic^3x^9)\arctan(cx^3)^2+b\arctan(cx^3)\left(2a+bcx^3+bc^3x^9+2bc^3x^9\log\left(1-\right.\right.}{9x^9}$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^2/x^10,x]
```

output
```
-1/9*(a^2 + a*b*c*x^3 + b^2*c^2*x^6 + b^2*(1 - I*c^3*x^9)*ArcTan[c*x^3]^2
+ b*ArcTan[c*x^3]*(2*a + b*c*x^3 + b*c^3*x^9 + 2*b*c^3*x^9*Log[1 - E^((2*I
)*ArcTan[c*x^3])]) + 2*a*b*c^3*x^9*Log[c*x^3] - a*b*c^3*x^9*Log[1 + c^2*x^
6] - I*b^2*c^3*x^9*PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/x^9
```

3.120.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5363, 5361, 5453, 5361, 264, 216, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx$$
$$\Big\downarrow \text{5363}$$
$$\frac{1}{3}\int \frac{(a+b\arctan(cx^3))^2}{x^{12}}dx^3$$
$$\Big\downarrow \text{5361}$$
$$\frac{1}{3}\left(\frac{2}{3}bc\int \frac{a+b\arctan(cx^3)}{x^9(c^2x^6+1)}dx^3-\frac{(a+b\arctan(cx^3))^2}{3x^9}\right)$$
$$\Big\downarrow \text{5453}$$
$$\frac{1}{3}\left(\frac{2}{3}bc\left(\int \frac{a+b\arctan(cx^3)}{x^9}dx^3-c^2\int \frac{a+b\arctan(cx^3)}{x^3(c^2x^6+1)}dx^3\right)-\frac{(a+b\arctan(cx^3))^2}{3x^9}\right)$$

$\Big\downarrow$ 5361

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx^3)}{x^3(c^2x^6+1)}dx^3\right)+\frac{1}{2}bc\int \frac{1}{x^6(c^2x^6+1)}dx^3-\frac{a+b\arctan(cx^3)}{2x^6}\right)-\frac{(a+b\arctan(cx^3)}{3x^9}\right.$$

$\Big\downarrow$ 264

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx^3)}{x^3(c^2x^6+1)}dx^3\right)+\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{c^2x^6+1}dx^3\right)-\frac{1}{x^3}\right)-\frac{a+b\arctan(cx^3)}{2x^6}\right)-\frac{(a+b}{}\right.$$

$\Big\downarrow$ 216

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx^3)}{x^3(c^2x^6+1)}dx^3\right)-\frac{a+b\arctan(cx^3)}{2x^6}+\frac{1}{2}bc\left(-c\arctan(cx^3)-\frac{1}{x^3}\right)\right)-\frac{(a+b\arctan}{3x^9}\right.$$

$\Big\downarrow$ 5459

$$\frac{1}{3}\left(-\frac{(a+b\arctan(cx^3))^2}{3x^9}+\frac{2}{3}bc\left(-\left(c^2\left(i\int \frac{a+b\arctan(cx^3)}{x^3(cx^3+i)}dx^3-\frac{i(a+b\arctan(cx^3))^2}{2b}\right)\right)-\frac{a+b\arctan}{2x^6}\right.\right.$$

$\Big\downarrow$ 5403

$$\frac{1}{3}\left(-\frac{(a+b\arctan(cx^3))^2}{3x^9}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx^3}\right)}{c^2x^6+1}dx^3-i\log\left(2-\frac{2}{1-icx^3}\right)(a+b\arctan(c\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 2897

$$\frac{1}{3}\left(-\frac{(a+b\arctan(cx^3))^2}{3x^9}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-icx^3}\right)(a+b\arctan(cx^3))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-ic}\right.\right.\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^3])^2/x^10,x]
```

output
```
(-1/3*(a + b*ArcTan[c*x^3])^2/x^9 + (2*b*c*(-1/2*(a + b*ArcTan[c*x^3])/x^6
 + (b*c*(-x^(-3) - c*ArcTan[c*x^3]))/2 - c^2*(((-1/2*I)*(a + b*ArcTan[c*x^
3])^2)/b + I*((-I)*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)] - (b*Pol
yLog[2, -1 + 2/(1 - I*c*x^3)])/2))))/3)/3
```

3.120. $\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx$

3.120.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.120. $\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx$

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.120.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.34 (sec) , antiderivative size = 11496, normalized size of antiderivative = 74.65

method	result	size
default	Expression too large to display	11496
parts	Expression too large to display	11496

input
```
int((a+b*arctan(c*x^3))^2/x^10,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.120.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx=\int \frac{(b\arctan(cx^3)+a)^2}{x^{10}}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x^10, x)
```

3.120.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(a+b\operatorname{atan}(cx^3))^2}{x^{10}}\,dx$$

input
```
integrate((a+b*atan(c*x**3))**2/x**10,x)
```

output
```
Integral((a + b*atan(c*x**3))**2/x**10, x)
```

3.120.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x^{10}}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="maxima")
```

output
```
1/9*((c^2*log(c^2*x^6 + 1) - c^2*log(x^6) - 1/x^6)*c - 2*arctan(c*x^3)/x^9
)*a*b + 1/144*(144*x^9*integrate(-1/48*(4*c^2*x^6*log(c^2*x^6 + 1) - 8*c*x
^3*arctan(c*x^3) - 36*(c^2*x^6 + 1)*arctan(c*x^3)^2 - 3*(c^2*x^6 + 1)*log(
c^2*x^6 + 1)^2)/(c^2*x^16 + x^10), x) - 4*arctan(c*x^3)^2 + log(c^2*x^6 +
1)^2)*b^2/x^9 - 1/9*a^2/x^9
```

3.120.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x^{10}}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2/x^10, x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^3))^2}{x^{10}}\,dx$$

input
```
int((a + b*atan(c*x^3))^2/x^10,x)
```

output
```
int((a + b*atan(c*x^3))^2/x^10, x)
```

3.121 $\int x^8(a+b\arctan(cx^3))^3\,dx$

3.121.1 Optimal result . . . 798
3.121.2 Mathematica [A] (verified) . . . 799
3.121.3 Rubi [A] (verified) . . . 799
3.121.4 Maple [F] . . . 803
3.121.5 Fricas [F] . . . 803
3.121.6 Sympy [F] . . . 804
3.121.7 Maxima [F] . . . 804
3.121.8 Giac [F] . . . 804
3.121.9 Mupad [F(-1)] . . . 805

3.121.1 Optimal result

Integrand size = 16, antiderivative size = 240

$$\begin{aligned}\int x^8(a+b\arctan(cx^3))^3\,dx &= \frac{ab^2x^3}{3c^2}+\frac{b^3x^3\arctan(cx^3)}{3c^2}-\frac{b(a+b\arctan(cx^3))^2}{6c^3}\\&\quad-\frac{bx^6(a+b\arctan(cx^3))^2}{6c}-\frac{i(a+b\arctan(cx^3))^3}{9c^3}\\&\quad+\frac{1}{9}x^9(a+b\arctan(cx^3))^3\\&\quad-\frac{b(a+b\arctan(cx^3))^2\log\left(\frac{2}{1+icx^3}\right)}{3c^3}-\frac{b^3\log(1+c^2x^6)}{6c^3}\\&\quad-\frac{ib^2(a+b\arctan(cx^3))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{3c^3}\\&\quad-\frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)}{6c^3}\end{aligned}$$

output
```
1/3*a*b^2*x^3/c^2+1/3*b^3*x^3*arctan(c*x^3)/c^2-1/6*b*(a+b*arctan(c*x^3))^
2/c^3-1/6*b*x^6*(a+b*arctan(c*x^3))^2/c-1/9*I*(a+b*arctan(c*x^3))^3/c^3+1/
9*x^9*(a+b*arctan(c*x^3))^3-1/3*b*(a+b*arctan(c*x^3))^2*ln(2/(1+I*c*x^3))/
c^3-1/6*b^3*ln(c^2*x^6+1)/c^3-1/3*I*b^2*(a+b*arctan(c*x^3))*polylog(2,1-2/
(1+I*c*x^3))/c^3-1/6*b^3*polylog(3,1-2/(1+I*c*x^3))/c^3
```

3.121.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.44

$$\int x^8 \left(a+b\arctan\left(cx^3\right)\right)^3 \, dx$$

$$= \frac{6ab^2cx^3-3a^2bc^2x^6+2a^3c^3x^9-6ab^2\arctan\left(cx^3\right)+6b^3cx^3\arctan\left(cx^3\right)-6ab^2c^2x^6\arctan\left(cx^3\right)+6a^2bc^3}{}$$

input
```
Integrate[x^8*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
(6*a*b^2*c*x^3 - 3*a^2*b*c^2*x^6 + 2*a^3*c^3*x^9 - 6*a*b^2*ArcTan[c*x^3] +
 6*b^3*c*x^3*ArcTan[c*x^3] - 6*a*b^2*c^2*x^6*ArcTan[c*x^3] + 6*a^2*b*c^3*x
^9*ArcTan[c*x^3] + (6*I)*a*b^2*ArcTan[c*x^3]^2 - 3*b^3*ArcTan[c*x^3]^2 - 3
*b^3*c^2*x^6*ArcTan[c*x^3]^2 + 6*a*b^2*c^3*x^9*ArcTan[c*x^3]^2 + (2*I)*b^3
*ArcTan[c*x^3]^3 + 2*b^3*c^3*x^9*ArcTan[c*x^3]^3 - 12*a*b^2*ArcTan[c*x^3]*
Log[1 + E^((2*I)*ArcTan[c*x^3])] - 6*b^3*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*
ArcTan[c*x^3])] + 3*a^2*b*Log[1 + c^2*x^6] - 3*b^3*Log[1 + c^2*x^6] + (6*I
)*b^2*(a + b*ArcTan[c*x^3])*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])] - 3*b^3*P
olyLog[3, -E^((2*I)*ArcTan[c*x^3])])/(18*c^3)
```

3.121.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {5363, 5361, 5451, 5361, 5451, 2009, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+b\arctan\left(cx^3\right)\right)^3 \, dx$$

$$\Big\downarrow \; 5363$$

$$\frac{1}{3}\int x^6 \left(a+b\arctan\left(cx^3\right)\right)^3 dx^3$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\int \frac{x^9\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}dx^3\right)$$

$$\downarrow 5451$$

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\int x^3\left(a+b\arctan\left(cx^3\right)\right)^2dx^3}{c^2}-\frac{\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}dx^3}{c^2}\right)\right)$$

$$\downarrow 5361$$

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\int\frac{x^6\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}-\frac{\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}d}{c^2}\right.\right.$$

$$\downarrow 5451$$

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{\int\left(a+b\arctan\left(cx^3\right)\right)dx^3}{c^2}-\frac{\int\frac{a+b\arctan\left(cx^3\right)}{c^2x^6+1}dx^3}{c^2}\right)}{c^2}-\right.\right.$$

$$\downarrow 2009$$

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{ax^3+bx^3\arctan\left(cx^3\right)-\frac{b\log\left(c^2x^6+1\right)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan\left(cx^3\right)}{c^2x^6+1}}{c^2}\right.}{c^2}\right.\right.$$

$$\downarrow 5419$$

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{ax^3+bx^3\arctan\left(cx^3\right)-\frac{b\log\left(c^2x^6+1\right)}{2c}}{c^2}-\frac{\left(a+b\arctan\left(cx^3\right.\right.}{2bc^3}\right.}{c^2}\right.\right.$$

$$\downarrow 5455$$

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{ax^3+bx^3\arctan\left(cx^3\right)-\frac{b\log\left(c^2x^6+1\right)}{2c}}{c^2}-\frac{\left(a+b\arctan\left(cx^3\right.\right.}{2bc^3}\right.}{c^2}\right.\right.$$

3.121. $\int x^8\left(a+b\arctan\left(cx^3\right)\right)^3\,dx$

$\downarrow$ 5379

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{ax^3+bx^3\arctan\left(cx^3\right)-\frac{b\log\left(c^2x^6+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx^3}{2bc^3}\right.}{c^2}\right.\right.$$

$\downarrow$ 5529

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{ax^3+bx^3\arctan\left(cx^3\right)-\frac{b\log\left(c^2x^6+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx^3}{2bc^3}\right.}{c^2}\right.\right.$$

$\downarrow$ 7164

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-bc\left(\frac{\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-bc\left(\frac{ax^3+bx^3\arctan\left(cx^3\right)-\frac{b\log\left(c^2x^6+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx^3}{2bc^3}\right.}{c^2}\right.\right.$$

input
```
Int[x^8*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
((x^9*(a + b*ArcTan[c*x^3])^3)/3 - b*c*(((x^6*(a + b*ArcTan[c*x^3])^2)/2 -
 b*c*(-1/2*(a + b*ArcTan[c*x^3])^2/(b*c^3) + (a*x^3 + b*x^3*ArcTan[c*x^3]
- (b*Log[1 + c^2*x^6])/(2*c))/c^2))/c^2 - (((-1/3*I)*(a + b*ArcTan[c*x^3])
^3)/(b*c^2) - (((a + b*ArcTan[c*x^3])^2*Log[2/(1 + I*c*x^3)])/c - 2*b*(((-
1/2*I)*(a + b*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c - (b*PolyL
og[3, 1 - 2/(1 + I*c*x^3)])/(4*c)))/c)/c^2))/3
```

3.121. $\int x^8\left(a+b\arctan\left(cx^3\right)\right)^3\,dx$

3.121.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.121. $\int x^8(a+b\arctan(cx^3))^3\,dx$

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.121.4 Maple [F]

$$\int x^8 \left(a+b\arctan\left(c\,x^3\right)\right)^3 dx$$

input
```
int(x^8*(a+b*arctan(c*x^3))^3,x)
```

output
```
int(x^8*(a+b*arctan(c*x^3))^3,x)
```

3.121.5 Fricas [F]

$$\int x^8 \left(a+b\arctan\left(cx^3\right)\right)^3 \, dx=\int \left(b\arctan\left(cx^3\right)+a\right)^3 x^8 \, dx$$

input
```
integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^8*arctan(c*x^3)^3 + 3*a*b^2*x^8*arctan(c*x^3)^2 + 3*a^2*b*x
^8*arctan(c*x^3) + a^3*x^8, x)
```

3.121.6 Sympy [F]

$$\int x^8 \left(a + b \arctan \left(cx^3\right)\right)^3 \, dx = \int x^8 \left(a + b \operatorname{atan} \left(cx^3\right)\right)^3 \, dx$$

input
```
integrate(x**8*(a+b*atan(c*x**3))**3,x)
```

output
```
Integral(x**8*(a + b*atan(c*x**3))**3, x)
```

3.121.7 Maxima [F]

$$\int x^8 \left(a + b \arctan \left(cx^3\right)\right)^3 \, dx = \int \left(b \arctan \left(cx^3\right) + a\right)^3 x^8 \, dx$$

input
```
integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

output
```
1/72*b^3*x^9*arctan(c*x^3)^3 - 1/96*b^3*x^9*arctan(c*x^3)*log(c^2*x^6 + 1)
^2 + 1/9*a^3*x^9 + 1/6*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/
c^4)*c)*a^2*b + integrate(1/32*(4*b^3*c^2*x^14*arctan(c*x^3)*log(c^2*x^6 +
 1) + 28*(b^3*c^2*x^14 + b^3*x^8)*arctan(c*x^3)^3 + 4*(24*a*b^2*c^2*x^14 -
 b^3*c*x^11 + 24*a*b^2*x^8)*arctan(c*x^3)^2 + (b^3*c*x^11 + 3*(b^3*c^2*x^1
4 + b^3*x^8)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^6 + 1), x)
```

3.121.8 Giac [F]

$$\int x^8 \left(a + b \arctan \left(cx^3\right)\right)^3 \, dx = \int \left(b \arctan \left(cx^3\right) + a\right)^3 x^8 \, dx$$

input
```
integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3*x^8, x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int x^8 (a + b \arctan(cx^3))^3 \, dx = \int x^8 \left(a + b \operatorname{atan}(c x^3)\right)^3 dx$$

input
```
int(x^8*(a + b*atan(c*x^3))^3,x)
```

output
```
int(x^8*(a + b*atan(c*x^3))^3, x)
```

3.122 $\int x^5(a+b\arctan(cx^3))^3\,dx$

3.122.1 Optimal result . . . 806
3.122.2 Mathematica [A] (verified) . . . 806
3.122.3 Rubi [A] (verified) . . . 807
3.122.4 Maple [C] (warning: unable to verify) . . . 810
3.122.5 Fricas [F] . . . 811
3.122.6 Sympy [F] . . . 812
3.122.7 Maxima [F] . . . 812
3.122.8 Giac [F] . . . 812
3.122.9 Mupad [F(-1)] . . . 813

3.122.1 Optimal result

Integrand size = 16, antiderivative size = 147

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=-\frac{ib\left(a+b\arctan\left(cx^3\right)\right)^2}{2c^2}-\frac{bx^3\left(a+b\arctan\left(cx^3\right)\right)^2}{2c}+\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{6c^2}+\frac{1}{6}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{b^2\left(a+b\arctan\left(cx^3\right)\right)\log\left(\frac{2}{1+icx^3}\right)}{c^2}-\frac{ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{2c^2}$$

output
```
-1/2*I*b*(a+b*arctan(c*x^3))^2/c^2-1/2*b*x^3*(a+b*arctan(c*x^3))^2/c+1/6*(
a+b*arctan(c*x^3))^3/c^2+1/6*x^6*(a+b*arctan(c*x^3))^3-b^2*(a+b*arctan(c*x
^3))*ln(2/(1+I*c*x^3))/c^2-1/2*I*b^3*polylog(2,1-2/(1+I*c*x^3))/c^2
```

3.122.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^3\,dx$$

$$=\frac{3b^2(a+ac^2x^6+b(i-cx^3))\arctan\left(cx^3\right)^2+b^3(1+c^2x^6)\arctan\left(cx^3\right)^3+3b\arctan\left(cx^3\right)\Big(a(a-2bcx^3+a}{}$$

input
```
Integrate[x^5*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
(3*b^2*(a + a*c^2*x^6 + b*(I - c*x^3))*ArcTan[c*x^3]^2 + b^3*(1 + c^2*x^6)
*ArcTan[c*x^3]^3 + 3*b*ArcTan[c*x^3]*(a*(a - 2*b*c*x^3 + a*c^2*x^6) - 2*b^
2*Log[1 + E^((2*I)*ArcTan[c*x^3])]) + a*(a*c*x^3*(-3*b + a*c*x^3) + 3*b^2*
Log[1 + c^2*x^6]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])])/(6*c^2
)
```

3.122.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5363, 5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b\arctan\left(cx^3\right)\right)^3 dx$$

$$\Big\downarrow \text{ 5363}$$

$$\frac{1}{3}\int x^3 \left(a+b\arctan\left(cx^3\right)\right)^3 dx^3$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\int \frac{x^6\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}dx^3\right)$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{\int \left(a+b\arctan\left(cx^3\right)\right)^2dx^3}{c^2}-\frac{\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 5345}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\int \frac{x^3\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}-\frac{\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}d}{c^2}\right.\right.$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\int\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)}{c^2x^6+1}dx^3}{c^2}-\frac{\left(a+b\arctan\left(cx^3\right)\right)}{3bc^3}\right.\right.$$

$$\Big\downarrow \text{ 5455}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3bc^3}+\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{\int\frac{a+b\arctan\left(cx^3\right)}{i-cx^3}dx}{c}\right.}{c^2}\right.\right.$$

$$\Big\downarrow \text{ 5379}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3bc^3}+\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{\log\left(\frac{2}{1+icx^3}\right)\left(a+b\,\mathrm{ar}\right.}{c}\right.}{c^2}\right.\right.$$

$$\Big\downarrow \text{ 2849}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3bc^3}+\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{ib\int\frac{\log\left(\frac{2}{icx^3+1}\right)}{1-\frac{2}{icx^3+1}}d\frac{}{ic}}{c}\right.}{}\right.\right.$$

$$\Big\downarrow \text{ 2752}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3bc^3}+\frac{x^3\left(a+b\arctan\left(cx^3\right)\right)^2-2bc\left(-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{2bc^2}\right.}{c}\right.\right.$$

3.122. $\int x^5\left(a+b\arctan\left(cx^3\right)\right)^3\,dx$

input
```
Int[x^5*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
((x^6*(a + b*ArcTan[c*x^3])^3)/2 - (3*b*c*(-1/3*(a + b*ArcTan[c*x^3])^3/(b
*c^3) + (x^3*(a + b*ArcTan[c*x^3])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^
3])^2)/(b*c^2) - (((a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/c + ((I/2)*
b*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c)/c))/c^2))/2)/3
```

3.122.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.122.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.57 (sec) , antiderivative size = 935, normalized size of antiderivative = 6.36

method	result	size
risch	Expression too large to display	935
default	Expression too large to display	11515
parts	Expression too large to display	11515

input
```
int(x^5*(a+b*arctan(c*x^3))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*b^2*(I*b*c^2*x^6*ln(1-I*c*x^3)+2*a*c^2*x^6-2*b*c*x^3+I*b*ln(1-I*c*x^
3)+2*I*b+2*a)/c^2*ln(1+I*c*x^3)^2+1/2*a*b^2/c^2*ln(c^2*x^6+1)+1/2*a^2*b/c^
2*arctan(c*x^3)-1/48*I*b^3*x^6*ln(1-I*c*x^3)^3+1/8*I/c^2*b^3*ln(c^2*x^6+1)
+1/6*a^3*x^6+1/8*I/c^2*b^3*ln(1-I*c*x^3)^2-1/2/c*a^2*b*x^3+1/8*b^3/c*x^3*l
n(1-I*c*x^3)^2+3/4*I/c*b^2*Sum(2/3*(ln(x-_alpha)*ln(1-I*c*x^3)+3*c*(-1/3*l
n(x-_alpha)*(ln((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=
1)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=1))+
ln((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2)-x+_alpha)/
RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2))+ln(1/2*(2*(I/
c)^(1/3)+x-_alpha)/(I/c)^(1/3)))/c-1/3*(dilog((RootOf(_Z^2+_Z*RootOf(c*_Z^
3-I)+RootOf(c*_Z^3-I)^2,index=1)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)
+RootOf(c*_Z^3-I)^2,index=1))+dilog((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootO
f(c*_Z^3-I)^2,index=2)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*
_Z^3-I)^2,index=2))+dilog(1/2*(2*(I/c)^(1/3)+x-_alpha)/(I/c)^(1/3)))/c))*b
/c,_alpha=RootOf(c*_Z^3-RootOf(_Z^2+1,index=1)))+1/4*I*b*a^2*x^6*ln(1-I*c*
x^3)-1/8*a*b^2*x^6*ln(1-I*c*x^3)^2-1/8/c^2*a*b^2*ln(1-I*c*x^3)^2+1/48*I*b^
3*(c^2*x^6+1)/c^2*ln(1+I*c*x^3)^3-1/2*I/c*a*b^2*x^3*ln(1-I*c*x^3)-1/4/c^2*
b^3*arctan(c*x^3)-1/48*I/c^2*b^3*ln(1-I*c*x^3)^3+(1/16*I*b^3*(c^2*x^6+1)/c
^2*ln(1-I*c*x^3)^2+1/16*b^2*(2*a*c*x^3-b)^2/c^2/a*ln(1-I*c*x^3)-1/16*b*(4*
I*a^3*c^2*x^6-8*I*a^2*b*c*x^3+4*I*ln(1-I*c*x^3)*a*b^2+4*I*a*b^2-4*ln(1-...
```

3.122.5 Fricas [F]

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int\left(b\arctan\left(cx^3\right)+a\right)^3x^5\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^5*arctan(c*x^3)^3 + 3*a*b^2*x^5*arctan(c*x^3)^2 + 3*a^2*b*x
^5*arctan(c*x^3) + a^3*x^5, x)
```

3.122.6 Sympy [F]

$$\int x^5 \left(a+b \arctan \left(c x^3\right)\right)^3 \, dx=\int x^5 \left(a+b \operatorname{atan}\left(c x^3\right)\right)^3 \, dx$$

input
```
integrate(x**5*(a+b*atan(c*x**3))**3,x)
```

output
```
Integral(x**5*(a + b*atan(c*x**3))**3, x)
```

3.122.7 Maxima [F]

$$\int x^5 \left(a+b \arctan \left(c x^3\right)\right)^3 \, dx=\int \left(b \arctan \left(c x^3\right)+a\right)^3 x^5 \, dx$$

input
```
integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

output
```
1/2*a*b^2*x^6*arctan(c*x^3)^2 + 1/6*a^3*x^6 + 1/2*(x^6*arctan(c*x^3) - c*(
x^3/c^2 - arctan(c*x^3)/c^3))*a^2*b - 1/2*(2*c*(x^3/c^2 - arctan(c*x^3)/c^
3)*arctan(c*x^3) + (arctan(c*x^3)^2 - log(6*c^5*x^6 + 6*c^3))/c^2)*a*b^2 +
 1/192*(4*x^6*arctan(c*x^3)^3 - 3*x^6*arctan(c*x^3)*log(c^2*x^6 + 1)^2 + 1
92*integrate(1/64*(12*c^2*x^11*arctan(c*x^3)*log(c^2*x^6 + 1) - 12*c*x^8*a
rctan(c*x^3)^2 + 56*(c^2*x^11 + x^5)*arctan(c*x^3)^3 + 3*(c*x^8 + 2*(c^2*x
^11 + x^5)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^6 + 1), x))*b^3
```

3.122.8 Giac [F]

$$\int x^5 \left(a+b \arctan \left(c x^3\right)\right)^3 \, dx=\int \left(b \arctan \left(c x^3\right)+a\right)^3 x^5 \, dx$$

input
```
integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3*x^5, x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int x^5 (a + b \arctan(cx^3))^3 \, dx = \int x^5 \left(a + b \operatorname{atan}(c x^3)\right)^3 dx$$

input
```
int(x^5*(a + b*atan(c*x^3))^3,x)
```

output
```
int(x^5*(a + b*atan(c*x^3))^3, x)
```

3.123 $\int x^2 (a + b \arctan(cx^3))^3 \, dx$

3.123.1 Optimal result 814
3.123.2 Mathematica [A] (verified) 814
3.123.3 Rubi [A] (verified) 815
3.123.4 Maple [B] (verified) 817
3.123.5 Fricas [F] 818
3.123.6 Sympy [F] 818
3.123.7 Maxima [F] 818
3.123.8 Giac [F] 819
3.123.9 Mupad [F(-1)] 819

3.123.1 Optimal result

Integrand size = 16, antiderivative size = 139

$$\int x^2 (a + b \arctan(cx^3))^3 \, dx = \frac{i(a + b \arctan(cx^3))^3}{3c} + \frac{1}{3} x^3 (a + b \arctan(cx^3))^3 + \frac{b(a + b \arctan(cx^3))^2 \log\left(\frac{2}{1+icx^3}\right)}{c} + \frac{ib^2(a + b \arctan(cx^3)) \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+icx^3}\right)}{c} + \frac{b^3 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1+icx^3}\right)}{2c}$$

output
```
1/3*I*(a+b*arctan(c*x^3))^3/c+1/3*x^3*(a+b*arctan(c*x^3))^3+b*(a+b*arctan(
c*x^3))^2*ln(2/(1+I*c*x^3))/c+I*b^2*(a+b*arctan(c*x^3))*polylog(2,1-2/(1+I
*c*x^3))/c+1/2*b^3*polylog(3,1-2/(1+I*c*x^3))/c
```

3.123.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.61

$$\int x^2 (a + b \arctan(cx^3))^3 \, dx = \frac{2a^3cx^3 + 6a^2bcx^3 \arctan(cx^3) - 6iab^2 \arctan(cx^3)^2 + 6ab^2cx^3 \arctan(cx^3)^2 - 2ib^3 \arctan(cx^3)^3 + 2b^3cx^3}{}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
(2*a^3*c*x^3 + 6*a^2*b*c*x^3*ArcTan[c*x^3] - (6*I)*a*b^2*ArcTan[c*x^3]^2 +
 6*a*b^2*c*x^3*ArcTan[c*x^3]^2 - (2*I)*b^3*ArcTan[c*x^3]^3 + 2*b^3*c*x^3*A
rcTan[c*x^3]^3 + 12*a*b^2*ArcTan[c*x^3]*Log[1 + E^((2*I)*ArcTan[c*x^3])] +
 6*b^3*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan[c*x^3])] - 3*a^2*b*Log[1 +
c^2*x^6] - (6*I)*b^2*(a + b*ArcTan[c*x^3])*PolyLog[2, -E^((2*I)*ArcTan[c*x
^3])] + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x^3])])/(6*c)
```

3.123.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5363, 5345, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \arctan \left(c x^3\right)\right)^3 \, dx$$

$$\Big\downarrow \ 5363$$

$$\frac{1}{3} \int \left(a+b \arctan \left(c x^3\right)\right)^3 dx^3$$

$$\Big\downarrow \ 5345$$

$$\frac{1}{3} \left(x^3 \left(a+b \arctan \left(c x^3\right)\right)^3-3 b c \int \frac{x^3 \left(a+b \arctan \left(c x^3\right)\right)^2}{c^2 x^6+1}dx^3\right)$$

$$\Big\downarrow \ 5455$$

$$\frac{1}{3} \left(x^3 \left(a+b \arctan \left(c x^3\right)\right)^3-3 b c \left(-\frac{\int \frac{\left(a+b \arctan \left(c x^3\right)\right)^2}{i-c x^3}dx^3}{c}-\frac{i \left(a+b \arctan \left(c x^3\right)\right)^3}{3 b c^2}\right)\right)$$

$$\Big\downarrow \ 5379$$

$$\frac{1}{3} \left(x^3 \left(a+b \arctan \left(c x^3\right)\right)^3-3 b c \left(-\frac{\frac{\log \left(\frac{2}{1+i c x^3}\right) \left(a+b \arctan \left(c x^3\right)\right)^2}{c}-2 b \int \frac{\left(a+b \arctan \left(c x^3\right)\right) \log \left(\frac{2}{i c x^3+1}\right)}{c^2 x^6+1}dx^3}{c}-\frac{i \left(a+b \arc\right.}{3}\right.\right.$$

$$\Big\downarrow \ 5529$$

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx^3}\right)\left(a+b\arctan(cx^3)\right)^2}{c}-2b\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3-\frac{i\,\text{PolyLog}\left(2,1-\right.}{}\right.}{c}\right.\right.$$

$\downarrow$ 7164

$$\frac{1}{3}\left(x^3\left(a+b\arctan\left(cx^3\right)\right)^3-3bc\left(-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{3bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx^3}\right)\left(a+b\arctan(cx^3)\right)^2}{c}-2b\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^3}\right.}{}\right.}{c}\right.\right.$$

input
```
Int[x^2*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
(x^3*(a + b*ArcTan[c*x^3])^3 - 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x^3])^3)/(
b*c^2) - (((a + b*ArcTan[c*x^3])^2*Log[2/(1 + I*c*x^3)])/c - 2*b*(((-1/2*I
)*(a + b*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c - (b*PolyLog[3,
 1 - 2/(1 + I*c*x^3)])/(4*c)))/c))/3
```

3.123.3.1 Defintions of rubi rules used

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.123.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(128) = 256$.

Time = 9.01 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.98

method	result
derivativedivides	$\frac{a^3cx^3+b^3\left(\arctan(cx^3)^3(cx^3+i)-2i\arctan(cx^3)^3+3\arctan(cx^3)^2\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)-3i\arctan(cx^3)\operatorname{polylog}\left(2,\right.\right.}{}$
default	$\frac{a^3cx^3+b^3\left(\arctan(cx^3)^3(cx^3+i)-2i\arctan(cx^3)^3+3\arctan(cx^3)^2\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)-3i\arctan(cx^3)\operatorname{polylog}\left(2,\right.\right.}{}$
parts	$\frac{a^3x^3}{3}+\frac{b^3\left(\arctan(cx^3)^3(cx^3+i)-2i\arctan(cx^3)^3+3\arctan(cx^3)^2\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)-3i\arctan(cx^3)\operatorname{polylog}\left(2\right.\right.}{3c}$

input
```
int(x^2*(a+b*arctan(c*x^3))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/c*(a^3*c*x^3+b^3*(arctan(c*x^3)^3*(c*x^3+I)-2*I*arctan(c*x^3)^3+3*arct
an(c*x^3)^2*ln(1+(1+I*c*x^3)^2/(c^2*x^6+1))-3*I*arctan(c*x^3)*polylog(2,-(
1+I*c*x^3)^2/(c^2*x^6+1))+3/2*polylog(3,-(1+I*c*x^3)^2/(c^2*x^6+1)))+3*a*b
^2*(arctan(c*x^3)^2*(c*x^3+I)+2*arctan(c*x^3)*ln(1+(1+I*c*x^3)^2/(c^2*x^6+
1))-2*I*arctan(c*x^3)^2-I*polylog(2,-(1+I*c*x^3)^2/(c^2*x^6+1)))+3*a^2*b*(
c*x^3*arctan(c*x^3)-1/2*ln(c^2*x^6+1)))
```

3.123.5 Fricas [F]

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int\left(b\arctan\left(cx^3\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arctan(c*x^3)^3 + 3*a*b^2*x^2*arctan(c*x^3)^2 + 3*a^2*b*x
^2*arctan(c*x^3) + a^3*x^2, x)
```

3.123.6 Sympy [F]

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int x^2\left(a+b\operatorname{atan}\left(cx^3\right)\right)^3\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x**3))**3,x)
```

output
```
Integral(x**2*(a + b*atan(c*x**3))**3, x)
```

3.123.7 Maxima [F]

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int\left(b\arctan\left(cx^3\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

output
```
1/24*b^3*x^3*arctan(c*x^3)^3 - 1/32*b^3*x^3*arctan(c*x^3)*log(c^2*x^6 + 1)
^2 + 1/3*a^3*x^3 + 7/96*b^3*arctan(c*x^3)^4/c + 28*b^3*c^2*integrate(1/32*
x^8*arctan(c*x^3)^3/(c^2*x^6 + 1), x) + 3*b^3*c^2*integrate(1/32*x^8*arcta
n(c*x^3)*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 96*a*b^2*c^2*integrate(1/3
2*x^8*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 12*b^3*c^2*integrate(1/32*x^8*ar
ctan(c*x^3)*log(c^2*x^6 + 1)/(c^2*x^6 + 1), x) + 1/3*a*b^2*arctan(c*x^3)^3
/c - 12*b^3*c*integrate(1/32*x^5*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 3*b^3
*c*integrate(1/32*x^5*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 3*b^3*integra
te(1/32*x^2*arctan(c*x^3)*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 1/2*(2*c*
x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*a^2*b/c
```

3.123.8 Giac [F]

$$\int x^2 \left(a+b \arctan \left(c x^3\right)\right)^3 \, dx=\int \left(b \arctan \left(c x^3\right)+a\right)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3*x^2, x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b \arctan \left(c x^3\right)\right)^3 \, dx=\int x^2 \, {\left(a+b \,\mathrm{atan}\left(c \,x^3\right)\right)}^3 \,d x$$

input
```
int(x^2*(a + b*atan(c*x^3))^3,x)
```

output
```
int(x^2*(a + b*atan(c*x^3))^3, x)
```

3.124 $\int \frac{(a+b\arctan(cx^3))^3}{x} dx$

3.124.1 Optimal result . . . 820
3.124.2 Mathematica [A] (verified) . . . 821
3.124.3 Rubi [A] (verified) . . . 822
3.124.4 Maple [F] . . . 824
3.124.5 Fricas [F] . . . 825
3.124.6 Sympy [F] . . . 825
3.124.7 Maxima [F] . . . 825
3.124.8 Giac [F] . . . 826
3.124.9 Mupad [F(-1)] . . . 826

3.124.1 Optimal result

Integrand size = 16, antiderivative size = 232

$$\begin{aligned}\int \frac{(a+b\arctan(cx^3))^3}{x} dx &= \frac{2}{3}(a+b\arctan(cx^3))^3 \operatorname{arctanh}\left(1-\frac{2}{1+icx^3}\right) \\ &- \frac{1}{2}ib(a+b\arctan(cx^3))^2 \operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right) \\ &+ \frac{1}{2}ib(a+b\arctan(cx^3))^2 \operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right) \\ &- \frac{1}{2}b^2(a+b\arctan(cx^3)) \operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right) \\ &+ \frac{1}{2}b^2(a+b\arctan(cx^3)) \operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^3}\right) \\ &+ \frac{1}{4}ib^3 \operatorname{PolyLog}\left(4,1-\frac{2}{1+icx^3}\right) \\ &- \frac{1}{4}ib^3 \operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx^3}\right)\end{aligned}$$

output
```
-2/3*(a+b*arctan(c*x^3))^3*arctanh(-1+2/(1+I*c*x^3))-1/2*I*b*(a+b*arctan(c
*x^3))^2*polylog(2,1-2/(1+I*c*x^3))+1/2*I*b*(a+b*arctan(c*x^3))^2*polylog(
2,-1+2/(1+I*c*x^3))-1/2*b^2*(a+b*arctan(c*x^3))*polylog(3,1-2/(1+I*c*x^3))
+1/2*b^2*(a+b*arctan(c*x^3))*polylog(3,-1+2/(1+I*c*x^3))+1/4*I*b^3*polylog
(4,1-2/(1+I*c*x^3))-1/4*I*b^3*polylog(4,-1+2/(1+I*c*x^3))
```

3.124.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.81

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = a^3\log(x)+\frac{1}{2}ia^2b\left(\operatorname{PolyLog}(2,-icx^3)-\operatorname{PolyLog}(2,icx^3)\right)+ab^2\left(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx^3)^3+\arctan(cx^3)^2\log\left(1-e^{-2i\arctan(cx^3)}\right)-\arctan(cx^3)^2\log\left(1+e^{2i\arctan(cx^3)}\right)+i\arctan(cx^3)\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx^3)}\right)+i\arctan(cx^3)\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx^3)}\right)+\frac{1}{2}\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx^3)}\right)-\frac{1}{2}\operatorname{PolyLog}\left(3,-e^{2i\arctan(cx^3)}\right)\right)-\frac{1}{192}ib^3\left(\pi^4-32\arctan(cx^3)^4+64i\arctan(cx^3)^3\log\left(1-e^{-2i\arctan(cx^3)}\right)-64i\arctan(cx^3)^3\log\left(1+e^{2i\arctan(cx^3)}\right)-96\arctan(cx^3)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx^3)}\right)-96\arctan(cx^3)^2\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx^3)}\right)+96i\arctan(cx^3)\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx^3)}\right)-96i\arctan(cx^3)\operatorname{PolyLog}\left(3,-e^{2i\arctan(cx^3)}\right)+48\operatorname{PolyLog}\left(4,e^{-2i\arctan(cx^3)}\right)+48\operatorname{PolyLog}\left(4,-e^{2i\arctan(cx^3)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^3/x,x]
```

output
```
a^3*Log[x] + (I/2)*a^2*b*(PolyLog[2, (-I)*c*x^3] - PolyLog[2, I*c*x^3]) +
a*b^2*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x^3]^3 + ArcTan[c*x^3]^2*Log[1
- E^((-2*I)*ArcTan[c*x^3])] - ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan[c*x^
3])] + I*ArcTan[c*x^3]*PolyLog[2, E^((-2*I)*ArcTan[c*x^3])] + I*ArcTan[c*x
^3]*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])] + PolyLog[3, E^((-2*I)*ArcTan[c*x
^3])]/2 - PolyLog[3, -E^((2*I)*ArcTan[c*x^3])]/2) - (I/192)*b^3*(Pi^4 - 32
*ArcTan[c*x^3]^4 + (64*I)*ArcTan[c*x^3]^3*Log[1 - E^((-2*I)*ArcTan[c*x^3])
] - (64*I)*ArcTan[c*x^3]^3*Log[1 + E^((2*I)*ArcTan[c*x^3])] - 96*ArcTan[c*
x^3]^2*PolyLog[2, E^((-2*I)*ArcTan[c*x^3])] - 96*ArcTan[c*x^3]^2*PolyLog[2
, -E^((2*I)*ArcTan[c*x^3])] + (96*I)*ArcTan[c*x^3]*PolyLog[3, E^((-2*I)*Ar
cTan[c*x^3])] - (96*I)*ArcTan[c*x^3]*PolyLog[3, -E^((2*I)*ArcTan[c*x^3])]
+ 48*PolyLog[4, E^((-2*I)*ArcTan[c*x^3])] + 48*PolyLog[4, -E^((2*I)*ArcTan
[c*x^3])])
```

3.124.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5359, 5357, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^3))^3}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{1}{3}\int \frac{(a+b\arctan(cx^3))^3}{x^3}dx^3$$

$$\Big\downarrow \text{5357}$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)(a+b\arctan(cx^3))^3-6bc\int \frac{(a+b\arctan(cx^3))^2\text{arctanh}\left(1-\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3\right)$$

$$\Big\downarrow \text{5523}$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)(a+b\arctan(cx^3))^3-6bc\left(\frac{1}{2}\int \frac{(a+b\arctan(cx^3))^2\log\left(2-\frac{2}{icx^3+1}\right)}{c^2x^6+1}dx^3-\frac{1}{2}\int\right.\right.$$

$\downarrow$ 5529

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)(a+b\arctan(cx^3))^3-6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)(a+b\arctan(cx^3))^2}{2c}-i\right.\right.\right.$$

$\downarrow$ 5533

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)(a+b\arctan(cx^3))^3-6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)(a+b\arctan(cx^3))^2}{2c}-i\right.\right.\right.$$

$\downarrow$ 7164

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)(a+b\arctan(cx^3))^3-6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)(a+b\arctan(cx^3))^2}{2c}-i\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^3])^3/x,x]
```

output
```
(2*(a + b*ArcTan[c*x^3])^3*ArcTanh[1 - 2/(1 + I*c*x^3)] - 6*b*c*((((I/2)*(
a + b*ArcTan[c*x^3])^2*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c - I*b*(((I/2)*(a
 + b*ArcTan[c*x^3])*PolyLog[3, 1 - 2/(1 + I*c*x^3)])/c + (b*PolyLog[4, 1 -
 2/(1 + I*c*x^3)])/(4*c)))/2 + (((-1/2*I)*(a + b*ArcTan[c*x^3])^2*PolyLog[
2, -1 + 2/(1 + I*c*x^3)])/c + I*b*(((I/2)*(a + b*ArcTan[c*x^3])*PolyLog[3,
 -1 + 2/(1 + I*c*x^3)])/c + (b*PolyLog[4, -1 + 2/(1 + I*c*x^3)])/(4*c)))/2
))/3
```

3.124.3.1 Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.124.4 Maple [F]

$$\int \frac{(a + b\arctan(c\,x^3))^3}{x}dx$$

input
```
int((a+b*arctan(c*x^3))^3/x,x)
```

output
```
int((a+b*arctan(c*x^3))^3/x,x)
```

3.124.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx=\int \frac{(b\arctan(cx^3)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c
*x^3) + a^3)/x, x)
```

3.124.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx=\int \frac{(a+b\,\mathrm{atan}(cx^3))^3}{x}\,dx$$

input
```
integrate((a+b*atan(c*x**3))**3/x,x)
```

output
```
Integral((a + b*atan(c*x**3))**3/x, x)
```

3.124.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx=\int \frac{(b\arctan(cx^3)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x^3)^3 + 3*b^3*arctan(c*x^3)*
log(c^2*x^6 + 1)^2 + 96*a*b^2*arctan(c*x^3)^2 + 96*a^2*b*arctan(c*x^3))/x,
 x)
```

3.124.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx=\int \frac{(b\arctan(cx^3)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3/x, x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x^3))^3}{x}\,dx$$

input
```
int((a + b*atan(c*x^3))^3/x,x)
```

output
```
int((a + b*atan(c*x^3))^3/x, x)
```

3.125 $\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx$

3.125.1 Optimal result . . . 827
3.125.2 Mathematica [A] (verified) . . . 828
3.125.3 Rubi [A] (verified) . . . 828
3.125.4 Maple [F] . . . 831
3.125.5 Fricas [F] . . . 831
3.125.6 Sympy [F] . . . 831
3.125.7 Maxima [F] . . . 832
3.125.8 Giac [F] . . . 832
3.125.9 Mupad [F(-1)] . . . 832

3.125.1 Optimal result

Integrand size = 16, antiderivative size = 133

$$\begin{aligned}\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = &-\frac{1}{3}ic\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3x^3}\\ &+bc\left(a+b\arctan\left(cx^3\right)\right)^2\log\left(2-\frac{2}{1-icx^3}\right)\\ &-ib^2c\left(a+b\arctan\left(cx^3\right)\right)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)\\ &+\frac{1}{2}b^3c\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx^3}\right)\end{aligned}$$

output
```
-1/3*I*c*(a+b*arctan(c*x^3))^3-1/3*(a+b*arctan(c*x^3))^3/x^3+b*c*(a+b*arct
an(c*x^3))^2*ln(2-2/(1-I*c*x^3))-I*b^2*c*(a+b*arctan(c*x^3))*polylog(2,-1+
2/(1-I*c*x^3))+1/2*b^3*c*polylog(3,-1+2/(1-I*c*x^3))
```

3.125.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.80

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = -\frac{a^3}{3x^3} - \frac{a^2b\arctan(cx^3)}{x^3} + 3a^2bc\log(x) - \frac{1}{2}a^2bc\log(1+c^2x^6) + ab^2c\left(\arctan(cx^3)\left(\left(-i-\frac{1}{cx^3}\right)\arctan(cx^3) + 2\log\left(1-e^{2i\arctan(cx^3)}\right)\right) - i\,\mathrm{PolyLog}\left(2,e^{2i\arctan(cx^3)}\right)\right) + \frac{1}{3}b^3c\left(-\frac{i\pi^3}{8} + i\arctan(cx^3)^3 - \frac{\arctan(cx^3)^3}{cx^3} + 3\arctan(cx^3)^2\log\left(1-e^{-2i\arctan(cx^3)}\right) + 3i\arctan(cx^3)\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(cx^3)}\right) + \frac{3}{2}\mathrm{PolyLog}\left(3,e^{-2i\arctan(cx^3)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^3/x^4,x]
```

output
```
-1/3*a^3/x^3 - (a^2*b*ArcTan[c*x^3])/x^3 + 3*a^2*b*c*Log[x] - (a^2*b*c*Log
[1 + c^2*x^6])/2 + a*b^2*c*(ArcTan[c*x^3]*((-I - 1/(c*x^3))*ArcTan[c*x^3]
+ 2*Log[1 - E^((2*I)*ArcTan[c*x^3])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x^3
])]) + (b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x^3]^3 - ArcTan[c*x^3]^3/(c*x^3)
 + 3*ArcTan[c*x^3]^2*Log[1 - E^((-2*I)*ArcTan[c*x^3])] + (3*I)*ArcTan[c*x^
3]*PolyLog[2, E^((-2*I)*ArcTan[c*x^3])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c
*x^3])])/2))/3
```

3.125.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5363, 5361, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx$$

$$\Big\downarrow \; 5363$$

$$\frac{1}{3}\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^6}dx^3$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{3}\left(3bc\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^3\left(c^2x^6+1\right)}dx^3-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^3}\right)$$

$$\Big\downarrow \; 5459$$

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^3}+3bc\left(i\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^3\left(cx^3+i\right)}dx^3-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{3b}\right)\right)$$

$$\Big\downarrow \; 5403$$

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^3}+3bc\left(i\left(2ibc\int \frac{\left(a+b\arctan\left(cx^3\right)\right)\log\left(2-\frac{2}{1-icx^3}\right)}{c^2x^6+1}dx^3-i\log\left(2-\frac{2}{1-icx^3}\right)(a\right.\right.\right.$$

$$\Big\downarrow \; 5527$$

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^3}+3bc\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx^3}-1\right)\left(a+b\arctan\left(cx^3\right)\right)}{2c}-\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,\right.}{c^2x^6}\right.\right.\right.\right.$$

$$\Big\downarrow \; 7164$$

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^3}+3bc\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx^3}-1\right)\left(a+b\arctan\left(cx^3\right)\right)}{2c}-\frac{b\,\text{PolyLog}\left(3,\frac{2}{1-icx^3}\right.}{4c}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^3])^3/x^4,x]
```

output
```
(-((a + b*ArcTan[c*x^3])^3/x^3) + 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x^3])^3
)/b + I*((-I)*(a + b*ArcTan[c*x^3])^2*Log[2 - 2/(1 - I*c*x^3)] + (2*I)*b*c
*(((I/2)*(a + b*ArcTan[c*x^3])*PolyLog[2, -1 + 2/(1 - I*c*x^3)])/c - (b*Po
lyLog[3, -1 + 2/(1 - I*c*x^3)])/(4*c)))))/3
```

3.125. $\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^4}\,dx$

3.125.3.1 Defintions of rubi rules used

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.125. $\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx$

3.125.4 Maple [F]

$$\int \frac{(a+b\arctan(c\,x^3))^3}{x^4}dx$$

input
```
int((a+b*arctan(c*x^3))^3/x^4,x)
```

output
```
int((a+b*arctan(c*x^3))^3/x^4,x)
```

3.125.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(b\arctan(cx^3)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c
*x^3) + a^3)/x^4, x)
```

3.125.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(a+b\operatorname{atan}(cx^3))^3}{x^4}\,dx$$

input
```
integrate((a+b*atan(c*x**3))**3/x**4,x)
```

output
```
Integral((a + b*atan(c*x**3))**3/x**4, x)
```

3.125.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(b\arctan(cx^3)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*a^2*b - 1/3*a
^3/x^3 - 1/96*(4*b^3*arctan(c*x^3)^3 - 3*b^3*arctan(c*x^3)*log(c^2*x^6 + 1
)^2 - 96*x^3*integrate(-1/32*(12*b^3*c^2*x^6*arctan(c*x^3)*log(c^2*x^6 + 1
) - 28*(b^3*c^2*x^6 + b^3)*arctan(c*x^3)^3 - 12*(8*a*b^2*c^2*x^6 + b^3*c*x
^3 + 8*a*b^2)*arctan(c*x^3)^2 + 3*(b^3*c*x^3 - (b^3*c^2*x^6 + b^3)*arctan(
c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^10 + x^4), x))/x^3
```

3.125.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(b\arctan(cx^3)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3/x^4, x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^3))^3}{x^4}\,dx$$

input
```
int((a + b*atan(c*x^3))^3/x^4,x)
```

output
```
int((a + b*atan(c*x^3))^3/x^4, x)
```

3.126 $\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx$

3.126.1 Optimal result 833
3.126.2 Mathematica [A] (verified) 833
3.126.3 Rubi [A] (verified) 834
3.126.4 Maple [C] (warning: unable to verify) 836
3.126.5 Fricas [F] 837
3.126.6 Sympy [F] 837
3.126.7 Maxima [F] 837
3.126.8 Giac [F] 838
3.126.9 Mupad [F(-1)] 838

3.126.1 Optimal result

Integrand size = 16, antiderivative size = 146

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx = -\frac{1}{2}ibc^2(a+b\arctan(cx^3))^2 - \frac{bc(a+b\arctan(cx^3))^2}{2x^3} - \frac{1}{6}c^2(a+b\arctan(cx^3))^3 - \frac{(a+b\arctan(cx^3))^3}{6x^6} + b^2c^2(a+b\arctan(cx^3))\log\left(2-\frac{2}{1-icx^3}\right) - \frac{1}{2}ib^3c^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)$$

output
```
-1/2*I*b*c^2*(a+b*arctan(c*x^3))^2-1/2*b*c*(a+b*arctan(c*x^3))^2/x^3-1/6*c
^2*(a+b*arctan(c*x^3))^3-1/6*(a+b*arctan(c*x^3))^3/x^6+b^2*c^2*(a+b*arctan
(c*x^3))*ln(2-2/(1-I*c*x^3))-1/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x^3))
```

3.126.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx =$$

$$-\frac{3b^2(a+ac^2x^6+bcx^3(1+icx^3))\arctan(cx^3)^2 + b^3(1+c^2x^6)\arctan(cx^3)^3 + 3b\arctan(cx^3)\left(a(a+2bc\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x^3])^3/x^7,x]
```

output
```
-1/6*(3*b^2*(a + a*c^2*x^6 + b*c*x^3*(1 + I*c*x^3))*ArcTan[c*x^3]^2 + b^3*
(1 + c^2*x^6)*ArcTan[c*x^3]^3 + 3*b*ArcTan[c*x^3]*(a*(a + 2*b*c*x^3 + a*c^
2*x^6) - 2*b^2*c^2*x^6*Log[1 - E^((2*I)*ArcTan[c*x^3])]) + a*(a*(a + 3*b*c
*x^3) - 6*b^2*c^2*x^6*Log[(c*x^3)/Sqrt[1 + c^2*x^6]]) + (3*I)*b^3*c^2*x^6*
PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/x^6
```

3.126.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5363, 5361, 5453, 5361, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^7}dx$$

$$\Big\downarrow \text{5363}$$

$$\frac{1}{3}\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^9}dx^3$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(\frac{3}{2}bc\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^6\left(c^2x^6+1\right)}dx^3-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{2x^6}\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{3}\left(\frac{3}{2}bc\left(\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^6}dx^3-c^2\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}dx^3\right)-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{2x^6}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\left(\frac{3}{2}bc\left(c^2\left(-\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{c^2x^6+1}dx^3\right)+2bc\int \frac{a+b\arctan\left(cx^3\right)}{x^3\left(c^2x^6+1\right)}dx^3-\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^3}\right)-\frac{(a+b\text{a}}{}\right.$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{3}\left(\frac{3}{2}bc\left(2bc\int\frac{a+b\arctan\left(cx^3\right)}{x^3\left(c^2x^6+1\right)}dx^3-\frac{c\left(a+b\arctan\left(cx^3\right)\right)^3}{3b}-\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x^3}\right)-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{2x^6}\right.$$

$$\Big\downarrow \text{5459}$$

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{2x^6}+\frac{3}{2}bc\left(2bc\left(i\int\frac{a+b\arctan\left(cx^3\right)}{x^3\left(cx^3+i\right)}dx^3-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{2b}\right)-\frac{c\left(a+b\arctan\left(c\right.\right.}{3b}\right.\right.$$

$$\Big\downarrow \text{5403}$$

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{2x^6}+\frac{3}{2}bc\left(2bc\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-icx^3}\right)}{c^2x^6+1}dx^3-i\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{3}\left(-\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{2x^6}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx^3}\right.\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x^3])^3/x^7,x]
```

output
```
(-1/2*(a + b*ArcTan[c*x^3])^3/x^6 + (3*b*c*(-((a + b*ArcTan[c*x^3])^2/x^3)
 - (c*(a + b*ArcTan[c*x^3])^3)/(3*b) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x^
3])^2)/b + I*((-I)*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)] - (b*Pol
yLog[2, -1 + 2/(1 - I*c*x^3)])/2))))/2)/3
```

3.126.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.126. $\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^7}\,dx$

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.126.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.88 (sec) , antiderivative size = 11581, normalized size of antiderivative = 79.32

method	result	size
default	Expression too large to display	11581
parts	Expression too large to display	11581

input
```
int((a+b*arctan(c*x^3))^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.126.5 Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx=\int \frac{(b\arctan(cx^3)+a)^3}{x^7}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c
*x^3) + a^3)/x^7, x)
```

3.126.6 Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx=\int \frac{(a+b\,\mathrm{atan}(cx^3))^3}{x^7}\,dx$$

input
```
integrate((a+b*atan(c*x**3))**3/x**7,x)
```

output
```
Integral((a + b*atan(c*x**3))**3/x**7, x)
```

3.126.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx=\int \frac{(b\arctan(cx^3)+a)^3}{x^7}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="maxima")
```

output
```
-1/2*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*a^2*b + 1/2*((arcta
n(c*x^3)^2 - log(c^2*x^6 + 1) + 6*log(x))*c^2 - 2*(c*arctan(c*x^3) + 1/x^3
)*c*arctan(c*x^3))*a*b^2 - 1/2*a*b^2*arctan(c*x^3)^2/x^6 + 1/192*(192*x^6*
integrate(-1/64*(12*c^2*x^6*arctan(c*x^3)*log(c^2*x^6 + 1) - 12*c*x^3*arct
an(c*x^3)^2 - 56*(c^2*x^6 + 1)*arctan(c*x^3)^3 + 3*(c*x^3 - 2*(c^2*x^6 + 1
)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^13 + x^7), x) - 4*arctan(c*x^3
)^3 + 3*arctan(c*x^3)*log(c^2*x^6 + 1)^2)*b^3/x^6 - 1/6*a^3/x^6
```

3.126.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx = \int \frac{(b\arctan(cx^3)+a)^3}{x^7}\,dx$$

input
```
integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3/x^7, x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^3))^3}{x^7}\,dx$$

input
```
int((a + b*atan(c*x^3))^3/x^7,x)
```

output
```
int((a + b*atan(c*x^3))^3/x^7, x)
```

3.127 $\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx$

3.127.1 Optimal result 839
3.127.2 Mathematica [N/A] 839
3.127.3 Rubi [N/A] 840
3.127.4 Maple [N/A] (verified) 840
3.127.5 Fricas [N/A] 841
3.127.6 Sympy [F(-1)] 841
3.127.7 Maxima [N/A] 841
3.127.8 Giac [N/A] 842
3.127.9 Mupad [N/A] 842

3.127.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx = \text{Int}\left((dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

3.127.2 Mathematica [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx = \int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^3, x]
```

3.127.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx$$
$$\Big\downarrow \ 5377$$
$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x^3])^3,x]
```

output
```
$Aborted
```

3.127.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.127.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^3\right)\right)^3 dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

3.127.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)^3(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c
*x^3) + a^3)*(d*x)^m, x)
```

3.127.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atan(c*x**3))**3,x)
```

output
```
Timed out
```

3.127.7 Maxima [N/A]

Not integrable

Time = 4.06 (sec) , antiderivative size = 407, normalized size of antiderivative = 22.61

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)^3(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x^3)^3 - 3*
b^3*d^m*x*x^m*arctan(c*x^3)*log(c^2*x^6 + 1)^2 + 32*(m + 1)*integrate(1/32
*(36*b^3*c^2*d^m*x^6*x^m*arctan(c*x^3)*log(c^2*x^6 + 1) + 28*((b^3*c^2*d^m
*m + b^3*c^2*d^m)*x^6 + b^3*d^m*m + b^3*d^m)*x^m*arctan(c*x^3)^3 - 12*(3*b
^3*c*d^m*x^3 - 8*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^6 - 8*a*b^2*d^m*m - 8
*a*b^2*d^m)*x^m*arctan(c*x^3)^2 + 96*((a^2*b*c^2*d^m*m + a^2*b*c^2*d^m)*x^
6 + a^2*b*d^m*m + a^2*b*d^m)*x^m*arctan(c*x^3) + 3*(3*b^3*c*d^m*x^3*x^m +
((b^3*c^2*d^m*m + b^3*c^2*d^m)*x^6 + b^3*d^m*m + b^3*d^m)*x^m*arctan(c*x^3
))*log(c^2*x^6 + 1)^2)/((c^2*m + c^2)*x^6 + m + 1), x))/(m + 1)
```

3.127.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 \,dx=\int \left(b\arctan\left(cx^3\right)+a\right)^3 (dx)^m \,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^3*(d*x)^m, x)
```

3.127.9 Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3 \,dx=\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^3 \,dx$$

input
```
int((d*x)^m*(a + b*atan(c*x^3))^3,x)
```

output
```
int((d*x)^m*(a + b*atan(c*x^3))^3, x)
```

3.128 $\int (dx)^m \left(a+b \arctan \left(c x^3\right)\right)^2 \, dx$

3.128.1 Optimal result 843
3.128.2 Mathematica [N/A] 843
3.128.3 Rubi [N/A] 844
3.128.4 Maple [N/A] (verified) 844
3.128.5 Fricas [N/A] 845
3.128.6 Sympy [F(-1)] 845
3.128.7 Maxima [N/A] 845
3.128.8 Giac [N/A] 846
3.128.9 Mupad [N/A] 846

3.128.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b \arctan \left(c x^3\right)\right)^2 \, dx=\text{Int}\left((dx)^m \left(a+b \arctan \left(c x^3\right)\right)^2,x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

3.128.2 Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b \arctan \left(c x^3\right)\right)^2 \, dx=\int (dx)^m \left(a+b \arctan \left(c x^3\right)\right)^2 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^2, x]
```

3.128.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 \, dx$$

$$\Big\downarrow \text{ 5377}$$

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x^3])^2,x]
```

output
```
$Aborted
```

3.128.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.128.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^3\right)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

3.128.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)^2 (dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)*(d*x)^m, x)
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atan(c*x**3))**2,x)
```

output
```
Timed out
```

3.128.7 Maxima [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 304, normalized size of antiderivative = 16.89

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)^2 (dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x^3)^2 - b^
2*d^m*x*x^m*log(c^2*x^6 + 1)^2 + 16*(m + 1)*integrate(1/16*(12*b^2*c^2*d^m
*x^6*x^m*log(c^2*x^6 + 1) + 12*((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^6 + b^2*d^
m*m + b^2*d^m)*x^m*arctan(c*x^3)^2 + ((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^6 +
b^2*d^m*m + b^2*d^m)*x^m*log(c^2*x^6 + 1)^2 - 8*(3*b^2*c*d^m*x^3 - 4*(a*b*
c^2*d^m*m + a*b*c^2*d^m)*x^6 - 4*a*b*d^m*m - 4*a*b*d^m)*x^m*arctan(c*x^3))
/((c^2*m + c^2)*x^6 + m + 1), x))/(m + 1)
```

3.128.8 Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)^2*(d*x)^m, x)
```

3.128.9 Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^2\,dx$$

input
```
int((d*x)^m*(a + b*atan(c*x^3))^2,x)
```

output
```
int((d*x)^m*(a + b*atan(c*x^3))^2, x)
```

3.129 $\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) dx$

3.129.1 Optimal result . . . 847
3.129.2 Mathematica [A] (verified) . . . 847
3.129.3 Rubi [A] (verified) . . . 848
3.129.4 Maple [F] . . . 849
3.129.5 Fricas [F] . . . 849
3.129.6 Sympy [F(-1)] . . . 849
3.129.7 Maxima [F] . . . 850
3.129.8 Giac [F] . . . 850
3.129.9 Mupad [F(-1)] . . . 850

3.129.1 Optimal result

Integrand size = 16, antiderivative size = 75

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) dx = \frac{(dx)^{1+m}\left(a+b\arctan\left(cx^3\right)\right)}{d(1+m)} - \frac{3bc(dx)^{4+m}\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{6},\frac{10+m}{6},-c^2x^6\right)}{d^4(1+m)(4+m)}$$

output
```
(d*x)^(1+m)*(a+b*arctan(c*x^3))/d/(1+m)-3*b*c*(d*x)^(4+m)*hypergeom([1, 2/
3+1/6*m],[5/3+1/6*m],-c^2*x^6)/d^4/(1+m)/(4+m)
```

3.129.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) dx = -\frac{x(dx)^m\left(-\left((4+m)\left(a+b\arctan\left(cx^3\right)\right)\right)+3bcx^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{6},\frac{10+m}{6},-c^2x^6\right)\right)}{(1+m)(4+m)}$$

input
```
Integrate[(d*x)^m*(a + b*ArcTan[c*x^3]),x]
```

output
```
-((x*(d*x)^m*(-((4 + m)*(a + b*ArcTan[c*x^3])) + 3*b*c*x^3*Hypergeometric2
F1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)]))/((1 + m)*(4 + m)))
```

3.129.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5373, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) \, dx$$

$$\Big\downarrow \text{5373}$$

$$\frac{(dx)^{m+1}\left(a+b\arctan\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc\int \frac{(dx)^{m+3}}{c^2x^6+1}dx}{d^3(m+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{(dx)^{m+1}\left(a+b\arctan\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc(dx)^{m+4}\,\text{Hypergeometric2F1}\left(1,\frac{m+4}{6},\frac{m+10}{6},-c^2x^6\right)}{d^4(m+1)(m+4)}$$

input
```
Int[(d*x)^m*(a + b*ArcTan[c*x^3]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcTan[c*x^3]))/(d*(1 + m)) - (3*b*c*(d*x)^(4 + m)*H
ypergeometric2F1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)])/(d^4*(1 + m)*(4 +
m))
```

3.129.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5373
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n/(
d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.129.4 Maple [F]

$$\int (dx)^m \left(a+b\arctan\left(c\,x^3\right)\right) dx$$

input
```
int((d*x)^m*(a+b*arctan(c*x^3)),x)
```

output
```
int((d*x)^m*(a+b*arctan(c*x^3)),x)
```

3.129.5 Fricas [F]

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) dx = \int \left(b\arctan\left(cx^3\right)+a\right)(dx)^m dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x^3) + a)*(d*x)^m, x)
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atan(c*x**3)),x)
```

output
```
Timed out
```

3.129.7 Maxima [F]

$$\int (dx)^m \left(a+b\arctan \left(cx^3\right)\right) \, dx = \int \left(b\arctan \left(cx^3\right)+a\right)(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
(d^m*x*x^m*arctan(c*x^3) - 3*(c*d^m*m + c*d^m)*integrate(x^3*x^m/((c^2*m +
 c^2)*x^6 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

3.129.8 Giac [F]

$$\int (dx)^m \left(a+b\arctan \left(cx^3\right)\right) \, dx = \int \left(b\arctan \left(cx^3\right)+a\right)(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^3) + a)*(d*x)^m, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan \left(cx^3\right)\right) \, dx = \int (d\,x)^m \left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right) \, dx$$

input
```
int((d*x)^m*(a + b*atan(c*x^3)),x)
```

output
```
int((d*x)^m*(a + b*atan(c*x^3)), x)
```

3.130 $\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx$

3.130.1 Optimal result . . . 851
3.130.2 Mathematica [N/A] . . . 851
3.130.3 Rubi [N/A] . . . 852
3.130.4 Maple [N/A] (verified) . . . 852
3.130.5 Fricas [N/A] . . . 853
3.130.6 Sympy [F(-1)] . . . 853
3.130.7 Maxima [N/A] . . . 853
3.130.8 Giac [N/A] . . . 854
3.130.9 Mupad [N/A] . . . 854

3.130.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx=\operatorname{Int}\left(\frac{(dx)^m}{a+b\arctan(cx^3)},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctan(c*x^3)),x)
```

3.130.2 Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx=\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^3]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^3]), x]
```

3.130.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx$$

$$\Big\downarrow \ 5377$$

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTan[c*x^3]),x]
```

output
```
$Aborted
```

3.130.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.130.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\arctan(c\,x^3)}dx$$

input
```
int((d*x)^m/(a+b*arctan(c*x^3)),x)
```

output
```
int((d*x)^m/(a+b*arctan(c*x^3)),x)
```

3.130.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx = \int \frac{(dx)^m}{b\arctan(cx^3)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arctan(c*x^3) + a), x)
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atan(c*x**3)),x)
```

output
```
Timed out
```

3.130.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx = \int \frac{(dx)^m}{b\arctan(cx^3)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arctan(c*x^3) + a), x)
```

3.130.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx = \int \frac{(dx)^m}{b\arctan(cx^3)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctan(c*x^3) + a), x)
```

3.130.9 Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx = \int \frac{(d\,x)^m}{a+b\,\mathrm{atan}\left(c\,x^3\right)}\,dx$$

input
```
int((d*x)^m/(a + b*atan(c*x^3)),x)
```

output
```
int((d*x)^m/(a + b*atan(c*x^3)), x)
```

3.131 $\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2} dx$

3.131.1 Optimal result . . . 855
3.131.2 Mathematica [N/A] . . . 855
3.131.3 Rubi [N/A] . . . 856
3.131.4 Maple [N/A] (verified) . . . 856
3.131.5 Fricas [N/A] . . . 857
3.131.6 Sympy [F(-1)] . . . 857
3.131.7 Maxima [N/A] . . . 857
3.131.8 Giac [N/A] . . . 858
3.131.9 Mupad [N/A] . . . 858

3.131.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2} dx = \text{Int}\left(\frac{(dx)^m}{(a+b\arctan(cx^3))^2}, x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

3.131.2 Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2} dx = \int \frac{(dx)^m}{(a+b\arctan(cx^3))^2} dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^3])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTan[c*x^3])^2, x]
```

3.131.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5377}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2} \, dx$$

$$\Big\downarrow \text{5377}$$

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTan[c*x^3])^2,x]
```

output
```
$Aborted
```

3.131.3.1 Defintions of rubi rules used

rule 5377
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Sy
mbol] :> Unintegrable[(d*x)^m*(a + b*ArcTan[c*x^n])^p, x] /; FreeQ[{a, b, c
, d, m, n, p}, x]
```

3.131.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b\arctan(c\,x^3))^2}dx$$

input
```
int((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

output
```
int((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

3.131.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^3)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2), x)
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atan(c*x**3))**2,x)
```

output
```
Timed out
```

3.131.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 7.33

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^3)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

output
```
-1/3*((c^2*d^m*x^6 + d^m)*x^m - 3*(b^2*c*x^2*arctan(c*x^3) + a*b*c*x^2)*in
tegrate(1/3*((c^2*d^m*m + 4*c^2*d^m)*x^6 + d^m*m - 2*d^m)*x^m/(b^2*c*x^3*a
rctan(c*x^3) + a*b*c*x^3), x))/(b^2*c*x^2*arctan(c*x^3) + a*b*c*x^2)
```

3.131.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a + b\arctan(cx^3))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^3) + a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctan(c*x^3) + a)^2, x)
```

3.131.9 Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a + b\arctan(cx^3))^2}\,dx = \int \frac{(d\,x)^m}{(a + b\,\mathrm{atan}(c\,x^3))^2}\,dx$$

input
```
int((d*x)^m/(a + b*atan(c*x^3))^2,x)
```

output
```
int((d*x)^m/(a + b*atan(c*x^3))^2, x)
```

3.132 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$

3.132.1 Optimal result 859
3.132.2 Mathematica [A] (verified) 859
3.132.3 Rubi [A] (verified) 860
3.132.4 Maple [A] (verified) 861
3.132.5 Fricas [A] (verification not implemented) 861
3.132.6 Sympy [A] (verification not implemented) 862
3.132.7 Maxima [A] (verification not implemented) 862
3.132.8 Giac [C] (verification not implemented) 862
3.132.9 Mupad [B] (verification not implemented) 863

3.132.1 Optimal result

Integrand size = 14, antiderivative size = 50

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc^4\arctan\left(\frac{x}{c}\right)$$

output
```
-1/4*b*c^3*x+1/12*b*c*x^3+1/4*x^4*(a+b*arctan(c/x))+1/4*b*c^4*arctan(x/c)
```

3.132.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{ax^4}{4}-\frac{1}{4}bc^4\arctan\left(\frac{c}{x}\right)+\frac{1}{4}bx^4\arctan\left(\frac{c}{x}\right)$$

input
```
Integrate[x^3*(a + b*ArcTan[c/x]),x]
```

output
```
-1/4*(b*c^3*x) + (b*c*x^3)/12 + (a*x^4)/4 - (b*c^4*ArcTan[c/x])/4 + (b*x^4
*ArcTan[c/x])/4
```

3.132.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 795, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right) \, dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{4} b c \int \frac{x^2}{\frac{c^2}{x^2}+1}dx+\frac{1}{4} x^4 \left(a+b \arctan \left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 795}$$

$$\frac{1}{4} b c \int \frac{x^4}{c^2+x^2}dx+\frac{1}{4} x^4 \left(a+b \arctan \left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{4} b c \int \left(\frac{c^4}{c^2+x^2}-c^2+x^2\right)dx+\frac{1}{4} x^4 \left(a+b \arctan \left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4} x^4 \left(a+b \arctan \left(\frac{c}{x}\right)\right)+\frac{1}{4} b c \left(c^3 \arctan \left(\frac{x}{c}\right)-c^2 x+\frac{x^3}{3}\right)$$

input
```
Int[x^3*(a + b*ArcTan[c/x]),x]
```

output
```
(x^4*(a + b*ArcTan[c/x]))/4 + (b*c*(-(c^2*x) + x^3/3 + c^3*ArcTan[x/c]))/4
```

3.132.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.132.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{x^4 \arctan\left(\frac{c}{x}\right) b}{4}-\frac{\arctan\left(\frac{c}{x}\right) b\, c^4}{4}+\frac{a\, x^4}{4}+\frac{b c\, x^3}{12}-\frac{b\, c^3 x}{4}$	46
parts	$\frac{a\, x^4}{4}+\frac{x^4 \arctan\left(\frac{c}{x}\right) b}{4}+\frac{b c\, x^3}{12}-\frac{b\, c^3 x}{4}+\frac{b\, c^4 \arctan\left(\frac{x}{c}\right)}{4}$	46
derivativedivides	$-c^4\left(-\frac{a\, x^4}{4c^4}+b\left(-\frac{x^4 \arctan\left(\frac{c}{x}\right)}{4c^4}-\frac{x^3}{12c^3}+\frac{x}{4c}+\frac{\arctan\left(\frac{c}{x}\right)}{4}\right)\right)$	55
default	$-c^4\left(-\frac{a\, x^4}{4c^4}+b\left(-\frac{x^4 \arctan\left(\frac{c}{x}\right)}{4c^4}-\frac{x^3}{12c^3}+\frac{x}{4c}+\frac{\arctan\left(\frac{c}{x}\right)}{4}\right)\right)$	55
risch	Expression too large to display	697

input
```
int(x^3*(a+b*arctan(c/x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arctan(c/x)*b-1/4*arctan(c/x)*b*c^4+1/4*a*x^4+1/12*b*c*x^3-1/4*b*c
^3*x
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=-\frac{1}{4}\,bc^3x+\frac{1}{12}\,bcx^3+\frac{1}{4}\,ax^4-\frac{1}{4}\,(bc^4-bx^4)\arctan\left(\frac{c}{x}\right)$$

input
```
integrate(x^3*(a+b*arctan(c/x)),x, algorithm="fricas")
```

output
```
-1/4*b*c^3*x + 1/12*b*c*x^3 + 1/4*a*x^4 - 1/4*(b*c^4 - b*x^4)*arctan(c/x)
```

3.132.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{ax^4}{4}-\frac{bc^4\operatorname{atan}\left(\frac{c}{x}\right)}{4}-\frac{bc^3x}{4}+\frac{bcx^3}{12}+\frac{bx^4\operatorname{atan}\left(\frac{c}{x}\right)}{4}$$

input
```
integrate(x**3*(a+b*atan(c/x)),x)
```

output
```
a*x**4/4 - b*c**4*atan(c/x)/4 - b*c**3*x/4 + b*c*x**3/12 + b*x**4*atan(c/x
)/4
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{4}\,ax^4+\frac{1}{12}\left(3\,x^4\arctan\left(\frac{c}{x}\right)+\left(3\,c^3\arctan\left(\frac{x}{c}\right)-3\,c^2x+x^3\right)c\right)b$$

input
```
integrate(x^3*(a+b*arctan(c/x)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/12*(3*x^4*arctan(c/x) + (3*c^3*arctan(x/c) - 3*c^2*x + x^3)*
c)*b
```

3.132.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.62

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{\left(6\,bc^5\arctan\left(\frac{c}{x}\right)-\frac{3i\,bc^9\log\left(\frac{i\,c}{x}-1\right)}{x^4}+\frac{3i\,bc^9\log\left(-\frac{i\,c}{x}-1\right)}{x^4}+6\,ac^5-\frac{6\,bc^8}{x^3}+\frac{2\,bc^6}{x}\right)x^4}{24\,c^5}$$

input
```
integrate(x^3*(a+b*arctan(c/x)),x, algorithm="giac")
```

output
```
1/24*(6*b*c^5*arctan(c/x) - 3*I*b*c^9*log(I*c/x - 1)/x^4 + 3*I*b*c^9*log(-
I*c/x - 1)/x^4 + 6*a*c^5 - 6*b*c^8/x^3 + 2*b*c^6/x)*x^4/c^5
```

3.132.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{a\,x^4}{4}-\frac{b\,c^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{4}+\frac{b\,x^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{4}+\frac{b\,c\,x^3}{12}-\frac{b\,c^3\,x}{4}$$

input
```
int(x^3*(a + b*atan(c/x)),x)
```

output
```
(a*x^4)/4 - (b*c^4*atan(c/x))/4 + (b*x^4*atan(c/x))/4 + (b*c*x^3)/12 - (b*
c^3*x)/4
```

3.133 $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$

3.133.1 Optimal result 864
3.133.2 Mathematica [A] (verified) 864
3.133.3 Rubi [A] (verified) 865
3.133.4 Maple [A] (verified) 866
3.133.5 Fricas [A] (verification not implemented) 867
3.133.6 Sympy [A] (verification not implemented) 867
3.133.7 Maxima [A] (verification not implemented) 867
3.133.8 Giac [A] (verification not implemented) 868
3.133.9 Mupad [B] (verification not implemented) 868

3.133.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{6}bcx^2+\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)$$

output
```
1/6*b*c*x^2+1/3*x^3*(a+b*arctan(c/x))-1/6*b*c^3*ln(c^2+x^2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{6}bcx^2+\frac{ax^3}{3}+\frac{1}{3}bx^3\arctan\left(\frac{c}{x}\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)$$

input
```
Integrate[x^2*(a + b*ArcTan[c/x]),x]
```

output
```
(b*c*x^2)/6 + (a*x^3)/3 + (b*x^3*ArcTan[c/x])/3 - (b*c^3*Log[c^2 + x^2])/6
```

3.133.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5361, 795, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b \arctan \left(\frac{c}{x}\right)\right) \, dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3} b c \int \frac{x}{\frac{c^2}{x^2}+1}dx+\frac{1}{3} x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 795}$$

$$\frac{1}{3} b c \int \frac{x^3}{c^2+x^2}dx+\frac{1}{3} x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{6} b c \int \frac{x^2}{c^2+x^2}dx^2+\frac{1}{3} x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{6} b c \int \left(1-\frac{c^2}{c^2+x^2}\right)dx^2+\frac{1}{3} x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)+\frac{1}{6} b c \left(x^2-c^2 \log \left(c^2+x^2\right)\right)$$

input
```
Int[x^2*(a + b*ArcTan[c/x]),x]
```

output
```
(x^3*(a + b*ArcTan[c/x]))/3 + (b*c*(x^2 - c^2*Log[c^2 + x^2]))/6
```

3.133.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.133.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$-\frac{b\,c^{3}\ln\left(c^{2}+x^{2}\right)}{6}+\frac{b\,x^{3}\arctan\left(\frac{c}{x}\right)}{3}+\frac{x^{3}a}{3}+\frac{bc\,x^{2}}{6}-\frac{b\,c^{3}}{6}$	47
parts	$\frac{x^{3}a}{3}-b\,c^{3}\left(-\frac{x^{3}\arctan\left(\frac{c}{x}\right)}{3c^{3}}+\frac{\ln\left(1+\frac{c^{2}}{x^{2}}\right)}{6}-\frac{x^{2}}{6c^{2}}-\frac{\ln\left(\frac{c}{x}\right)}{3}\right)$	57
derivativedivides	$-c^{3}\left(-\frac{a\,x^{3}}{3c^{3}}+b\left(-\frac{x^{3}\arctan\left(\frac{c}{x}\right)}{3c^{3}}+\frac{\ln\left(1+\frac{c^{2}}{x^{2}}\right)}{6}-\frac{x^{2}}{6c^{2}}-\frac{\ln\left(\frac{c}{x}\right)}{3}\right)\right)$	61
default	$-c^{3}\left(-\frac{a\,x^{3}}{3c^{3}}+b\left(-\frac{x^{3}\arctan\left(\frac{c}{x}\right)}{3c^{3}}+\frac{\ln\left(1+\frac{c^{2}}{x^{2}}\right)}{6}-\frac{x^{2}}{6c^{2}}-\frac{\ln\left(\frac{c}{x}\right)}{3}\right)\right)$	61
risch	Expression too large to display	692

input
```
int(x^2*(a+b*arctan(c/x)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*b*c^3*ln(c^2+x^2)+1/3*b*x^3*arctan(c/x)+1/3*x^3*a+1/6*b*c*x^2-1/6*b*c
^3
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{3}\,bx^3\arctan\left(\frac{c}{x}\right)-\frac{1}{6}\,bc^3\log\left(c^2+x^2\right)+\frac{1}{6}\,bcx^2+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(a+b*arctan(c/x)),x, algorithm="fricas")
```

output
```
1/3*b*x^3*arctan(c/x) - 1/6*b*c^3*log(c^2 + x^2) + 1/6*b*c*x^2 + 1/3*a*x^3
```

3.133.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{ax^3}{3}-\frac{bc^3\log\left(c^2+x^2\right)}{6}+\frac{bcx^2}{6}+\frac{bx^3\operatorname{atan}\left(\frac{c}{x}\right)}{3}$$

input
```
integrate(x**2*(a+b*atan(c/x)),x)
```

output
```
a*x**3/3 - b*c**3*log(c**2 + x**2)/6 + b*c*x**2/6 + b*x**3*atan(c/x)/3
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\arctan\left(\frac{c}{x}\right)-\left(c^2\log\left(c^2+x^2\right)-x^2\right)c\right)b$$

input
```
integrate(x^2*(a+b*arctan(c/x)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*b
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{\left(2\,bc^4\arctan\left(\frac{c}{x}\right)-\frac{bc^7\log\left(\frac{c^2}{x^2}+1\right)}{x^3}+\frac{2\,bc^7\log\left(\frac{c}{x}\right)}{x^3}+2\,ac^4+\frac{bc^5}{x}\right)x^3}{6\,c^4}$$

input
```
integrate(x^2*(a+b*arctan(c/x)),x, algorithm="giac")
```

output
```
1/6*(2*b*c^4*arctan(c/x) - b*c^7*log(c^2/x^2 + 1)/x^3 + 2*b*c^7*log(c/x)/x
^3 + 2*a*c^4 + b*c^5/x)*x^3/c^4
```

3.133.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{a\,x^3}{3}+\frac{b\,x^3\,\mathrm{atan}\left(\frac{c}{x}\right)}{3}-\frac{b\,c^3\,\ln\left(c^2+x^2\right)}{6}+\frac{b\,c\,x^2}{6}$$

input
```
int(x^2*(a + b*atan(c/x)),x)
```

output
```
(a*x^3)/3 + (b*x^3*atan(c/x))/3 - (b*c^3*log(c^2 + x^2))/6 + (b*c*x^2)/6
```

3.134 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$

3.134.1 Optimal result 869
3.134.2 Mathematica [A] (verified) 869
3.134.3 Rubi [A] (verified) 870
3.134.4 Maple [A] (verified) 871
3.134.5 Fricas [A] (verification not implemented) 872
3.134.6 Sympy [A] (verification not implemented) 872
3.134.7 Maxima [A] (verification not implemented) 872
3.134.8 Giac [C] (verification not implemented) 873
3.134.9 Mupad [B] (verification not implemented) 873

3.134.1 Optimal result

Integrand size = 12, antiderivative size = 39

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\arctan\left(\frac{x}{c}\right)$$

output
```
1/2*b*c*x+1/2*x^2*(a+b*arctan(c/x))-1/2*b*c^2*arctan(x/c)
```

3.134.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{bcx}{2}+\frac{ax^2}{2}+\frac{1}{2}bc^2\arctan\left(\frac{c}{x}\right)+\frac{1}{2}bx^2\arctan\left(\frac{c}{x}\right)$$

input
```
Integrate[x*(a + b*ArcTan[c/x]),x]
```

output
```
(b*c*x)/2 + (a*x^2)/2 + (b*c^2*ArcTan[c/x])/2 + (b*x^2*ArcTan[c/x])/2
```

3.134.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5361, 772, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}bc\int \frac{1}{\frac{c^2}{x^2}+1}dx+\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{772}$$

$$\frac{1}{2}bc\int \frac{x^2}{c^2+x^2}dx+\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}bc\left(x-c^2\int \frac{1}{c^2+x^2}dx\right)+\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(x-c\arctan\left(\frac{x}{c}\right)\right)$$

input
```
Int[x*(a + b*ArcTan[c/x]),x]
```

output
```
(x^2*(a + b*ArcTan[c/x]))/2 + (b*c*(x - c*ArcTan[x/c]))/2
```

3.134.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.134.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

method	result	size
parts	$\frac{a\,x^2}{2}+\frac{\arctan\left(\frac{c}{x}\right)b\,x^2}{2}-\frac{b\,c^2\arctan\left(\frac{x}{c}\right)}{2}+\frac{xbc}{2}$	37
parallelrisch	$\frac{\arctan\left(\frac{c}{x}\right)b\,x^2}{2}+\frac{\arctan\left(\frac{c}{x}\right)b\,c^2}{2}+\frac{a\,x^2}{2}+\frac{xbc}{2}-\frac{a\,c^2}{2}$	43
derivativedivides	$-c^2\left(-\frac{a\,x^2}{2c^2}+b\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)}{2c^2}-\frac{x}{2c}-\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)\right)$	47
default	$-c^2\left(-\frac{a\,x^2}{2c^2}+b\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)}{2c^2}-\frac{x}{2c}-\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)\right)$	47
risch	Expression too large to display	688

input
```
int(x*(a+b*arctan(c/x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/2*arctan(c/x)*b*x^2-1/2*b*c^2*arctan(x/c)+1/2*x*b*c
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{2}\,bcx+\frac{1}{2}\,ax^2+\frac{1}{2}\left(bc^2+bx^2\right)\arctan\left(\frac{c}{x}\right)$$

input
```
integrate(x*(a+b*arctan(c/x)),x, algorithm="fricas")
```

output
```
1/2*b*c*x + 1/2*a*x^2 + 1/2*(b*c^2 + b*x^2)*arctan(c/x)
```

3.134.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{ax^2}{2}+\frac{bc^2\operatorname{atan}\left(\frac{c}{x}\right)}{2}+\frac{bcx}{2}+\frac{bx^2\operatorname{atan}\left(\frac{c}{x}\right)}{2}$$

input
```
integrate(x*(a+b*atan(c/x)),x)
```

output
```
a*x**2/2 + b*c**2*atan(c/x)/2 + b*c*x/2 + b*x**2*atan(c/x)/2
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\arctan\left(\frac{c}{x}\right)-\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\right)b$$

input
```
integrate(x*(a+b*arctan(c/x)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/2*(x^2*arctan(c/x) - (c*arctan(x/c) - x)*c)*b
```

3.134.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx = \frac{\left(2\,bc^3\arctan\left(\frac{c}{x}\right)-\frac{i\,bc^5\log\left(\frac{i\,c}{x}+1\right)}{x^2}+\frac{i\,bc^5\log\left(-\frac{i\,c}{x}+1\right)}{x^2}+2\,ac^3+\frac{2\,bc^4}{x}\right)x^2}{4\,c^3}$$

input
```
integrate(x*(a+b*arctan(c/x)),x, algorithm="giac")
```

output
```
1/4*(2*b*c^3*arctan(c/x) - I*b*c^5*log(I*c/x + 1)/x^2 + I*b*c^5*log(-I*c/x
 + 1)/x^2 + 2*a*c^3 + 2*b*c^4/x)*x^2/c^3
```

3.134.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx = \frac{a\,x^2}{2}+\frac{b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{b\,x^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{b\,c\,x}{2}$$

input
```
int(x*(a + b*atan(c/x)),x)
```

output
```
(a*x^2)/2 + (b*c^2*atan(c/x))/2 + (b*x^2*atan(c/x))/2 + (b*c*x)/2
```

3.135 $\int \left(a+b \arctan \left(\frac{c}{x}\right)\right) \, dx$

3.135.1 Optimal result . . . 874
3.135.2 Mathematica [A] (verified) . . . 874
3.135.3 Rubi [A] (verified) . . . 875
3.135.4 Maple [A] (verified) . . . 875
3.135.5 Fricas [A] (verification not implemented) . . . 876
3.135.6 Sympy [A] (verification not implemented) . . . 876
3.135.7 Maxima [A] (verification not implemented) . . . 876
3.135.8 Giac [A] (verification not implemented) . . . 877
3.135.9 Mupad [B] (verification not implemented) . . . 877

3.135.1 Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \left(a+b \arctan \left(\frac{c}{x}\right)\right) \, dx=a x+b x \arctan \left(\frac{c}{x}\right)+\frac{1}{2} b c \log \left(c^2+x^2\right)$$

output
```
a*x+b*x*arctan(c/x)+1/2*b*c*ln(c^2+x^2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a+b \arctan \left(\frac{c}{x}\right)\right) \, dx=a x+b x \arctan \left(\frac{c}{x}\right)+\frac{1}{2} b c \log \left(c^2+x^2\right)$$

input
```
Integrate[a + b*ArcTan[c/x],x]
```

output
```
a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

3.135.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+bx\arctan\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2+x^2\right)$$

input
```
Int[a + b*ArcTan[c/x],x]
```

output
```
a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

3.135.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.135.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$b\left(\frac{c\ln\left(c^2+x^2\right)}{2}+\arctan\left(\frac{c}{x}\right)x\right)+ax$	27
default	$ax-bc\left(-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)$	40
parts	$ax-bc\left(-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)$	40
derivativedivides	$-c\left(-\frac{ax}{c}+b\left(-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)\right)$	45
risch	Expression too large to display	642

input
```
int(a+b*arctan(c/x),x,method=_RETURNVERBOSE)
```

output
```
b*(1/2*c*ln(c^2+x^2)+arctan(c/x)*x)+a*x
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx = bx\arctan\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2+x^2\right)+ax$$

input
```
integrate(a+b*arctan(c/x),x, algorithm="fricas")
```

output
```
b*x*arctan(c/x) + 1/2*b*c*log(c^2 + x^2) + a*x
```

3.135.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx = ax+b\left(\frac{c\log\left(c^2+x^2\right)}{2}+x\operatorname{atan}\left(\frac{c}{x}\right)\right)$$

input
```
integrate(a+b*atan(c/x),x)
```

output
```
a*x + b*(c*log(c**2 + x**2)/2 + x*atan(c/x))
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx = \frac{1}{2}\left(2\,x\arctan\left(\frac{c}{x}\right)+c\log\left(c^2+x^2\right)\right)b+ax$$

input
```
integrate(a+b*arctan(c/x),x, algorithm="maxima")
```

output
```
1/2*(2*x*arctan(c/x) + c*log(c^2 + x^2))*b + a*x
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx = ax + \frac{\left(c^2\left(\log\left(\frac{c^2}{x^2}+1\right)-\log\left(\frac{c^2}{x^2}\right)\right)+2\,cx\arctan\left(\frac{c}{x}\right)\right)b}{2\,c}$$

input
```
integrate(a+b*arctan(c/x),x, algorithm="giac")
```

output
```
a*x + 1/2*(c^2*(log(c^2/x^2 + 1) - log(c^2/x^2)) + 2*c*x*arctan(c/x))*b/c
```

3.135.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx = a\,x + b\,x\,\mathrm{atan}\left(\frac{c}{x}\right) + \frac{b\,c\,\ln\left(c^2+x^2\right)}{2}$$

input
```
int(a + b*atan(c/x),x)
```

output
```
a*x + b*x*atan(c/x) + (b*c*log(c^2 + x^2))/2
```

3.136 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx$

3.136.1 Optimal result 878
3.136.2 Mathematica [A] (verified) 878
3.136.3 Rubi [A] (verified) 879
3.136.4 Maple [B] (verified) 880
3.136.5 Fricas [F] 880
3.136.6 Sympy [F] 881
3.136.7 Maxima [F] 881
3.136.8 Giac [B] (verification not implemented) 881
3.136.9 Mupad [B] (verification not implemented) 882

3.136.1 Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx = a\log(x) - \frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{ic}{x}\right) + \frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{ic}{x}\right)$$

output
```
a*ln(x)-1/2*I*b*polylog(2,-I*c/x)+1/2*I*b*polylog(2,I*c/x)
```

3.136.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx = a\log(x) - \frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{ic}{x}\right) + \frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{ic}{x}\right)$$

input
```
Integrate[(a + b*ArcTan[c/x])/x,x]
```

output
```
a*Log[x] - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*c)/x]
```

3.136.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx$$

$$\Big\downarrow \; 5359$$

$$-\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \; 5355$$

$$-\frac{1}{2}ib\int x\log\left(1-\frac{ic}{x}\right)d\frac{1}{x}+\frac{1}{2}ib\int x\log\left(\frac{ic}{x}+1\right)d\frac{1}{x}-a\log\left(\frac{1}{x}\right)$$

$$\Big\downarrow \; 2838$$

$$-a\log\left(\frac{1}{x}\right)-\frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{ic}{x}\right)$$

input
```
Int[(a + b*ArcTan[c/x])/x,x]
```

output
```
-(a*Log[x^(-1)]) - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*
c)/x]
```

3.136.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

3.136.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.

Time = 1.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

method	result
parts	$a\ln\left(x\right)+b\left(-\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{ic}{x}\right)}{2}+\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)}{2}-\frac{i\,\mathrm{dilog}\left(1+\frac{ic}{x}\right)}{2}+\frac{i\,\mathrm{dilog}(1-}{2}$
derivativedivides	$-a\ln\left(\frac{c}{x}\right)-b\left(\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)+\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{ic}{x}\right)}{2}-\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)}{2}+\frac{i\,\mathrm{dilog}\left(1+\frac{ic}{x}\right)}{2}-\frac{i\,\mathrm{dilog}(1-}{2}$
default	$-a\ln\left(\frac{c}{x}\right)-b\left(\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)+\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{ic}{x}\right)}{2}-\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)}{2}+\frac{i\,\mathrm{dilog}\left(1+\frac{ic}{x}\right)}{2}-\frac{i\,\mathrm{dilog}(1-}{2}$
risch	Expression too large to display

input
```
int((a+b*arctan(c/x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*(-ln(c/x)*arctan(c/x)-1/2*I*ln(c/x)*ln(1+I*c/x)+1/2*I*ln(c/x)*ln
(1-I*c/x)-1/2*I*dilog(1+I*c/x)+1/2*I*dilog(1-I*c/x))
```

3.136.5 Fricas [F]

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx=\int \frac{b\arctan\left(\frac{c}{x}\right)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))/x,x, algorithm="fricas")
```

output
```
integral((b*arctan(c/x) + a)/x, x)
```

3.136.6 Sympy [F]

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx=\int \frac{a+b\operatorname{atan}\left(\frac{c}{x}\right)}{x}\,dx$$

input
```
integrate((a+b*atan(c/x))/x,x)
```

output
```
Integral((a + b*atan(c/x))/x, x)
```

3.136.7 Maxima [F]

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx=\int \frac{b\arctan\left(\frac{c}{x}\right)+a}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))/x,x, algorithm="maxima")
```

output
```
b*integrate(arctan2(c, x)/x, x) + a*log(x)
```

3.136.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(25) = 50$.

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx=-\frac{\left(2\,bc^4\arctan\left(\frac{c}{x}\right)-\frac{i\,bc^6\log\left(\frac{i\,c}{x}+1\right)}{x^2}+\frac{i\,bc^6\log\left(-\frac{i\,c}{x}+1\right)}{x^2}+2\,ac^4+\frac{2\,bc^5}{x}\right)x^2}{4\,c^5}$$

input
```
integrate((a+b*arctan(c/x))/x,x, algorithm="giac")
```

output
```
-1/4*(2*b*c^4*arctan(c/x) - I*b*c^6*log(I*c/x + 1)/x^2 + I*b*c^6*log(-I*c/
x + 1)/x^2 + 2*a*c^4 + 2*b*c^5/x)*x^2/c^5
```

3.136.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx = a\,\ln\left(x\right)+\frac{b\left(\mathrm{Li}_2\left(1-\frac{c\,1\mathrm{i}}{x}\right)-\mathrm{Li}_2\left(1+\frac{c\,1\mathrm{i}}{x}\right)\right)\,1\mathrm{i}}{2}$$

input
```
int((a + b*atan(c/x))/x,x)
```

output
```
(b*(dilog(1 - (c*1i)/x) - dilog((c*1i)/x + 1))*1i)/2 + a*log(x)
```

3.137 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx$

3.137.1 Optimal result 883
3.137.2 Mathematica [A] (verified) 883
3.137.3 Rubi [A] (verified) 884
3.137.4 Maple [A] (verified) 885
3.137.5 Fricas [A] (verification not implemented) 885
3.137.6 Sympy [A] (verification not implemented) 886
3.137.7 Maxima [A] (verification not implemented) 886
3.137.8 Giac [A] (verification not implemented) 886
3.137.9 Mupad [B] (verification not implemented) 887

3.137.1 Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx=-\frac{a+b\arctan\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(1+\frac{c^2}{x^2}\right)}{2c}$$

output
```
(-a-b*arctan(c/x))/x+1/2*b*ln(1+c^2/x^2)/c
```

3.137.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx=-\frac{a}{x}-\frac{b\arctan\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(1+\frac{c^2}{x^2}\right)}{2c}$$

input
```
Integrate[(a + b*ArcTan[c/x])/x^2,x]
```

output
```
-(a/x) - (b*ArcTan[c/x])/x + (b*Log[1 + c^2/x^2])/(2*c)
```

3.137.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5361, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}dx$$

$$\Big\downarrow \text{5361}$$

$$-bc\int \frac{1}{\left(\frac{c^2}{x^2}+1\right)x^3}dx-\frac{a+b\arctan\left(\frac{c}{x}\right)}{x}$$

$$\Big\downarrow \text{792}$$

$$\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}-\frac{a+b\arctan\left(\frac{c}{x}\right)}{x}$$

input
```
Int[(a + b*ArcTan[c/x])/x^2,x]
```

output
```
-((a + b*ArcTan[c/x])/x) + (b*Log[1 + c^2/x^2])/(2*c)
```

3.137.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.137.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
parts	$-\frac{a}{x}-\frac{b \arctan\left(\frac{c}{x}\right)}{x}+\frac{b \ln\left(1+\frac{c^{2}}{x^{2}}\right)}{2c}$	36
derivativedivides	$-\frac{\frac{ca}{x}+b\left(\frac{c \arctan\left(\frac{c}{x}\right)}{x}-\frac{\ln\left(1+\frac{c^{2}}{x^{2}}\right)}{2}\right)}{c}$	39
default	$-\frac{\frac{ca}{x}+b\left(\frac{c \arctan\left(\frac{c}{x}\right)}{x}-\frac{\ln\left(1+\frac{c^{2}}{x^{2}}\right)}{2}\right)}{c}$	39
parallelrisch	$-\frac{2xb \ln\left(x\right)-b \ln\left(c^{2}+x^{2}\right) x+2 \arctan\left(\frac{c}{x}\right) bc+2ac}{2cx}$	42
risch	Expression too large to display	652

input
```
int((a+b*arctan(c/x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x-b/x*arctan(c/x)+1/2*b*ln(1+c^2/x^2)/c
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx=-\frac{2\,bc\arctan\left(\frac{c}{x}\right)-bx\log\left(c^2+x^2\right)+2\,bx\log\left(x\right)+2\,ac}{2\,cx}$$

input
```
integrate((a+b*arctan(c/x))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*c*arctan(c/x) - b*x*log(c^2 + x^2) + 2*b*x*log(x) + 2*a*c)/(c*x)
```

3.137.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx=\begin{cases}-\frac{a}{x}-\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{x}-\frac{b\log(x)}{c}+\frac{b\log(c^2+x^2)}{2c} & \text{for } c\neq 0\\ -\frac{a}{x} & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*atan(c/x))/x**2,x)
```

output
```
Piecewise((-a/x - b*atan(c/x)/x - b*log(x)/c + b*log(c**2 + x**2)/(2*c), N
e(c, 0)), (-a/x, True))
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx=-\frac{b\left(\frac{2\,c\arctan\left(\frac{c}{x}\right)}{x}-\log\left(\frac{c^2}{x^2}+1\right)\right)}{2\,c}-\frac{a}{x}$$

input
```
integrate((a+b*arctan(c/x))/x^2,x, algorithm="maxima")
```

output
```
-1/2*b*(2*c*arctan(c/x)/x - log(c^2/x^2 + 1))/c - a/x
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx=-\frac{\frac{2\,bc\arctan\left(\frac{c}{x}\right)}{x}-b\log\left(\frac{c^2}{x^2}+1\right)+\frac{2\,ac}{x}}{2\,c}$$

input
```
integrate((a+b*arctan(c/x))/x^2,x, algorithm="giac")
```

output
```
-1/2*(2*b*c*arctan(c/x)/x - b*log(c^2/x^2 + 1) + 2*a*c/x)/c
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx=\frac{\frac{b\,\ln\left(c^2+x^2\right)}{2}-b\,\ln\left(x\right)}{c}-\frac{a\,c+b\,c\,\mathrm{atan}\left(\frac{c}{x}\right)}{c\,x}$$

input
```
int((a + b*atan(c/x))/x^2,x)
```

output
```
((b*log(c^2 + x^2))/2 - b*log(x))/c - (a*c + b*c*atan(c/x))/(c*x)
```

3.138 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx$

3.138.1 Optimal result . . . 888
3.138.2 Mathematica [A] (verified) . . . 888
3.138.3 Rubi [A] (verified) . . . 889
3.138.4 Maple [A] (verified) . . . 890
3.138.5 Fricas [A] (verification not implemented) . . . 891
3.138.6 Sympy [A] (verification not implemented) . . . 891
3.138.7 Maxima [A] (verification not implemented) . . . 891
3.138.8 Giac [C] (verification not implemented) . . . 892
3.138.9 Mupad [B] (verification not implemented) . . . 892

3.138.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx=\frac{b}{2cx}-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}+\frac{b\arctan\left(\frac{x}{c}\right)}{2c^2}$$

output
```
1/2*b/c/x+1/2*(-a-b*arctan(c/x))/x^2+1/2*b*arctan(x/c)/c^2
```

3.138.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx=-\frac{a}{2x^2}+\frac{b}{2cx}-\frac{b\arctan\left(\frac{c}{x}\right)}{2x^2}+\frac{b\arctan\left(\frac{x}{c}\right)}{2c^2}$$

input
```
Integrate[(a + b*ArcTan[c/x])/x^3,x]
```

output
```
-1/2*a/x^2 + b/(2*c*x) - (b*ArcTan[c/x])/(2*x^2) + (b*ArcTan[x/c])/(2*c^2)
```

3.138.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5361, 795, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}dx$$

$$\Big\downarrow \text{ 5361}$$

$$-\frac{1}{2}bc\int \frac{1}{\left(\frac{c^2}{x^2}+1\right)x^4}dx-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 795}$$

$$-\frac{1}{2}bc\int \frac{1}{x^2\left(c^2+x^2\right)}dx-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{1}{2}bc\left(-\frac{\int \frac{1}{c^2+x^2}dx}{c^2}-\frac{1}{c^2x}\right)-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{1}{2}bc\left(-\frac{\arctan\left(\frac{x}{c}\right)}{c^3}-\frac{1}{c^2x}\right)$$

input
```
Int[(a + b*ArcTan[c/x])/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c/x])/x^2 - (b*c*(-(1/(c^2*x)) - ArcTan[x/c]/c^3))/2
```

3.138.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.138.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
parts	$-\frac{a}{2x^2}-\frac{b\arctan\left(\frac{c}{x}\right)}{2x^2}+\frac{b}{2cx}+\frac{b\arctan\left(\frac{x}{c}\right)}{2c^2}$	41
parallelrisch	$-\frac{\arctan\left(\frac{c}{x}\right)b\,x^2+\arctan\left(\frac{c}{x}\right)b\,c^2-xbc+a\,c^2}{2x^2c^2}$	42
derivativedivides	$-\frac{\frac{a\,c^2}{2x^2}+b\left(\frac{c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{c}{2x}+\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)}{c^2}$	47
default	$-\frac{\frac{a\,c^2}{2x^2}+b\left(\frac{c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{c}{2x}+\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)}{c^2}$	47
risch	Expression too large to display	709

input
```
int((a+b*arctan(c/x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2-1/2*b/x^2*arctan(c/x)+1/2*b/c/x+1/2*b*arctan(x/c)/c^2
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{a + b \arctan\left(\frac{c}{x}\right)}{x^3}\,dx = -\frac{ac^2 - bcx + (bc^2 + bx^2)\arctan\left(\frac{c}{x}\right)}{2\,c^2x^2}$$

input
```
integrate((a+b*arctan(c/x))/x^3,x, algorithm="fricas")
```

output
```
-1/2*(a*c^2 - b*c*x + (b*c^2 + b*x^2)*arctan(c/x))/(c^2*x^2)
```

3.138.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{a + b \arctan\left(\frac{c}{x}\right)}{x^3}\,dx = \begin{cases} -\frac{a}{2x^2} - \frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{2x^2} + \frac{b}{2cx} - \frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{2c^2} & \text{for } c \neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atan(c/x))/x**3,x)
```

output
```
Piecewise((-a/(2*x**2) - b*atan(c/x)/(2*x**2) + b/(2*c*x) - b*atan(c/x)/(2
*c**2), Ne(c, 0)), (-a/(2*x**2), True))
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{a + b \arctan\left(\frac{c}{x}\right)}{x^3}\,dx = \frac{1}{2}\left(c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3} + \frac{1}{c^2x}\right) - \frac{\arctan\left(\frac{c}{x}\right)}{x^2}\right)b - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*arctan(c/x))/x^3,x, algorithm="maxima")
```

output
```
1/2*(c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*b - 1/2*a/x^2
```

3.138.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx = -\frac{i\left(-\frac{2i\,bc^2\arctan\left(\frac{c}{x}\right)}{x^2}+b\log\left(\frac{i\,c}{x}-1\right)-b\log\left(-\frac{i\,c}{x}-1\right)-\frac{2i\,ac^2}{x^2}+\frac{2i\,bc}{x}\right)}{4\,c^2}$$

input
```
integrate((a+b*arctan(c/x))/x^3,x, algorithm="giac")
```

output
```
-1/4*I*(-2*I*b*c^2*arctan(c/x)/x^2 + b*log(I*c/x - 1) - b*log(-I*c/x - 1)
- 2*I*a*c^2/x^2 + 2*I*b*c/x)/c^2
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx = \frac{b\,c\,\mathrm{atan}\left(\frac{x}{\sqrt{c^2}}\right)}{2\,(c^2)^{3/2}}-\frac{\frac{a\,c^2}{2}+\frac{b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}-\frac{b\,c\,x}{2}}{c^2\,x^2}$$

input
```
int((a + b*atan(c/x))/x^3,x)
```

output
```
(b*c*atan(x/(c^2)^(1/2)))/(2*(c^2)^(3/2)) - ((a*c^2)/2 + (b*c^2*atan(c/x))
/2 - (b*c*x)/2)/(c^2*x^2)
```

3.139 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx$

3.139.1 Optimal result . . . 893
3.139.2 Mathematica [A] (verified) . . . 893
3.139.3 Rubi [A] (verified) . . . 894
3.139.4 Maple [A] (verified) . . . 895
3.139.5 Fricas [A] (verification not implemented) . . . 896
3.139.6 Sympy [A] (verification not implemented) . . . 896
3.139.7 Maxima [A] (verification not implemented) . . . 897
3.139.8 Giac [A] (verification not implemented) . . . 897
3.139.9 Mupad [B] (verification not implemented) . . . 897

3.139.1 Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = \frac{b}{6cx^2} - \frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3} + \frac{b\log(x)}{3c^3} - \frac{b\log\left(c^2+x^2\right)}{6c^3}$$

output
```
1/6*b/c/x^2+1/3*(-a-b*arctan(c/x))/x^3+1/3*b*ln(x)/c^3-1/6*b*ln(c^2+x^2)/c
^3
```

3.139.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = -\frac{a}{3x^3} + \frac{b}{6cx^2} - \frac{b\arctan\left(\frac{c}{x}\right)}{3x^3} + \frac{b\log(x)}{3c^3} - \frac{b\log\left(c^2+x^2\right)}{6c^3}$$

input
```
Integrate[(a + b*ArcTan[c/x])/x^4,x]
```

output
```
-1/3*a/x^3 + b/(6*c*x^2) - (b*ArcTan[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) -
(b*Log[c^2 + x^2])/(6*c^3)
```

3.139.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5361, 795, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}dx$$

$$\Big\downarrow \text{5361}$$

$$-\frac{1}{3}bc\int \frac{1}{\left(\frac{c^2}{x^2}+1\right)x^5}dx-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{795}$$

$$-\frac{1}{3}bc\int \frac{1}{x^3\left(c^2+x^2\right)}dx-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{6}bc\int \frac{1}{x^4\left(c^2+x^2\right)}dx^2-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{54}$$

$$-\frac{1}{6}bc\int \left(-\frac{1}{c^4x^2}+\frac{1}{c^2x^4}+\frac{1}{c^4\left(c^2+x^2\right)}\right)dx^2-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}bc\left(-\frac{\log\left(x^2\right)}{c^4}-\frac{1}{c^2x^2}+\frac{\log\left(c^2+x^2\right)}{c^4}\right)$$

input
```
Int[(a + b*ArcTan[c/x])/x^4,x]
```

output
```
-1/3*(a + b*ArcTan[c/x])/x^3 - (b*c*(-(1/(c^2*x^2)) - Log[x^2]/c^4 + Log[c
^2 + x^2]/c^4))/6
```

3.139.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.139.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result	size
parts	$-\frac{a}{3x^3}-\frac{b\arctan\left(\frac{c}{x}\right)}{3x^3}+\frac{b}{6c\,x^2}-\frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{6c^3}$	45
derivativedivides	$-\frac{\frac{a\,c^3}{3x^3}+b\left(\frac{c^3\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{c^2}{6x^2}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{6}\right)}{c^3}$	53
default	$-\frac{\frac{a\,c^3}{3x^3}+b\left(\frac{c^3\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{c^2}{6x^2}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{6}\right)}{c^3}$	53
parallelrisch	$\frac{2b\ln(x)x^3-b\ln(c^2+x^2)x^3-2b\,c^3\arctan\left(\frac{c}{x}\right)+b\,c^2x-2a\,c^3}{6x^3c^3}$	56
risch	Expression too large to display	705

3.139. $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx$

input
```
int((a+b*arctan(c/x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3-1/3*b/x^3*arctan(c/x)+1/6*b/c/x^2-1/6*b/c^3*ln(1+c^2/x^2)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = -\frac{2\,bc^3\arctan\left(\frac{c}{x}\right)+bx^3\log\left(c^2+x^2\right)-2\,bx^3\log\left(x\right)+2\,ac^3-bc^2x}{6\,c^3x^3}$$

input
```
integrate((a+b*arctan(c/x))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(2*b*c^3*arctan(c/x) + b*x^3*log(c^2 + x^2) - 2*b*x^3*log(x) + 2*a*c^
3 - b*c^2*x)/(c^3*x^3)
```

3.139.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = \begin{cases} -\frac{a}{3x^3}-\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{3x^3}+\frac{b}{6cx^2}+\frac{b\log\left(x\right)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3} & \text{for}\ c\neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atan(c/x))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3) - b*atan(c/x)/(3*x**3) + b/(6*c*x**2) + b*log(x)/(3
*c**3) - b*log(c**2 + x**2)/(6*c**3), Ne(c, 0)), (-a/(3*x**3), True))
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{a + b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = -\frac{1}{6}\left(c\left(\frac{\log\left(c^2+x^2\right)}{c^4} - \frac{\log\left(x^2\right)}{c^4} - \frac{1}{c^2x^2}\right) + \frac{2\arctan\left(\frac{c}{x}\right)}{x^3}\right)b - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctan(c/x))/x^4,x, algorithm="maxima")
```

output
```
-1/6*(c*(log(c^2 + x^2)/c^4 - log(x^2)/c^4 - 1/(c^2*x^2)) + 2*arctan(c/x)/
x^3)*b - 1/3*a/x^3
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{a + b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = -\frac{\frac{2\,bc^3\arctan\left(\frac{c}{x}\right)}{x^3} + b\log\left(\frac{c^2}{x^2}+1\right) + \frac{2\,ac^3}{x^3} - \frac{bc^2}{x^2}}{6\,c^3}$$

input
```
integrate((a+b*arctan(c/x))/x^4,x, algorithm="giac")
```

output
```
-1/6*(2*b*c^3*arctan(c/x)/x^3 + b*log(c^2/x^2 + 1) + 2*a*c^3/x^3 - b*c^2/x
^2)/c^3
```

3.139.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{a + b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = \frac{\frac{b\,x^3\,\ln(x)}{3} + \frac{b\,c^2\,x}{6} - \frac{b\,x^3\,\ln\left(c^2+x^2\right)}{6}}{c^3\,x^3} - \frac{\frac{a}{3} + \frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{3}}{x^3}$$

input
```
int((a + b*atan(c/x))/x^4,x)
```

output
```
((b*x^3*log(x))/3 + (b*c^2*x)/6 - (b*x^3*log(c^2 + x^2))/6)/(c^3*x^3) - (a
/3 + (b*atan(c/x))/3)/x^3
```

3.140 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

3.140.1 Optimal result . . . 898
3.140.2 Mathematica [A] (verified) . . . 898
3.140.3 Rubi [A] (warning: unable to verify) . . . 899
3.140.4 Maple [A] (verified) . . . 902
3.140.5 Fricas [A] (verification not implemented) . . . 903
3.140.6 Sympy [A] (verification not implemented) . . . 903
3.140.7 Maxima [A] (verification not implemented) . . . 904
3.140.8 Giac [F] . . . 904
3.140.9 Mupad [B] (verification not implemented) . . . 905

3.140.1 Optimal result

Integrand size = 16, antiderivative size = 122

$$\begin{aligned}\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx &= \frac{1}{12}b^2c^2x^2-\frac{1}{2}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\ &\quad+\frac{1}{6}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\ &\quad-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\ &\quad-\frac{1}{3}b^2c^4\log\left(1+\frac{c^2}{x^2}\right)-\frac{2}{3}b^2c^4\log(x)\end{aligned}$$

output
```
1/12*b^2*c^2*x^2-1/2*b*c^3*x*(a+b*arccot(x/c))+1/6*b*c*x^3*(a+b*arccot(x/c
))-1/4*c^4*(a+b*arccot(x/c))^2+1/4*x^4*(a+b*arccot(x/c))^2-1/3*b^2*c^4*ln(
1+c^2/x^2)-2/3*b^2*c^4*ln(x)
```

3.140.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$\begin{aligned}\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx &= \frac{1}{12}\Bigg(x\left(b^2c^2x+3a^2x^3+2abc\left(-3c^2+x^2\right)\right)\\ &\quad+2b\left(bcx\left(-3c^2+x^2\right)+3a\left(-c^4+x^4\right)\right)\arctan\left(\frac{c}{x}\right)\\ &\quad+3b^2\left(-c^4+x^4\right)\arctan\left(\frac{c}{x}\right)^2-4b^2c^4\log\left(c^2+x^2\right)\Bigg)\end{aligned}$$

input
```
Integrate[x^3*(a + b*ArcTan[c/x])^2,x]
```

output
```
(x*(b^2*c^2*x + 3*a^2*x^3 + 2*a*b*c*(-3*c^2 + x^2)) + 2*b*(b*c*x*(-3*c^2 +
 x^2) + 3*a*(-c^4 + x^4))*ArcTan[c/x] + 3*b^2*(-c^4 + x^4)*ArcTan[c/x]^2 -
 4*b^2*c^4*Log[c^2 + x^2])/12
```

3.140.3 Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5363, 5361, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2dx$$

$$\Big\downarrow \; 5363$$

$$-\int x^5\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\int \frac{x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}$$

$$\Big\downarrow \; 5453$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(\int x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}-c^2\int \frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\int \frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}+\frac{1}{3}bc\int \frac{x^3}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \; 243$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\int \frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}+\frac{1}{6}bc\int \frac{x^2}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \ 54$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\int\frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}+\frac{1}{6}bc\int\left(\frac{c^4}{\frac{c^2}{x^2}+1}-xc^2+x^2\right)d\frac{1}{x^2}-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\int\frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc\left(c^2\log\left(\frac{c^2}{x^2}+1\right)-c^2\log\left(\frac{1}{x^2}\right)-x\right)\right)$$

$$\Big\downarrow \ 5453$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\left(\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}-c^2\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc\left(c^2\log\left(\frac{c}{x}\right.\right.\right.$$

$$\Big\downarrow \ 5361$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\int\frac{x}{\frac{c^2}{x^2}+1}d\frac{1}{x}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right.$$

$$\Big\downarrow \ 243$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\int\frac{x}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right.\right.\right.$$

$$\Big\downarrow \ 47$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(\int xd\frac{1}{x^2}-c^2\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{1}{3}x^3\left(\right.\right.$$

$$\Big\downarrow \ 14$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(-c^2\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-c^2\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{1}{3}x^3\right.$$

$$\Big\downarrow \ 16$$

3.140. $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{1}{2}bc\left(-c^2\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(\frac{c^2}{x^2}+1\right)\right)\right)-\frac{1}{3}x^3\left(a\right.\right.$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{1}{2}bc\left(-c^2\left(-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(\frac{c^2}{x^2}+1\right)\right)\right)-\frac{1}{3}x^3\left(a+b\,\mathrm{arc}\right.\right.$$

input
```
Int[x^3*(a + b*ArcTan[c/x])^2,x]
```

output
```
(x^4*(a + b*ArcTan[c/x])^2)/4 - (b*c*(-1/3*(x^3*(a + b*ArcTan[c/x])) + (b*
c*(-x + c^2*Log[1 + c^2/x^2] - c^2*Log[x^(-2)]))/6 - c^2*(-(x*(a + b*ArcTa
n[c/x])) - (c*(a + b*ArcTan[c/x])^2)/(2*b) + (b*c*(-Log[1 + c^2/x^2] + Log
[x^(-2)]))/2)))/2
```

3.140.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_^2))^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.140.4 Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.16

method	result
parts	$\frac{a^2x^4}{4}-b^2c^4\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^2}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)}{6c^3}+\frac{x\arctan\left(\frac{c}{x}\right)}{2c}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2}{12c^2}-\frac{2\ln}{3}\right.$
derivativedivides	$-c^4\left(-\frac{a^2x^4}{4c^4}+b^2\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^2}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)}{6c^3}+\frac{x\arctan\left(\frac{c}{x}\right)}{2c}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2}{12c^2}\right.\right.$
default	$-c^4\left(-\frac{a^2x^4}{4c^4}+b^2\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^2}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)}{6c^3}+\frac{x\arctan\left(\frac{c}{x}\right)}{2c}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2}{12c^2}\right.\right.$
parallelrisch	$\frac{x^4\arctan\left(\frac{c}{x}\right)^2b^2}{4}-\frac{\arctan\left(\frac{c}{x}\right)^2b^2c^4}{4}-\frac{b^2c^4\ln\left(c^2+x^2\right)}{3}+\frac{x^4\arctan\left(\frac{c}{x}\right)ab}{2}+\frac{x^3\arctan\left(\frac{c}{x}\right)b^2c}{6}-\frac{x\arctan\left(\frac{c}{x}\right)b^2c}{2}$
risch	Expression too large to display

input
```
int(x^3*(a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4-b^2*c^4*(-1/4/c^4*x^4*arctan(c/x)^2+1/4*arctan(c/x)^2-1/6/c^3*
x^3*arctan(c/x)+1/2/c*x*arctan(c/x)+1/3*ln(1+c^2/x^2)-1/12/c^2*x^2-2/3*ln(
c/x))+1/2*x^4*arctan(c/x)*a*b+1/6*a*b*c*x^3-1/2*a*b*c^3*x+1/2*a*b*c^4*arct
an(x/c)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{1}{2}\,abc^4\arctan\left(\frac{x}{c}\right)-\frac{1}{3}\,b^2c^4\log\left(c^2+x^2\right)-\frac{1}{2}\,abc^3x+\frac{1}{12}\,b^2c^2x^2+\frac{1}{6}\,abcx^3+\frac{1}{4}\,a^2x^4-\frac{1}{4}\left(b^2c^4-b^2x^4\right)\arctan\left(\frac{c}{x}\right)^2-\frac{1}{6}\left(3\,b^2c^3x-b^2cx^3-3\,abx^4\right)\arctan\left(\frac{c}{x}\right)$$

input
```
integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

output
```
1/2*a*b*c^4*arctan(x/c) - 1/3*b^2*c^4*log(c^2 + x^2) - 1/2*a*b*c^3*x + 1/1
2*b^2*c^2*x^2 + 1/6*a*b*c*x^3 + 1/4*a^2*x^4 - 1/4*(b^2*c^4 - b^2*x^4)*arct
an(c/x)^2 - 1/6*(3*b^2*c^3*x - b^2*c*x^3 - 3*a*b*x^4)*arctan(c/x)
```

3.140.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.18

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{a^2x^4}{4}-\frac{abc^4\operatorname{atan}\left(\frac{c}{x}\right)}{2}-\frac{abc^3x}{2}+\frac{abcx^3}{6}+\frac{abx^4\operatorname{atan}\left(\frac{c}{x}\right)}{2}-\frac{b^2c^4\log\left(c^2+x^2\right)}{3}-\frac{b^2c^4\operatorname{atan}^2\left(\frac{c}{x}\right)}{4}-\frac{b^2c^3x\operatorname{atan}\left(\frac{c}{x}\right)}{2}+\frac{b^2c^2x^2}{12}+\frac{b^2cx^3\operatorname{atan}\left(\frac{c}{x}\right)}{6}+\frac{b^2x^4\operatorname{atan}^2\left(\frac{c}{x}\right)}{4}$$

input
```
integrate(x**3*(a+b*atan(c/x))**2,x)
```

output
```
a**2*x**4/4 - a*b*c**4*atan(c/x)/2 - a*b*c**3*x/2 + a*b*c*x**3/6 + a*b*x**
4*atan(c/x)/2 - b**2*c**4*log(c**2 + x**2)/3 - b**2*c**4*atan(c/x)**2/4 -
b**2*c**3*x*atan(c/x)/2 + b**2*c**2*x**2/12 + b**2*c*x**3*atan(c/x)/6 + b*
*2*x**4*atan(c/x)**2/4
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.10

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$$
$$=\frac{1}{4}\,b^2x^4\arctan\left(\frac{c}{x}\right)^2+\frac{1}{4}\,a^2x^4+\frac{1}{6}\left(3\,x^4\arctan\left(\frac{c}{x}\right)+\left(3\,c^3\arctan\left(\frac{x}{c}\right)-3\,c^2x+x^3\right)c\right)ab$$
$$+\frac{1}{12}\left(\left(3\,c^2\arctan\left(\frac{x}{c}\right)^2-4\,c^2\log\left(c^2+x^2\right)+x^2\right)c^2+2\left(3\,c^3\arctan\left(\frac{x}{c}\right)-3\,c^2x+x^3\right)c\arctan\left(\frac{c}{x}\right)\right)$$

input
```
integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arctan(c/x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arctan(c/x) + (3*c^3*
arctan(x/c) - 3*c^2*x + x^3)*c)*a*b + 1/12*((3*c^2*arctan(x/c)^2 - 4*c^2*l
og(c^2 + x^2) + x^2)*c^2 + 2*(3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c*arctan(
c/x))*b^2
```

3.140.8 Giac [F]

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^2*x^3, x)
```

3.140.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.15

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{a^2\,x^4}{4}-\frac{b^2\,c^4\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{4}-\frac{b^2\,c^4\,\ln\left(c^2+x^2\right)}{3}+\frac{b^2\,x^4\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{4}+\frac{b^2\,c^2\,x^2}{12}+\frac{b^2\,c\,x^3\,\mathrm{atan}\left(\frac{c}{x}\right)}{6}-\frac{b^2\,c^3\,x\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{a\,b\,c\,x^3}{6}-\frac{a\,b\,c^3\,x}{2}-\frac{a\,b\,c^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{a\,b\,x^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}$$

input
```
int(x^3*(a + b*atan(c/x))^2,x)
```

output
```
(a^2*x^4)/4 - (b^2*c^4*atan(c/x)^2)/4 - (b^2*c^4*log(c^2 + x^2))/3 + (b^2*
x^4*atan(c/x)^2)/4 + (b^2*c^2*x^2)/12 + (b^2*c*x^3*atan(c/x))/6 - (b^2*c^3
*x*atan(c/x))/2 + (a*b*c*x^3)/6 - (a*b*c^3*x)/2 - (a*b*c^4*atan(c/x))/2 +
(a*b*x^4*atan(c/x))/2
```

3.141 $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

3.141.1 Optimal result . . . 906
3.141.2 Mathematica [A] (verified) . . . 906
3.141.3 Rubi [A] (verified) . . . 907
3.141.4 Maple [B] (verified) . . . 910
3.141.5 Fricas [F] . . . 911
3.141.6 Sympy [F] . . . 911
3.141.7 Maxima [F] . . . 911
3.141.8 Giac [F] . . . 912
3.141.9 Mupad [F(-1)] . . . 912

3.141.1 Optimal result

Integrand size = 16, antiderivative size = 152

$$\begin{aligned}\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=&\frac{1}{3}b^2c^2x+\frac{1}{3}b^2c^3\cot^{-1}\left(\frac{x}{c}\right)+\frac{1}{3}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\&-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\&+\frac{2}{3}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\\&-\frac{1}{3}ib^2c^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)\end{aligned}$$

output
```
1/3*b^2*c^2*x+1/3*b^2*c^3*arccot(x/c)+1/3*b*c*x^2*(a+b*arccot(x/c))-1/3*I*
c^3*(a+b*arccot(x/c))^2+1/3*x^3*(a+b*arccot(x/c))^2+2/3*b*c^3*(a+b*arccot(
x/c))*ln(2-2/(1-I*c/x))-1/3*I*b^2*c^3*polylog(2,-1+2/(1-I*c/x))
```

3.141.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{1}{3}\Bigg(&b^2c^2x+abcx^2+a^2x^3+b^2\left(-ic^3+x^3\right)\arctan\left(\frac{c}{x}\right)^2\\&+b\arctan\left(\frac{c}{x}\right)\left(2ax^3+bc(c^2+x^2)\right)\\&+2bc^3\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)-abc^3\log\left(1+\frac{c^2}{x^2}\right)\\&+2abc^3\log\left(\frac{c}{x}\right)-ib^2c^3\operatorname{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\Bigg)\end{aligned}$$

input
```
Integrate[x^2*(a + b*ArcTan[c/x])^2,x]
```

output
```
(b^2*c^2*x + a*b*c*x^2 + a^2*x^3 + b^2*((-I)*c^3 + x^3)*ArcTan[c/x]^2 + b*
ArcTan[c/x]*(2*a*x^3 + b*c*(c^2 + x^2) + 2*b*c^3*Log[1 - E^((2*I)*ArcTan[c
/x])]) - a*b*c^3*Log[1 + c^2/x^2] + 2*a*b*c^3*Log[c/x] - I*b^2*c^3*PolyLog
[2, E^((2*I)*ArcTan[c/x])])/3
```

3.141.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5363, 5361, 5453, 5361, 264, 216, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx$$

$$\Big\downarrow \text{5363}$$

$$-\int x^4 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 d\frac{1}{x}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}x^3 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\int \frac{x^3 \left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{3}x^3 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(\int x^3 \left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}-c^2\int \frac{x \left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}x^3 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{2}{3}bc\left(c^2\left(-\int \frac{x \left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\int \frac{x^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{2}x^2 \left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{2}{3}bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(c^2\left(-\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-x\right)-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{2}{3}bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(-c\arctan\left(\frac{c}{x}\right)-x\right)\right)$$

$$\Big\downarrow \; 5459$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{2}{3}bc\left(-\left(c^2\left(i\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c}{x}+i}d\frac{1}{x}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right)\right)-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(-c\arctan\right.\right.$$

$$\Big\downarrow \; 5403$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{2}{3}bc\left(-\left(c^2\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right)\right)\right.$$

$$\Big\downarrow \; 2897$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)\right)-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right.\right.\right.$$

input
```
Int[x^2*(a + b*ArcTan[c/x])^2,x]
```

output
```
(x^3*(a + b*ArcTan[c/x])^2)/3 - (2*b*c*(-1/2*(x^2*(a + b*ArcTan[c/x])) + (
b*c*(-x - c*ArcTan[c/x]))/2 - c^2*(((-1/2*I)*(a + b*ArcTan[c/x])^2)/b + I*
((-I)*(a + b*ArcTan[c/x])*Log[2 - 2/(1 - (I*c)/x)] - (b*PolyLog[2, -1 + 2/
(1 - (I*c)/x)])/2))))/3
```

3.141. $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

3.141.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.141.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(134) = 268$.

Time = 3.78 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.36

method	result
derivativedivides	$-c^3\left(-\frac{a^2x^3}{3c^3}+b^2\left(-\frac{x^3\arctan\left(\frac{c}{x}\right)^2}{3c^3}+\frac{\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2\arctan\left(\frac{c}{x}\right)}{3c^2}-\frac{2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)}{3}+\frac{i\left(\mathrm{l}\right.}{}\right.\right.$
default	$-c^3\left(-\frac{a^2x^3}{3c^3}+b^2\left(-\frac{x^3\arctan\left(\frac{c}{x}\right)^2}{3c^3}+\frac{\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2\arctan\left(\frac{c}{x}\right)}{3c^2}-\frac{2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)}{3}+\frac{i\left(\mathrm{l}\right.}{}\right.\right.$
parts	$\frac{a^2x^3}{3}+\frac{b^2x^3\arctan\left(\frac{c}{x}\right)^2}{3}+\frac{c\,b^2x^2\arctan\left(\frac{c}{x}\right)}{3}+\frac{2c^3b^2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)}{3}-\frac{c^3b^2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{ic^3b^2\ln}{1}$
risch	Expression too large to display

input
```
int(x^2*(a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

output
```
-c^3*(-1/3*a^2/c^3*x^3+b^2*(-1/3/c^3*x^3*arctan(c/x)^2+1/3*arctan(c/x)*ln(
1+c^2/x^2)-1/3/c^2*x^2*arctan(c/x)-2/3*ln(c/x)*arctan(c/x)+1/6*I*(ln(c/x-I
)*ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c/x+I))-ln(c/x-I)*ln(-1/2*I*
(c/x+I)))-1/6*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln(c/x+I)^2-dilog(1/2*I*(c/x-
I))-ln(c/x+I)*ln(1/2*I*(c/x-I)))-1/3*arctan(c/x)-1/3*x/c-1/3*I*ln(c/x)*ln(
1+I*c/x)+1/3*I*ln(c/x)*ln(1-I*c/x)-1/3*I*dilog(1+I*c/x)+1/3*I*dilog(1-I*c/
x))+2*a*b*(-1/3/c^3*x^3*arctan(c/x)+1/6*ln(1+c^2/x^2)-1/6/c^2*x^2-1/3*ln(c
/x)))
```

3.141.5 Fricas [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctan(c/x)^2 + 2*a*b*x^2*arctan(c/x) + a^2*x^2, x)
```

3.141.6 Sympy [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int x^2\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^2\,dx$$

input
```
integrate(x**2*(a+b*atan(c/x))**2,x)
```

output
```
Integral(x**2*(a + b*atan(c/x))**2, x)
```

3.141.7 Maxima [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/3*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*a*b +
 1/48*(4*x^3*arctan2(c, x)^2 - x^3*log(c^2 + x^2)^2 + 48*integrate(1/48*(3
6*c^2*x^2*arctan2(c, x)^2 + 36*x^4*arctan2(c, x)^2 + 8*c*x^3*arctan2(c, x)
 + 4*x^4*log(c^2 + x^2) + 3*(c^2*x^2 + x^4)*log(c^2 + x^2)^2)/(c^2 + x^2),
 x))*b^2
```

3.141.8 Giac [F]

$$\int x^2 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx = \int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^2 x^2 dx$$

input
```
integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^2*x^2, x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx = \int x^2 \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2 dx$$

input
```
int(x^2*(a + b*atan(c/x))^2,x)
```

output
```
int(x^2*(a + b*atan(c/x))^2, x)
```

3.142 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

3.142.1 Optimal result 913
3.142.2 Mathematica [A] (verified) 913
3.142.3 Rubi [A] (verified) 914
3.142.4 Maple [A] (verified) 916
3.142.5 Fricas [A] (verification not implemented) 917
3.142.6 Sympy [A] (verification not implemented) 917
3.142.7 Maxima [A] (verification not implemented) 918
3.142.8 Giac [F] 918
3.142.9 Mupad [B] (verification not implemented) 918

3.142.1 Optimal result

Integrand size = 14, antiderivative size = 82

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}b^2c^2\log\left(1+\frac{c^2}{x^2}\right)+b^2c^2\log(x)$$

output
```
b*c*x*(a+b*arccot(x/c))+1/2*c^2*(a+b*arccot(x/c))^2+1/2*x^2*(a+b*arccot(x/
c))^2+1/2*b^2*c^2*ln(1+c^2/x^2)+b^2*c^2*ln(x)
```

3.142.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{1}{2}\left(ax(2bc+ax)+2b\left(bcx+a\left(c^2+x^2\right)\right)\arctan\left(\frac{c}{x}\right)+b^2\left(c^2+x^2\right)\arctan\left(\frac{c}{x}\right)^2+b^2c^2\log\left(c^2+x^2\right)\right)$$

input
```
Integrate[x*(a + b*ArcTan[c/x])^2,x]
```

output
```
(a*x*(2*b*c + a*x) + 2*b*(b*c*x + a*(c^2 + x^2))*ArcTan[c/x] + b^2*(c^2 +
x^2)*ArcTan[c/x]^2 + b^2*c^2*Log[c^2 + x^2])/2
```

3.142.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5363, 5361, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2dx$$

$$\Big\downarrow \text{5363}$$

$$-\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\int \frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\left(\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}-c^2\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\left(c^2\left(-\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\int \frac{x}{\frac{c^2}{x^2}+1}d\frac{1}{x}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\left(c^2\left(-\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\int \frac{x}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-bc\left(c^2\left(-\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(\int xd\frac{1}{x^2}-c^2\int \frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-c^2\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(\frac{c^2}{x^2}+1\right)\right)\right)$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2-$$
$$bc\left(-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(\frac{c^2}{x^2}+1\right)\right)\right)$$

input
```
Int[x*(a + b*ArcTan[c/x])^2,x]
```

output
```
(x^2*(a + b*ArcTan[c/x])^2)/2 - b*c*(-(x*(a + b*ArcTan[c/x])) - (c*(a + b*
ArcTan[c/x])^2)/(2*b) + (b*c*(-Log[1 + c^2/x^2] + Log[x^(-2)]))/2)
```

3.142.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.142.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.28

method	result
parts	$\frac{a^2x^2}{2}-b^2c^2\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^2}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)-ab\,c^2\arctan$
parallelrisch	$\frac{x^2\arctan\left(\frac{c}{x}\right)^2b^2}{2}+\frac{\arctan\left(\frac{c}{x}\right)^2b^2c^2}{2}+\frac{b^2c^2\ln\left(c^2+x^2\right)}{2}+x^2\arctan\left(\frac{c}{x}\right)ab+x\arctan\left(\frac{c}{x}\right)b^2c+\text{arct}$
derivativedivides	$-c^2\left(-\frac{a^2x^2}{2c^2}+b^2\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^2}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)-\frac{ab\,x^2\,\text{arc}}{c^2}\right.$
default	$-c^2\left(-\frac{a^2x^2}{2c^2}+b^2\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^2}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)-\frac{ab\,x^2\,\text{arc}}{c^2}\right.$
risch	Expression too large to display

input
```
int(x*(a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2-b^2*c^2*(-1/2/c^2*x^2*arctan(c/x)^2-1/2*arctan(c/x)^2-1/c*x*ar
ctan(c/x)-1/2*ln(1+c^2/x^2)+ln(c/x))-a*b*c^2*arctan(x/c)+x^2*arctan(c/x)*a
*b+a*b*c*x
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=-abc^2\arctan\left(\frac{x}{c}\right)+\frac{1}{2}\,b^2c^2\log\left(c^2+x^2\right)+abcx+\frac{1}{2}\,a^2x^2+\frac{1}{2}\left(b^2c^2+b^2x^2\right)\arctan\left(\frac{c}{x}\right)^2+\left(b^2cx+abx^2\right)\arctan\left(\frac{c}{x}\right)$$

input
```
integrate(x*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

output
```
-a*b*c^2*arctan(x/c) + 1/2*b^2*c^2*log(c^2 + x^2) + a*b*c*x + 1/2*a^2*x^2
+ 1/2*(b^2*c^2 + b^2*x^2)*arctan(c/x)^2 + (b^2*c*x + a*b*x^2)*arctan(c/x)
```

3.142.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{a^2x^2}{2}+abc^2\,\mathrm{atan}\left(\frac{c}{x}\right)+abcx+abx^2\,\mathrm{atan}\left(\frac{c}{x}\right)+\frac{b^2c^2\log\left(c^2+x^2\right)}{2}+\frac{b^2c^2\,\mathrm{atan}^2\left(\frac{c}{x}\right)}{2}+b^2cx\,\mathrm{atan}\left(\frac{c}{x}\right)+\frac{b^2x^2\,\mathrm{atan}^2\left(\frac{c}{x}\right)}{2}$$

input
```
integrate(x*(a+b*atan(c/x))**2,x)
```

output
```
a**2*x**2/2 + a*b*c**2*atan(c/x) + a*b*c*x + a*b*x**2*atan(c/x) + b**2*c**
2*log(c**2 + x**2)/2 + b**2*c**2*atan(c/x)**2/2 + b**2*c*x*atan(c/x) + b**
2*x**2*atan(c/x)**2/2
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx = \frac{1}{2}\,b^2x^2\arctan\left(\frac{c}{x}\right)^2+\frac{1}{2}\,a^2x^2+\left(x^2\arctan\left(\frac{c}{x}\right)-\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\right)ab - \frac{1}{2}\left(\left(\arctan\left(\frac{x}{c}\right)^2-\log\left(c^2+x^2\right)\right)c^2+2\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\arctan\left(\frac{c}{x}\right)\right)b^2$$

input
```
integrate(x*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*arctan(c/x)^2 + 1/2*a^2*x^2 + (x^2*arctan(c/x) - (c*arctan(x/c
) - x)*c)*a*b - 1/2*((arctan(x/c)^2 - log(c^2 + x^2))*c^2 + 2*(c*arctan(x/
c) - x)*c*arctan(c/x))*b^2
```

3.142.8 Giac [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^2*x, x)
```

3.142.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{a^2\,x^2}{2}+\frac{b^2\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+\frac{b^2\,c^2\,\ln\left(c^2+x^2\right)}{2}+\frac{b^2\,x^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2} + a\,b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)+a\,b\,x^2\,\mathrm{atan}\left(\frac{c}{x}\right)+b^2\,c\,x\,\mathrm{atan}\left(\frac{c}{x}\right)+a\,b\,c\,x$$

input
```
int(x*(a + b*atan(c/x))^2,x)
```

output
```
(a^2*x^2)/2 + (b^2*c^2*atan(c/x)^2)/2 + (b^2*c^2*log(c^2 + x^2))/2 + (b^2*
x^2*atan(c/x)^2)/2 + a*b*c^2*atan(c/x) + a*b*x^2*atan(c/x) + b^2*c*x*atan(
c/x) + a*b*c*x
```

3.142. $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

3.143 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx$

3.143.1 Optimal result . . . 920
3.143.2 Mathematica [A] (verified) . . . 920
3.143.3 Rubi [A] (verified) . . . 921
3.143.4 Maple [B] (verified) . . . 923
3.143.5 Fricas [F] . . . 924
3.143.6 Sympy [F] . . . 925
3.143.7 Maxima [F] . . . 925
3.143.8 Giac [F] . . . 925
3.143.9 Mupad [F(-1)] . . . 926

3.143.1 Optimal result

Integrand size = 12, antiderivative size = 83

$$\begin{aligned}\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx = {} & ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\ & -2bc\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c+ix}\right) \\ & +ib^2c\,\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)\end{aligned}$$

output
```
I*c*(a+b*arccot(x/c))^2+x*(a+b*arccot(x/c))^2-2*b*c*(a+b*arccot(x/c))*ln(2
*c/(c+I*x))+I*b^2*c*polylog(2,1-2*c/(c+I*x))
```

3.143.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx = {} & b^2(ic+x)\arctan\left(\frac{c}{x}\right)^2 \\ & +2b\arctan\left(\frac{c}{x}\right)\left(ax-bc\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) \\ & +a\left(ax+bc\log\left(1+\frac{c^2}{x^2}\right)-2bc\log\left(\frac{c}{x}\right)\right) \\ & +ib^2c\,\text{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\end{aligned}$$

input
```
Integrate[(a + b*ArcTan[c/x])^2,x]
```

output
```
b^2*(I*c + x)*ArcTan[c/x]^2 + 2*b*ArcTan[c/x]*(a*x - b*c*Log[1 - E^((2*I)*
ArcTan[c/x])]) + a*(a*x + b*c*Log[1 + c^2/x^2] - 2*b*c*Log[c/x]) + I*b^2*c
*PolyLog[2, E^((2*I)*ArcTan[c/x])]
```

3.143.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5349, 5346, 27, 5456, 27, 5380, 27, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2dx$$

$$\Big\downarrow \ 5349$$

$$\int \left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2dx$$

$$\Big\downarrow \ 5346$$

$$\frac{2b\int \frac{c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c^2+x^2}dx}{c}+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2$$

$$\Big\downarrow \ 27$$

$$2bc\int \frac{x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c^2+x^2}dx+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2$$

$$\Big\downarrow \ 5456$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+2bc\left(\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}-\frac{\int \frac{c\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{ic-x}dx}{c}\right)$$

$$\Big\downarrow \ 27$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+2bc\left(\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}-\int \frac{a+b\cot^{-1}\left(\frac{x}{c}\right)}{ic-x}dx\right)$$

$$\Big\downarrow \ 5380$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+2bc\left(-\frac{b\int\frac{c^2\log\left(\frac{2c}{c+ix}\right)}{c^2+x^2}dx}{c}+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}-\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\right)$$

$\Big\downarrow$ 27

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+2bc\left(-bc\int\frac{\log\left(\frac{2c}{c+ix}\right)}{c^2+x^2}dx+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}-\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\right)$$

$\Big\downarrow$ 2849

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+2bc\left(ibc\int\frac{\log\left(\frac{2c}{c+ix}\right)}{1-\frac{2c}{c+ix}}d\frac{1}{c+ix}+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}-\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\right)$$

$\Big\downarrow$ 2752

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+2bc\left(\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}-\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)\right)$$

input
```
Int[(a + b*ArcTan[c/x])^2,x]
```

output
```
x*(a + b*ArcCot[x/c])^2 + 2*b*c*(((I/2)*(a + b*ArcCot[x/c])^2)/b - (a + b*
ArcCot[x/c])*Log[(2*c)/(c + I*x)] + (I/2)*b*PolyLog[2, 1 - (2*c)/(c + I*x)
])
```

3.143.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

3.143. $\int\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5349
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*A
rcCot[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.143.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(79) = 158$.

Time = 3.62 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.71

method	result
parts	$a^2x-b^2c\left(-\frac{x\arctan\left(\frac{c}{x}\right)^2}{c}+2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)+i\ln\left(\frac{c}{x}\right)\ln\left(1\right.\right.$
derivativedivides	$-c\left(-\frac{a^2x}{c}+b^2\left(-\frac{x\arctan\left(\frac{c}{x}\right)^2}{c}+2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)+i\ln\left(\frac{c}{x}\right)\right.\right.$
default	$-c\left(-\frac{a^2x}{c}+b^2\left(-\frac{x\arctan\left(\frac{c}{x}\right)^2}{c}+2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)+i\ln\left(\frac{c}{x}\right)\right.\right.$
risch	Expression too large to display

input
```
int((a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-b^2*c*(-1/c*x*arctan(c/x)^2+2*ln(c/x)*arctan(c/x)-arctan(c/x)*ln(1+c
^2/x^2)+I*ln(c/x)*ln(1+I*c/x)-I*ln(c/x)*ln(1-I*c/x)+I*dilog(1+I*c/x)-I*dil
og(1-I*c/x)-1/2*I*(ln(c/x-I)*ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c
/x+I))-ln(c/x-I)*ln(-1/2*I*(c/x+I)))+1/2*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln
(c/x+I)^2-dilog(1/2*I*(c/x-I))-ln(c/x+I)*ln(1/2*I*(c/x-I))))-2*a*b*c*(-1/c
*x*arctan(c/x)-1/2*ln(1+c^2/x^2)+ln(c/x))
```

3.143.5 Fricas [F]

$$\int\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arctan(c/x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2, x)
```

3.143.6 Sympy [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2\,dx$$

input
```
integrate((a+b*atan(c/x))**2,x)
```

output
```
Integral((a + b*atan(c/x))**2, x)
```

3.143.7 Maxima [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arctan(c/x))^2,x, algorithm="maxima")
```

output
```
(2*x*arctan(c/x) + c*log(c^2 + x^2))*a*b + 1/16*(12*c*arctan(c/x)^2*arctan
(x/c) + 4*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*c^2 + 4*x*arct
an2(c, x)^2 + 16*c^2*integrate(1/16*log(c^2 + x^2)^2/(c^2 + x^2), x) - x*l
og(c^2 + x^2)^2 + 128*c*integrate(1/16*x*arctan(c/x)/(c^2 + x^2), x) + 192
*integrate(1/16*x^2*arctan(c/x)^2/(c^2 + x^2), x) + 16*integrate(1/16*x^2*
log(c^2 + x^2)^2/(c^2 + x^2), x) + 64*integrate(1/16*x^2*log(c^2 + x^2)/(c
^2 + x^2), x))*b^2 + a^2*x
```

3.143.8 Giac [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arctan(c/x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^2, x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \left(a + b\arctan\left(\frac{c}{x}\right)\right)^2 dx = \int \left(a + b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2 dx$$

input
```
int((a + b*atan(c/x))^2,x)
```

output
```
int((a + b*atan(c/x))^2, x)
```

3.144 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx$

3.144.1 Optimal result . . . 927
3.144.2 Mathematica [A] (verified) . . . 928
3.144.3 Rubi [A] (verified) . . . 928
3.144.4 Maple [C] (warning: unable to verify) . . . 930
3.144.5 Fricas [F] . . . 931
3.144.6 Sympy [F] . . . 931
3.144.7 Maxima [F] . . . 932
3.144.8 Giac [F] . . . 932
3.144.9 Mupad [F(-1)] . . . 932

3.144.1 Optimal result

Integrand size = 16, antiderivative size = 148

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx = -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right) + ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right) - ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right) + \frac{1}{2}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right) - \frac{1}{2}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{ic}{x}}\right)$$

output
```
2*(a+b*arccot(x/c))^2*arctanh(-1+2/(1+I*c/x))+I*b*(a+b*arccot(x/c))*polylo
g(2,1-2/(1+I*c/x))-I*b*(a+b*arccot(x/c))*polylog(2,-1+2/(1+I*c/x))+1/2*b^2
*polylog(3,1-2/(1+I*c/x))-1/2*b^2*polylog(3,-1+2/(1+I*c/x))
```

3.144.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.37

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx = a^2\log(x)-iab\left(\operatorname{PolyLog}\left(2,-\frac{ic}{x}\right)-\operatorname{PolyLog}\left(2,\frac{ic}{x}\right)\right)+b^2\left(\frac{i\pi^3}{24}-\frac{2}{3}i\arctan\left(\frac{c}{x}\right)^3-\arctan\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right)+\arctan\left(\frac{c}{x}\right)^2\log\left(1+e^{2i\arctan\left(\frac{c}{x}\right)}\right)-i\arctan\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)-i\arctan\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(2,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)-\frac{1}{2}\operatorname{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)+\frac{1}{2}\operatorname{PolyLog}\left(3,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c/x])^2/x,x]
```

output
```
a^2*Log[x] - I*a*b*(PolyLog[2, ((-I)*c)/x] - PolyLog[2, (I*c)/x]) + b^2*((
I/24)*Pi^3 - ((2*I)/3)*ArcTan[c/x]^3 - ArcTan[c/x]^2*Log[1 - E^((-2*I)*Arc
Tan[c/x])] + ArcTan[c/x]^2*Log[1 + E^((2*I)*ArcTan[c/x])] - I*ArcTan[c/x]*
PolyLog[2, E^((-2*I)*ArcTan[c/x])] - I*ArcTan[c/x]*PolyLog[2, -E^((2*I)*Ar
cTan[c/x])] - PolyLog[3, E^((-2*I)*ArcTan[c/x])]/2 + PolyLog[3, -E^((2*I)*
ArcTan[c/x])]/2)
```

3.144.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5359, 5357, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$-\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}$$

$$\Big\downarrow \text{5357}$$

$$4bc\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)\operatorname{arctanh}\left(1-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-2\operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2$$

$$\Big\downarrow \text{5523}$$

$$4bc\left(\frac{1}{2}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)\log\left(2-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{2}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-$$
$$2\operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2$$

$$\Big\downarrow \text{5529}$$

$$4bc\left(\frac{1}{2}\left(\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}-\frac{1}{2}ib\int \frac{\operatorname{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}\left(\frac{1}{2}ib\int \frac{\operatorname{PolyLog}}{\ldots}\right.\right.$$
$$2\operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2$$

$$\Big\downarrow \text{7164}$$

$$4bc\left(\frac{1}{2}\left(\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}+\frac{b\operatorname{PolyLog}\left(3,1-\frac{2}{\frac{ic}{x}+1}\right)}{4c}\right)+\frac{1}{2}\left(-\frac{i\operatorname{PolyLog}\left(2,\frac{2}{\frac{ic}{x}+1}-1\ldots\right)}{2c\ldots}\right.\right.$$
$$2\operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2$$

input
```
Int[(a + b*ArcTan[c/x])^2/x,x]
```

output
```
-2*(a + b*ArcTan[c/x])^2*ArcTanh[1 - 2/(1 + (I*c)/x)] + 4*b*c*((((I/2)*(a
+ b*ArcTan[c/x])*PolyLog[2, 1 - 2/(1 + (I*c)/x)])/c + (b*PolyLog[3, 1 - 2/
(1 + (I*c)/x)])/(4*c))/2 + (((-1/2*I)*(a + b*ArcTan[c/x])*PolyLog[2, -1 +
2/(1 + (I*c)/x)])/c - (b*PolyLog[3, -1 + 2/(1 + (I*c)/x)])/(4*c))/2)
```

3.144. $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx$

3.144.3.1 Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.144.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 1106, normalized size of antiderivative = 7.47

Expression too large to display

input
```
int((a+b*arctan(c/x))^2/x,x)
```

output
```
-a^2*ln(c/x)-b^2*(ln(c/x)*arctan(c/x)^2+I*arctan(c/x)*polylog(2,-(1+I*c/x)
^2/(1+c^2/x^2))-1/2*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))-arctan(c/x)^2*ln((
1+I*c/x)^2/(1+c^2/x^2)-1)+arctan(c/x)^2*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))-
2*I*arctan(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))+2*polylog(3,(1+I*c/
x)/(1+c^2/x^2)^(1/2))+arctan(c/x)^2*ln((1+I*c/x)/(1+c^2/x^2)^(1/2)+1)-2*I*
arctan(c/x)*polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))+2*polylog(3,-(1+I*c/x)
/(1+c^2/x^2)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x
)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x
^2)+1))-csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2-cs
gn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I
*c/x)^2/(1+c^2/x^2)+1))^2+csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I
*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1
+c^2/x^2)-1))+csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+
1))^3-csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*cs
gn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))-csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I
*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c
^2/x^2)+1))^2+csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1)
)^3+1)*arctan(c/x)^2)-2*a*b*(ln(c/x)*arctan(c/x)+1/2*I*ln(c/x)*ln(1+I*c/x)
-1/2*I*ln(c/x)*ln(1-I*c/x)+1/2*I*dilog(1+I*c/x)-1/2*I*dilog(1-I*c/x))
```

3.144.5 Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2)/x, x)
```

3.144.6 Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

input
```
integrate((a+b*atan(c/x))**2/x,x)
```

output
```
Integral((a + b*atan(c/x))**2/x, x)
```

3.144.7 Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + 1/16*integrate((12*b^2*arctan2(c, x)^2 + b^2*log(c^2 + x^2)^2
 + 32*a*b*arctan2(c, x))/x, x)
```

3.144.8 Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^2/x, x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

input
```
int((a + b*atan(c/x))^2/x,x)
```

output
```
int((a + b*atan(c/x))^2/x, x)
```

3.145 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$

3.145.1 Optimal result . . . 933
3.145.2 Mathematica [A] (verified) . . . 933
3.145.3 Rubi [A] (verified) . . . 934
3.145.4 Maple [A] (verified) . . . 936
3.145.5 Fricas [F] . . . 937
3.145.6 Sympy [F] . . . 937
3.145.7 Maxima [F] . . . 937
3.145.8 Giac [F] . . . 938
3.145.9 Mupad [F(-1)] . . . 938

3.145.1 Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x}-\frac{2b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c}-\frac{ib^2\,\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{c}$$

output
```
-I*(a+b*arccot(x/c))^2/c-(a+b*arccot(x/c))^2/x-2*b*(a+b*arccot(x/c))*ln(2/
(1+I*c/x))/c-I*b^2*polylog(2,1-2/(1+I*c/x))/c
```

3.145.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = -\frac{b^2(c-ix)\arctan\left(\frac{c}{x}\right)^2+2b\arctan\left(\frac{c}{x}\right)\left(ac+bx\log\left(1+e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)+a\left(ac+2bx\log\left(\frac{1}{\sqrt{1+\frac{c^2}{x^2}}}\right)\right)-}{cx}$$

input
```
Integrate[(a + b*ArcTan[c/x])^2/x^2,x]
```

output
```
-((b^2*(c - I*x)*ArcTan[c/x]^2 + 2*b*ArcTan[c/x]*(a*c + b*x*Log[1 + E^((2*
I)*ArcTan[c/x])]) + a*(a*c + 2*b*x*Log[1/Sqrt[1 + c^2/x^2]]) - I*b^2*x*Pol
yLog[2, -E^((2*I)*ArcTan[c/x])])/(c*x))
```

3.145.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5363, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}dx$$

$$\Big\downarrow \ 5363$$

$$-\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}$$

$$\Big\downarrow \ 5345$$

$$2bc\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\left(\frac{c^2}{x^2}+1\right)x}d\frac{1}{x}-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}$$

$$\Big\downarrow \ 5455$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}+2bc\left(-\frac{\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{i-\frac{c}{x}}d\frac{1}{x}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)$$

$$\Big\downarrow \ 5379$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}+2bc\left(-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c}-b\int \frac{\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)$$

$$\Big\downarrow \ 2849$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}+2bc\left(-\frac{-\frac{ib\int\frac{\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{1-\frac{2}{\frac{ic}{x}+1}}d\frac{1}{\frac{ic}{x}+1}}{c}+\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)$$

$$\Big\downarrow \ 2752$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}+2bc\left(-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)}{2c}}{c}\right)$$

input
```
Int[(a + b*ArcTan[c/x])^2/x^2,x]
```

output
```
-((a + b*ArcTan[c/x])^2/x) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c/x])^2)/(b*c^
2) - (((a + b*ArcTan[c/x])*Log[2/(1 + (I*c)/x)])/c + ((I/2)*b*PolyLog[2, 1
 - 2/(1 + (I*c)/x)])/c)/c)
```

3.145.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

3.145. $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.145.4 Maple [A] (verified)

Time = 6.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.48

method	result
derivativedivides	$-\frac{\frac{c a^2}{x} - i\arctan\left(\frac{c}{x}\right)^2 b^2 + \frac{\arctan\left(\frac{c}{x}\right)^2 b^2 c}{x} + 2\arctan\left(\frac{c}{x}\right)\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)b^2 - i\,\mathrm{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)b^2 + \frac{2abc\arctan\left(\frac{c}{x}\right.\cdots}{x}}{c}$
default	$-\frac{\frac{c a^2}{x} - i\arctan\left(\frac{c}{x}\right)^2 b^2 + \frac{\arctan\left(\frac{c}{x}\right)^2 b^2 c}{x} + 2\arctan\left(\frac{c}{x}\right)\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)b^2 - i\,\mathrm{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)b^2 + \frac{2abc\arctan\left(\frac{c}{x}\right.\cdots}{x}}{c}$
parts	$-\frac{a^2}{x} - \frac{b^2\arctan\left(\frac{c}{x}\right)^2}{x} + \frac{ib^2\arctan\left(\frac{c}{x}\right)^2}{c} - \frac{2b^2\arctan\left(\frac{c}{x}\right)\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)}{c} + \frac{ib^2\,\mathrm{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)}{c} - 2\cdots$

input
```
int((a+b*arctan(c/x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c*(c/x*a^2-I*arctan(c/x)^2*b^2+arctan(c/x)^2*b^2*c/x+2*arctan(c/x)*ln((
1+I*c/x)^2/(1+c^2/x^2)+1)*b^2-I*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))*b^2+2*
a*b*c/x*arctan(c/x)-a*b*ln(1+c^2/x^2))
```

3.145.5 Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2)/x^2, x)
```

3.145.6 Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*atan(c/x))**2/x**2,x)
```

output
```
Integral((a + b*atan(c/x))**2/x**2, x)
```

3.145.7 Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="maxima")
```

output
```
1/16*(4*(48*c^2*integrate(1/16*arctan(c/x)^2/(c^2*x^2 + x^4), x) + 4*c^2*i
ntegrate(1/16*log(c^2 + x^2)^2/(c^2*x^2 + x^4), x) + 3*arctan(c/x)^2*arcta
n(x/c)/c + 3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c - 32*c*integrat
e(1/16*x*arctan(c/x)/(c^2*x^2 + x^4), x) + 4*integrate(1/16*x^2*log(c^2 +
x^2)^2/(c^2*x^2 + x^4), x) - 16*integrate(1/16*x^2*log(c^2 + x^2)/(c^2*x^2
 + x^4), x))*x - 4*arctan2(c, x)^2 + log(c^2 + x^2)^2)*b^2/x - a*b*(2*c*ar
ctan(c/x)/x - log(c^2/x^2 + 1))/c - a^2/x
```

3.145.8 Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^2/x^2, x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

input
```
int((a + b*atan(c/x))^2/x^2,x)
```

output
```
int((a + b*atan(c/x))^2/x^2, x)
```

3.146 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$

3.146.1 Optimal result 939
3.146.2 Mathematica [A] (verified) 939
3.146.3 Rubi [A] (verified) 940
3.146.4 Maple [A] (verified) 941
3.146.5 Fricas [A] (verification not implemented) 942
3.146.6 Sympy [A] (verification not implemented) 943
3.146.7 Maxima [A] (verification not implemented) 943
3.146.8 Giac [C] (verification not implemented) 944
3.146.9 Mupad [B] (verification not implemented) 944

3.146.1 Optimal result

Integrand size = 16, antiderivative size = 84

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \frac{ab}{cx}+\frac{b^2\cot^{-1}\left(\frac{x}{c}\right)}{cx}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}-\frac{b^2\log\left(1+\frac{c^2}{x^2}\right)}{2c^2}$$

output
```
a*b/c/x+b^2*arccot(x/c)/c/x-1/2*(a+b*arccot(x/c))^2/c^2-1/2*(a+b*arccot(x/
c))^2/x^2-1/2*b^2*ln(1+c^2/x^2)/c^2
```

3.146.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = -\frac{a^2c^2-2abcx+2bc(ac-bx)\arctan\left(\frac{c}{x}\right)+b^2(c^2+x^2)\arctan\left(\frac{c}{x}\right)^2-2abx^2\arctan\left(\frac{x}{c}\right)-2b^2x^2\log(x)+}{2c^2x^2}$$

input
```
Integrate[(a + b*ArcTan[c/x])^2/x^3,x]
```

output
```
-1/2*(a^2*c^2 - 2*a*b*c*x + 2*b*c*(a*c - b*x)*ArcTan[c/x] + b^2*(c^2 + x^2
)*ArcTan[c/x]^2 - 2*a*b*x^2*ArcTan[x/c] - 2*b^2*x^2*Log[x] + b^2*x^2*Log[c
^2 + x^2])/(c^2*x^2)
```

3.146.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5363, 5361, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}dx$$

$$\Big\downarrow \text{5363}$$

$$-\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{5361}$$

$$bc\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\left(\frac{c^2}{x^2}+1\right)x^2}d\frac{1}{x}-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2x^2}$$

$$\Big\downarrow \text{5451}$$

$$bc\left(\frac{\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}}{c^2}-\frac{\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c^2}\right)-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$bc\left(\frac{\frac{a}{x}+\frac{b\arctan\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}}{c^2}-\frac{\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c^2}\right)-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2x^2}$$

$$\Big\downarrow \text{5419}$$

$$bc\left(\frac{\frac{a}{x}+\frac{b\arctan\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}}{c^2}-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^3}\right)-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2x^2}$$

input
```
Int[(a + b*ArcTan[c/x])^2/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c/x])^2/x^2 + b*c*(-1/2*(a + b*ArcTan[c/x])^2/(b*c^3) +
 (a/x + (b*ArcTan[c/x])/x - (b*Log[1 + c^2/x^2])/(2*c))/c^2)
```

3.146.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

3.146.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.23

method	result
parts	$-\frac{a^2}{2x^2}-\frac{b^2\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^2}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{c\arctan\left(\frac{c}{x}\right)}{x}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}\right)}{c^2}-\frac{ab\arctan\left(\frac{c}{x}\right)}{x^2}+\frac{ab}{cx}+\frac{ab\arctan\left(\frac{x}{c}\right)}{c^2}$
derivativedivides	$-\frac{\frac{a^2c^2}{2x^2}+b^2\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^2}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{c\arctan\left(\frac{c}{x}\right)}{x}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}\right)+2ab\left(\frac{c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{c}{2x}+\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)}{c^2}$
default	$-\frac{\frac{a^2c^2}{2x^2}+b^2\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^2}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{c\arctan\left(\frac{c}{x}\right)}{x}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}\right)+2ab\left(\frac{c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{c}{2x}+\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)}{c^2}$
parallelrisch	$\frac{-x^2\arctan\left(\frac{c}{x}\right)^2b^2-\arctan\left(\frac{c}{x}\right)^2b^2c^2+2b^2\ln(x)x^2-b^2\ln(c^2+x^2)x^2-2x^2\arctan\left(\frac{c}{x}\right)ab+2x\arctan\left(\frac{c}{x}\right)b^2c-2\arctan\left(\frac{c}{x}\right)a}{2x^2c^2}$
risch	Expression too large to display

input
```
int((a+b*arctan(c/x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2-b^2/c^2*(1/2*c^2/x^2*arctan(c/x)^2+1/2*arctan(c/x)^2-c/x*arct
an(c/x)+1/2*ln(1+c^2/x^2))-a*b*arctan(c/x)/x^2+a*b/c/x+a*b/c^2*arctan(x/c)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$$
$$=\frac{2\,abx^2\arctan\left(\frac{x}{c}\right)-b^2x^2\log\left(c^2+x^2\right)+2\,b^2x^2\log\left(x\right)-a^2c^2+2\,abcx-\left(b^2c^2+b^2x^2\right)\arctan\left(\frac{c}{x}\right)^2-2\,(abc}{2\,c^2x^2}$$

input
```
integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*x^2*arctan(x/c) - b^2*x^2*log(c^2 + x^2) + 2*b^2*x^2*log(x) - a
^2*c^2 + 2*a*b*c*x - (b^2*c^2 + b^2*x^2)*arctan(c/x)^2 - 2*(a*b*c^2 - b^2*
c*x)*arctan(c/x))/(c^2*x^2)
```

3.146.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$$
$$=\begin{cases} -\frac{a^2}{2x^2}-\frac{ab\,\mathrm{atan}\left(\frac{c}{x}\right)}{x^2}+\frac{ab}{cx}-\frac{ab\,\mathrm{atan}\left(\frac{c}{x}\right)}{c^2}-\frac{b^2\,\mathrm{atan}^2\left(\frac{c}{x}\right)}{2x^2}+\frac{b^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{cx}+\frac{b^2\log(x)}{c^2}-\frac{b^2\log(c^2+x^2)}{2c^2}-\frac{b^2\,\mathrm{atan}^2\left(\frac{c}{x}\right)}{2c^2} & \text{for } c \neq \\ -\frac{a^2}{2x^2} & \text{otherw} \end{cases}$$

input
```
integrate((a+b*atan(c/x))**2/x**3,x)
```

output
```
Piecewise((-a**2/(2*x**2) - a*b*atan(c/x)/x**2 + a*b/(c*x) - a*b*atan(c/x)
/c**2 - b**2*atan(c/x)**2/(2*x**2) + b**2*atan(c/x)/(c*x) + b**2*log(x)/c*
*2 - b**2*log(c**2 + x**2)/(2*c**2) - b**2*atan(c/x)**2/(2*c**2), Ne(c, 0)
), (-a**2/(2*x**2), True))
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$$
$$=\left(c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)-\frac{\arctan\left(\frac{c}{x}\right)}{x^2}\right)ab$$
$$+\frac{1}{2}\left(2\,c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)\arctan\left(\frac{c}{x}\right)+\frac{\arctan\left(\frac{x}{c}\right)^2-\log\left(c^2+x^2\right)+2\,\log\left(x\right)}{c^2}\right)b^2$$
$$-\frac{b^2\arctan\left(\frac{c}{x}\right)^2}{2\,x^2}-\frac{a^2}{2\,x^2}$$

input
```
integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="maxima")
```

output
```
(c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*a*b + 1/2*(2*c*(arctan
(x/c)/c^3 + 1/(c^2*x))*arctan(c/x) + (arctan(x/c)^2 - log(c^2 + x^2) + 2*l
og(x))/c^2)*b^2 - 1/2*b^2*arctan(c/x)^2/x^2 - 1/2*a^2/x^2
```

3.146.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.63

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx =$$
$$-\frac{b^2\arctan\left(\frac{c}{x}\right)^2+\frac{b^2c^2\arctan\left(\frac{c}{x}\right)^2}{x^2}+\frac{2\,abc^2\arctan\left(\frac{c}{x}\right)}{x^2}-\frac{2\,b^2c\arctan\left(\frac{c}{x}\right)}{x}+i\,ab\log\left(\frac{i\,c}{x}-1\right)+b^2\log\left(\frac{i\,c}{x}-1\right)-i\,ab\log}{2\,c^2}$$

input
```
integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="giac")
```

output
```
-1/2*(b^2*arctan(c/x)^2 + b^2*c^2*arctan(c/x)^2/x^2 + 2*a*b*c^2*arctan(c/x
)/x^2 - 2*b^2*c*arctan(c/x)/x + I*a*b*log(I*c/x - 1) + b^2*log(I*c/x - 1)
- I*a*b*log(-I*c/x - 1) + b^2*log(-I*c/x - 1) + a^2*c^2/x^2 - 2*a*b*c/x)/c
^2
```

3.146.9 Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$$
$$=\frac{b^2\,\ln\left(x\right)-\frac{b^2\,\ln(x+c\,1\mathrm{i})}{2}-\frac{b^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+\frac{b^2\,\ln\left(\frac{1}{-x+c\,1\mathrm{i}}\right)}{2}+\frac{a\,b\,\ln(x+c\,1\mathrm{i})\,1\mathrm{i}}{2}-\frac{a\,b\,\ln(-x+c\,1\mathrm{i})\,1\mathrm{i}}{2}}{c^2}$$
$$-\frac{\frac{a^2\,c^2}{2}-x\,\left(c\,\mathrm{atan}\left(\frac{c}{x}\right)\,b^2+a\,c\,b\right)+\frac{b^2\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+a\,b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{c^2\,x^2}$$

input
```
int((a + b*atan(c/x))^2/x^3,x)
```

output
```
(b^2*log(x) - (b^2*log(c*1i + x))/2 - (b^2*atan(c/x)^2)/2 + (b^2*log(1/(c*
1i - x)))/2 + (a*b*log(c*1i + x)*1i)/2 - (a*b*log(c*1i - x)*1i)/2)/c^2 - (
(a^2*c^2)/2 - x*(b^2*c*atan(c/x) + a*b*c) + (b^2*c^2*atan(c/x)^2)/2 + a*b*
c^2*atan(c/x))/(c^2*x^2)
```

3.147 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$

3.147.1 Optimal result 945
3.147.2 Mathematica [A] (verified) 946
3.147.3 Rubi [A] (verified) 946
3.147.4 Maple [B] (verified) 950
3.147.5 Fricas [F] 951
3.147.6 Sympy [F] 951
3.147.7 Maxima [F] 952
3.147.8 Giac [F] 952
3.147.9 Mupad [F(-1)] 953

3.147.1 Optimal result

Integrand size = 16, antiderivative size = 214

$$\begin{aligned}\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx &= \frac{1}{4}b^3c^3x+\frac{1}{4}b^3c^4\cot^{-1}\left(\frac{x}{c}\right)+\frac{1}{4}b^2c^2x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\ &\quad -ibc^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{3}{4}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\ &\quad +\frac{1}{4}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\ &\quad -\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\ &\quad +2b^2c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\\ &\quad -ib^3c^4\,\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)\end{aligned}$$

output
```
1/4*b^3*c^3*x+1/4*b^3*c^4*arccot(x/c)+1/4*b^2*c^2*x^2*(a+b*arccot(x/c))-I*
b*c^4*(a+b*arccot(x/c))^2-3/4*b*c^3*x*(a+b*arccot(x/c))^2+1/4*b*c*x^3*(a+b
*arccot(x/c))^2-1/4*c^4*(a+b*arccot(x/c))^3+1/4*x^4*(a+b*arccot(x/c))^3+2*
b^2*c^4*(a+b*arccot(x/c))*ln(2-2/(1-I*c/x))-I*b^3*c^4*polylog(2,-1+2/(1-I*
c/x))
```

3.147.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.18

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\frac{1}{4}\left(ab^2c^4-3a^2bc^3x+b^3c^3x+ab^2c^2x^2+a^2bcx^3+a^3x^4\right.$$
$$+b^2\left(bc\left(-4ic^3-3c^2x+x^3\right)+3a\left(-c^4+x^4\right)\right)\arctan\left(\frac{c}{x}\right)^2$$
$$+b^3\left(-c^4+x^4\right)\arctan\left(\frac{c}{x}\right)^3$$
$$+b\arctan\left(\frac{c}{x}\right)\left(2abcx\left(-3c^2+x^2\right)+b^2c^2\left(c^2+x^2\right)\right.$$
$$\left.+3a^2\left(-c^4+x^4\right)+8b^2c^4\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$
$$+8ab^2c^4\log\left(\frac{c}{\sqrt{1+\frac{c^2}{x^2}}\,x}\right)$$
$$\left.-4ib^3c^4\,\mathrm{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[x^3*(a + b*ArcTan[c/x])^3,x]
```

output
```
(a*b^2*c^4 - 3*a^2*b*c^3*x + b^3*c^3*x + a*b^2*c^2*x^2 + a^2*b*c*x^3 + a^3
*x^4 + b^2*(b*c*((-4*I)*c^3 - 3*c^2*x + x^3) + 3*a*(-c^4 + x^4))*ArcTan[c/
x]^2 + b^3*(-c^4 + x^4)*ArcTan[c/x]^3 + b*ArcTan[c/x]*(2*a*b*c*x*(-3*c^2 +
 x^2) + b^2*c^2*(c^2 + x^2) + 3*a^2*(-c^4 + x^4) + 8*b^2*c^4*Log[1 - E^((2
*I)*ArcTan[c/x])]) + 8*a*b^2*c^4*Log[c/(Sqrt[1 + c^2/x^2]*x)] - (4*I)*b^3*
c^4*PolyLog[2, E^((2*I)*ArcTan[c/x])])/4
```

3.147.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.41, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5363, 5361, 5453, 5361, 5453, 5361, 264, 216, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 dx$$

$$\Big\downarrow \text{5363}$$

$$-\int x^5 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 d\frac{1}{x}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 - \frac{3}{4}bc\int \frac{x^4 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{4}x^4 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 - \frac{3}{4}bc\left(\int x^4 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^2 d\frac{1}{x} - c^2\int \frac{x^2 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 -$$
$$\frac{3}{4}bc\left(-c^2\int \frac{x^2 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x} + \frac{2}{3}bc\int \frac{x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x} - \frac{1}{3}x^3\left(a+b \arctan \left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{4}x^4 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 -$$
$$\frac{3}{4}bc\left(-c^2\left(\int x^2 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^2 d\frac{1}{x} - c^2\int \frac{\left(a+b \arctan \left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right) + \frac{2}{3}bc\left(\int x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right) d\frac{1}{x} -\right.\right.$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}x^4 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 -$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\left(-\int \frac{x \left(a+b \arctan \left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right) + \frac{1}{2}bc\int \frac{x^2}{\frac{c^2}{x^2}+1}d\frac{1}{x} - \frac{1}{2}x^2\left(a+b \arctan \left(\frac{c}{x}\right)\right)\right) - c^2\left(c^2\left(-\int \frac{(a+\right.\right.\right.$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{4}x^4 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 -$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\left(-\int \frac{x \left(a+b \arctan \left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right) + \frac{1}{2}bc\left(c^2\left(-\int \frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right) - x\right) - \frac{1}{2}x^2\left(a+b \arctan \left(\frac{c}{x}\right)\right)\right) - c^2\right.$$

$$\Big\downarrow \text{216}$$

3.147. $\int x^3 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 dx$

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(-c\arctan\left(\frac{c}{x}\right)-x\right)\right)-c^2\left(c^2\left(\right.\right.\right.$$

$\downarrow$ 5419

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(-c^2\left(2bc\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3b}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\right)+\frac{2}{3}bc\left(c^2\left(-\int\frac{x(a}{}\right.\right.\right.$$

$\downarrow$ 5459

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(-\left(c^2\left(i\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c}{x}+i}d\frac{1}{x}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right)\right)-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(-c\,\mathrm{ar}\right.\right.\right.$$

$\downarrow$ 5403

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(-c^2\left(2bc\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right.\right.\right.$$

$\downarrow$ 2897

$$\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)\right)-\frac{i\left(a+b\arctan\left(\right.\right.}{2b}\right.\right.\right.\right.$$

input
```
Int[x^3*(a + b*ArcTan[c/x])^3,x]
```

output
```
(x^4*(a + b*ArcTan[c/x])^3)/4 - (3*b*c*(-1/3*(x^3*(a + b*ArcTan[c/x])^2) -
 c^2*(-(x*(a + b*ArcTan[c/x])^2) - (c*(a + b*ArcTan[c/x])^3)/(3*b) + 2*b*c
*(((-1/2*I)*(a + b*ArcTan[c/x])^2)/b + I*((-I)*(a + b*ArcTan[c/x])*Log[2 -
 2/(1 - (I*c)/x)] - (b*PolyLog[2, -1 + 2/(1 - (I*c)/x)])/2))) + (2*b*c*(-1
/2*(x^2*(a + b*ArcTan[c/x])) + (b*c*(-x - c*ArcTan[c/x]))/2 - c^2*(((-1/2*
I)*(a + b*ArcTan[c/x])^2)/b + I*((-I)*(a + b*ArcTan[c/x])*Log[2 - 2/(1 - (
I*c)/x)] - (b*PolyLog[2, -1 + 2/(1 - (I*c)/x)])/2))))/3))/4
```

3.147. $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$

3.147.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

3.147. $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.147.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(196) = 392$.

Time = 14.52 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.24

method	result
derivativedivides	$-c^4\left(-\frac{a^3x^4}{4c^4}+b^3\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^3}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^3}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)^2}{4c^3}+\frac{3x\arctan\left(\frac{c}{x}\right)^2}{4c}+\arctan\left(\frac{c}{x}\right)\ln\right.\right.$
default	$-c^4\left(-\frac{a^3x^4}{4c^4}+b^3\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^3}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^3}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)^2}{4c^3}+\frac{3x\arctan\left(\frac{c}{x}\right)^2}{4c}+\arctan\left(\frac{c}{x}\right)\ln\right.\right.$
parts	$\frac{a^2bc\,x^3}{4}+\frac{a^3x^4}{4}+\frac{b^3c^3x}{4}-\frac{c^4b^3\arctan\left(\frac{x}{c}\right)}{4}-ic^4b^3\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)-\frac{ic^4b^3\ln\left(\frac{c}{x}-i\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+ic$
risch	Expression too large to display

input
```
int(x^3*(a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

output
```
-c^4*(-1/4*a^3/c^4*x^4+b^3*(-1/4/c^4*x^4*arctan(c/x)^3+1/4*arctan(c/x)^3-1
/4/c^3*x^3*arctan(c/x)^2+3/4/c*x*arctan(c/x)^2+arctan(c/x)*ln(1+c^2/x^2)-1
/4/c^2*x^2*arctan(c/x)-2*ln(c/x)*arctan(c/x)-1/4*arctan(c/x)-1/4*x/c-I*ln(
c/x)*ln(1+I*c/x)+I*ln(c/x)*ln(1-I*c/x)-I*dilog(1+I*c/x)+I*dilog(1-I*c/x)+1
/2*I*(ln(c/x-I)*ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c/x+I))-ln(c/x
-I)*ln(-1/2*I*(c/x+I)))-1/2*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln(c/x+I)^2-dil
og(1/2*I*(c/x-I))-ln(c/x+I)*ln(1/2*I*(c/x-I))))+3*a*b^2*(-1/4/c^4*x^4*arct
an(c/x)^2+1/4*arctan(c/x)^2-1/6/c^3*x^3*arctan(c/x)+1/2/c*x*arctan(c/x)+1/
3*ln(1+c^2/x^2)-1/12/c^2*x^2-2/3*ln(c/x))+3*a^2*b*(-1/4/c^4*x^4*arctan(c/x
)-1/12/c^3*x^3+1/4*x/c+1/4*arctan(c/x)))
```

3.147.5 Fricas [F]

$$\int x^3 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx=\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3 x^3\, dx$$

input
```
integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arctan(c/x)^3 + 3*a*b^2*x^3*arctan(c/x)^2 + 3*a^2*b*x^3*a
rctan(c/x) + a^3*x^3, x)
```

3.147.6 Sympy [F]

$$\int x^3 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx=\int x^3 \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3 dx$$

input
```
integrate(x**3*(a+b*atan(c/x))**3,x)
```

output
```
Integral(x**3*(a + b*atan(c/x))**3, x)
```

3.147.7 Maxima [F]

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

output
```
3/4*a*b^2*x^4*arctan(c/x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arctan(c/x) + (3*c^
3*arctan(x/c) - 3*c^2*x + x^3)*c)*a^2*b + 1/4*((3*c^2*arctan(x/c)^2 - 4*c^
2*log(c^2 + x^2) + x^2)*c^2 + 2*(3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c*arct
an(c/x))*a*b^2 - 1/64*(12*c^4*arctan(c/x)^2*arctan(x/c) + 8*c^4*arctan2(c,
 x)^3 - 8*x^4*arctan2(c, x)^3 + 4*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(
x/c)^3/c)*c^5 + 12*c^3*x*arctan2(c, x)^2 - 4*c*x^3*arctan2(c, x)^2 + 192*c
^5*integrate(1/64*log(c^2 + x^2)^2/(c^2 + x^2), x) + 1536*c^4*integrate(1/
64*x*arctan(c/x)/(c^2 + x^2), x) + 768*c^3*integrate(1/64*x^2*log(c^2 + x^
2)/(c^2 + x^2), x) - 2048*c^2*integrate(1/64*x^3*arctan(c/x)^3/(c^2 + x^2)
, x) - 512*c^2*integrate(1/64*x^3*arctan(c/x)/(c^2 + x^2), x) - (3*c^3*x -
 c*x^3)*log(c^2 + x^2)^2 - 768*c*integrate(1/64*x^4*arctan(c/x)^2/(c^2 + x
^2), x) - 192*c*integrate(1/64*x^4*log(c^2 + x^2)^2/(c^2 + x^2), x) - 256*
c*integrate(1/64*x^4*log(c^2 + x^2)/(c^2 + x^2), x) - 2048*integrate(1/64*
x^5*arctan(c/x)^3/(c^2 + x^2), x))*b^3
```

3.147.8 Giac [F]

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3*x^3, x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x^3\,\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
int(x^3*(a + b*atan(c/x))^3,x)
```

output
```
int(x^3*(a + b*atan(c/x))^3, x)
```

3.148 $\int x^2 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 \, dx$

3.148.1 Optimal result . . . 954
3.148.2 Mathematica [A] (verified) . . . 955
3.148.3 Rubi [A] (verified) . . . 956
3.148.4 Maple [C] (warning: unable to verify) . . . 960
3.148.5 Fricas [F] . . . 961
3.148.6 Sympy [F] . . . 961
3.148.7 Maxima [F] . . . 961
3.148.8 Giac [F] . . . 962
3.148.9 Mupad [F(-1)] . . . 962

3.148.1 Optimal result

Integrand size = 16, antiderivative size = 229

$$\begin{aligned}\int x^2 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 \, dx = {} & b^2c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\ & +\frac{1}{2}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\ & -\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 \\ & +bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(2-\frac{2}{1-\frac{ic}{x}}\right) \\ & +\frac{1}{2}b^3c^3\log\left(1+\frac{c^2}{x^2}\right)+b^3c^3\log(x) \\ & -ib^2c^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right) \\ & +\frac{1}{2}b^3c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-\frac{ic}{x}}\right)\end{aligned}$$

output
```
b^2*c^2*x*(a+b*arccot(x/c))+1/2*b*c^3*(a+b*arccot(x/c))^2+1/2*b*c*x^2*(a+b
*arccot(x/c))^2-1/3*I*c^3*(a+b*arccot(x/c))^3+1/3*x^3*(a+b*arccot(x/c))^3+
b*c^3*(a+b*arccot(x/c))^2*ln(2-2/(1-I*c/x))+1/2*b^3*c^3*ln(1+c^2/x^2)+b^3*
c^3*ln(x)-I*b^2*c^3*(a+b*arccot(x/c))*polylog(2,-1+2/(1-I*c/x))+1/2*b^3*c^
3*polylog(3,-1+2/(1-I*c/x))
```

3.148.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.47

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx=\frac{1}{6}\left(3a^2bcx^2+2a^3x^3+6a^2bx^3\arctan\left(\frac{c}{x}\right)-3a^2bc^3\log\left(c^2+x^2\right)\right.$$
$$+6ab^2\left(c^2x+\left(-ic^3+x^3\right)\arctan\left(\frac{c}{x}\right)^2\right.$$
$$+c\arctan\left(\frac{c}{x}\right)\left(c^2+x^2+2c^2\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$
$$\left.-ic^3\operatorname{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$
$$+\frac{1}{4}b^3\left(-ic^3\pi^3+24c^2x\arctan\left(\frac{c}{x}\right)+12c^3\arctan\left(\frac{c}{x}\right)^2\right.$$
$$+12cx^2\arctan\left(\frac{c}{x}\right)^2+8ic^3\arctan\left(\frac{c}{x}\right)^3+8x^3\arctan\left(\frac{c}{x}\right)^3$$
$$+24c^3\arctan\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right)$$
$$-24c^3\log\left(\frac{1}{\sqrt{1+\frac{c^2}{x^2}}}\right)-24c^3\log\left(\frac{c}{x}\right)$$
$$+24ic^3\arctan\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)$$
$$\left.\left.+12c^3\operatorname{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\right)\right)$$

input
```
Integrate[x^2*(a + b*ArcTan[c/x])^3,x]
```

output
```
(3*a^2*b*c*x^2 + 2*a^3*x^3 + 6*a^2*b*x^3*ArcTan[c/x] - 3*a^2*b*c^3*Log[c^2
 + x^2] + 6*a*b^2*(c^2*x + ((-I)*c^3 + x^3)*ArcTan[c/x]^2 + c*ArcTan[c/x]*
(c^2 + x^2 + 2*c^2*Log[1 - E^((2*I)*ArcTan[c/x])]) - I*c^3*PolyLog[2, E^((
2*I)*ArcTan[c/x])]) + (b^3*((-I)*c^3*Pi^3 + 24*c^2*x*ArcTan[c/x] + 12*c^3*
ArcTan[c/x]^2 + 12*c*x^2*ArcTan[c/x]^2 + (8*I)*c^3*ArcTan[c/x]^3 + 8*x^3*A
rcTan[c/x]^3 + 24*c^3*ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcTan[c/x])] - 24*c
^3*Log[1/Sqrt[1 + c^2/x^2]] - 24*c^3*Log[c/x] + (24*I)*c^3*ArcTan[c/x]*Pol
yLog[2, E^((-2*I)*ArcTan[c/x])] + 12*c^3*PolyLog[3, E^((-2*I)*ArcTan[c/x])
]))/4)/6
```

3.148.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.938, Rules used = {5363, 5361, 5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3dx$$

$$\Big\downarrow \text{ 5363}$$

$$-\int x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3d\frac{1}{x}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-bc\int\frac{x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-bc\left(\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}-c^2\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3- bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\int\frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3- bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)d\frac{1}{x}-c^2\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-\frac{1}{2}x^2\left(a\right.\right.$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3- bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\int\frac{x}{\frac{c^2}{x^2}+1}d\frac{1}{x}-x\left(a+b\arctan\right.\right.\right.$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\int\frac{x}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}-x\left(a+b\arcta\right.\right.\right.$$

$\downarrow$ 47

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(\int xd\frac{1}{x^2}-c^2\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}\right)\right.\right.$$

$\downarrow$ 14

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-c^2\int\frac{1}{\frac{c^2}{x^2}+1}d\frac{1}{x^2}\right.\right.\right.$$

$\downarrow$ 16

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(c^2\left(-\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\right.\right.\right.\right.$$

$\downarrow$ 5419

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+bc\left(-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\right.\right.\right.$$

$\downarrow$ 5459

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(-\left(c^2\left(i\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c}{x}+i}d\frac{1}{x}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3b}\right)\right)+bc\left(-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}-x\left(a+b\arctan\right.\right.\right.$$

$\downarrow$ 5403

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(-\left(c^2\left(i\left(2ibc\int\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\right)-\frac{i(a+\right.\right.\right.\right.$$

$\downarrow$ 5527

3.148. $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3- bc\left(-\left(c^2\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}-\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-i\log\left(2\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3- bc\left(-\left(c^2\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}-\frac{b\,\text{PolyLog}\left(3,\frac{2}{1-\frac{ic}{x}}-1\right)}{4c}\right)-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right.\right.\right.\right.\right.$$

input
```
Int[x^2*(a + b*ArcTan[c/x])^3,x]
```

output
```
(x^3*(a + b*ArcTan[c/x])^3)/3 - b*c*(-1/2*(x^2*(a + b*ArcTan[c/x])^2) + b*
c*(-(x*(a + b*ArcTan[c/x])) - (c*(a + b*ArcTan[c/x])^2)/(2*b) + (b*c*(-Log
[1 + c^2/x^2] + Log[x^(-2)]))/2) - c^2*(((-1/3*I)*(a + b*ArcTan[c/x])^3)/b
 + I*((-I)*(a + b*ArcTan[c/x])^2*Log[2 - 2/(1 - (I*c)/x)] + (2*I)*b*c*(((I
/2)*(a + b*ArcTan[c/x])*PolyLog[2, -1 + 2/(1 - (I*c)/x)])/c - (b*PolyLog[3
, -1 + 2/(1 - (I*c)/x)])/(4*c)))))
```

3.148.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

3.148. $\int x^2 \left(a + b \arctan\left(\frac{c}{x}\right)\right)^3 \, dx$

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.148.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.89 (sec) , antiderivative size = 2634, normalized size of antiderivative = 11.50

Expression too large to display

input
```
int(x^2*(a+b*arctan(c/x))^3,x)
```

output
```
1/4*I*c^3*b^3*arctan(c/x)^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)^2)*csgn(I*(
(1+I*c/x)^2/(1+c^2/x^2)+1))^2*Pi-1/2*I*c^3*b^3*arctan(c/x)^2*csgn(I*((1+I*
c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^
2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*Pi+1/2*I*c^3*b^3*arctan(c/x)^2*cs
gn(I*(1+I*c/x)/(1+c^2/x^2)^(1/2))*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))^2*Pi+1/2
*I*c^3*b^3*arctan(c/x)^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(
1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1
))*Pi+1/4*I*c^3*b^3*arctan(c/x)^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/((1+I*c/x
)^2/(1+c^2/x^2)+1)^2)^2*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)*Pi-1/2*I*c^3
*b^3*arctan(c/x)^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/
x^2)+1))^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*Pi+a*b^2*c^2*x+1/3*a^3*x^3-
1/2*I*c^3*b^3*arctan(c/x)^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^
2/(1+c^2/x^2)+1))^2*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*Pi-1/4*I*c^3*b^3*a
rctan(c/x)^2*csgn(I*(1+I*c/x)/(1+c^2/x^2)^(1/2))^2*csgn(I*(1+I*c/x)^2/(1+c
^2/x^2))*Pi+1/2*a^2*b*c*x^2-c^3*a*b^2*arctan(x/c)+c^3*a^2*b*ln(c/x)-1/2*c^
3*a^2*b*ln(1+c^2/x^2)+I*c^3*b^3*arctan(c/x)+c^3*b^3*arctan(c/x)^2*ln(2)+c^
3*b^3*ln(c/x)*arctan(c/x)^2-1/2*c^3*b^3*arctan(c/x)^2*ln(1+c^2/x^2)+c^3*b^
3*arctan(c/x)^2*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))+c^3*b^3*arctan(c/x)^2*ln
((1+I*c/x)/(1+c^2/x^2)^(1/2))-c^3*b^3*arctan(c/x)^2*ln((1+I*c/x)^2/(1+c^2/
x^2)-1)+c^3*b^3*arctan(c/x)^2*ln((1+I*c/x)/(1+c^2/x^2)^(1/2)+1)-1/3*I*c...
```

3.148.5 Fricas [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arctan(c/x)^3 + 3*a*b^2*x^2*arctan(c/x)^2 + 3*a^2*b*x^2*a
rctan(c/x) + a^3*x^2, x)
```

3.148.6 Sympy [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x^2\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
integrate(x**2*(a+b*atan(c/x))**3,x)
```

output
```
Integral(x**2*(a + b*atan(c/x))**3, x)
```

3.148.7 Maxima [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

output
```
1/24*b^3*x^3*arctan2(c, x)^3 - 1/32*b^3*x^3*arctan2(c, x)*log(c^2 + x^2)^2
 + 1/3*a^3*x^3 + 1/2*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*a^
2*b + integrate(1/32*(4*b^3*c*x^3*arctan2(c, x)^2 + 4*b^3*x^4*arctan2(c, x
)*log(c^2 + x^2) + 4*(7*b^3*arctan2(c, x)^3 + 24*a*b^2*arctan2(c, x)^2)*x^
4 + 4*(7*b^3*c^2*arctan2(c, x)^3 + 24*a*b^2*c^2*arctan2(c, x)^2)*x^2 + (3*
b^3*c^2*x^2*arctan2(c, x) + 3*b^3*x^4*arctan2(c, x) - b^3*c*x^3)*log(c^2 +
 x^2)^2)/(c^2 + x^2), x)
```

3.148.8 Giac [F]

$$\int x^2 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3*x^2, x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int x^2 \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3 dx$$

input
```
int(x^2*(a + b*atan(c/x))^3,x)
```

output
```
int(x^2*(a + b*atan(c/x))^3, x)
```

3.149 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$

3.149.1 Optimal result 963
3.149.2 Mathematica [A] (verified) 964
3.149.3 Rubi [A] (verified) 964
3.149.4 Maple [B] (verified) 967
3.149.5 Fricas [F] 968
3.149.6 Sympy [F] 968
3.149.7 Maxima [F] 969
3.149.8 Giac [F] 969
3.149.9 Mupad [F(-1)] 969

3.149.1 Optimal result

Integrand size = 14, antiderivative size = 145

$$\begin{aligned}\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx &= \frac{3}{2}ibc^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{3}{2}bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\ &\quad+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\ &\quad-3b^2c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\\ &\quad+\frac{3}{2}ib^3c^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)\end{aligned}$$

output

```
3/2*I*b*c^2*(a+b*arccot(x/c))^2+3/2*b*c*x*(a+b*arccot(x/c))^2+1/2*c^2*(a+b
*arccot(x/c))^3+1/2*x^2*(a+b*arccot(x/c))^3-3*b^2*c^2*(a+b*arccot(x/c))*ln
(2-2/(1-I*c/x))+3/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c/x))
```

3.149.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.20

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \frac{1}{2}\left(3b^2(bc(ic+x)+a(c^2+x^2))\arctan\left(\frac{c}{x}\right)^2 + b^3(c^2+x^2)\arctan\left(\frac{c}{x}\right)^3 + 3b\arctan\left(\frac{c}{x}\right)\left(a(2bcx+a(c^2+x^2)) - 2b^2c^2\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) + a\left(ax(3bc+ax)-6b^2c^2\log\left(\frac{c}{\sqrt{1+\frac{c^2}{x^2}}x}\right)\right) + 3ib^3c^2\,\text{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[x*(a + b*ArcTan[c/x])^3,x]
```

output
```
(3*b^2*(b*c*(I*c + x) + a*(c^2 + x^2))*ArcTan[c/x]^2 + b^3*(c^2 + x^2)*Arc
Tan[c/x]^3 + 3*b*ArcTan[c/x]*(a*(2*b*c*x + a*(c^2 + x^2)) - 2*b^2*c^2*Log[
1 - E^((2*I)*ArcTan[c/x])]) + a*(a*x*(3*b*c + a*x) - 6*b^2*c^2*Log[c/(Sqrt
[1 + c^2/x^2]*x)]) + (3*I)*b^3*c^2*PolyLog[2, E^((2*I)*ArcTan[c/x])])/2
```

3.149.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5363, 5361, 5453, 5361, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx$$

$$\Big\downarrow \text{ 5363}$$

$$-\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 d\frac{1}{x}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\int\frac{x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}$$

$$\Big\downarrow \text{5453}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\left(\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}-c^2\int\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\left(c^2\left(-\int\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+2bc\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{5419}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\left(2bc\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3b}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{5459}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\left(2bc\left(i\int\frac{x\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{\frac{c}{x}+i}d\frac{1}{x}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right)-\frac{c\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3b}-x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{5403}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\left(2bc\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)\right)-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right)-\frac{c(\ldots$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)\right)-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2b}\right)\right.$$

input
```
Int[x*(a + b*ArcTan[c/x])^3,x]
```

3.149. $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$

output
```
(x^2*(a + b*ArcTan[c/x])^3)/2 - (3*b*c*(-(x*(a + b*ArcTan[c/x])^2) - (c*(a
 + b*ArcTan[c/x])^3)/(3*b) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c/x])^2)/b + I
*((-I)*(a + b*ArcTan[c/x])*Log[2 - 2/(1 - (I*c)/x)] - (b*PolyLog[2, -1 + 2
/(1 - (I*c)/x)])/2))))/2
```

3.149.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

3.149.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(131) = 262$.

Time = 8.16 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.79

method	result
derivativedivides	$-c^2\left(-\frac{a^3x^2}{2c^2}+b^3\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^3}{2c^2}-\frac{3x\arctan\left(\frac{c}{x}\right)^2}{2c}-\frac{\arctan\left(\frac{c}{x}\right)^3}{2}+3\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\frac{3\arct\right.\right.$
default	$-c^2\left(-\frac{a^3x^2}{2c^2}+b^3\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^3}{2c^2}-\frac{3x\arctan\left(\frac{c}{x}\right)^2}{2c}-\frac{\arctan\left(\frac{c}{x}\right)^3}{2}+3\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\frac{3\arct\right.\right.$
parts	$\frac{a^3x^2}{2}-b^3c^2\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^3}{2c^2}-\frac{3x\arctan\left(\frac{c}{x}\right)^2}{2c}-\frac{\arctan\left(\frac{c}{x}\right)^3}{2}+3\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\frac{3\arctan\left(\frac{c}{x}\right)\ln}{2}\right.$
risch	Expression too large to display

input
```
int(x*(a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

output
```
-c^2*(-1/2*a^3/c^2*x^2+b^3*(-1/2/c^2*x^2*arctan(c/x)^3-3/2/c*x*arctan(c/x)
^2-1/2*arctan(c/x)^3+3*ln(c/x)*arctan(c/x)-3/2*arctan(c/x)*ln(1+c^2/x^2)+3
/2*I*ln(c/x)*ln(1+I*c/x)-3/2*I*ln(c/x)*ln(1-I*c/x)+3/2*I*dilog(1+I*c/x)-3/
2*I*dilog(1-I*c/x)-3/4*I*(ln(c/x-I)*ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1
/2*I*(c/x+I))-ln(c/x-I)*ln(-1/2*I*(c/x+I)))+3/4*I*(ln(c/x+I)*ln(1+c^2/x^2)
-1/2*ln(c/x+I)^2-dilog(1/2*I*(c/x-I))-ln(c/x+I)*ln(1/2*I*(c/x-I))))+3*a*b^
2*(-1/2/c^2*x^2*arctan(c/x)^2-1/2*arctan(c/x)^2-1/c*x*arctan(c/x)-1/2*ln(1
+c^2/x^2)+ln(c/x))+3*a^2*b*(-1/2/c^2*x^2*arctan(c/x)-1/2*x/c-1/2*arctan(c/
x)))
```

3.149.5 Fricas [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arctan(c/x)^3 + 3*a*b^2*x*arctan(c/x)^2 + 3*a^2*b*x*arctan(
c/x) + a^3*x, x)
```

3.149.6 Sympy [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
integrate(x*(a+b*atan(c/x))**3,x)
```

output
```
Integral(x*(a + b*atan(c/x))**3, x)
```

3.149.7 Maxima [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3 x\,dx$$

input
```
integrate(x*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

output
```
3/2*a*b^2*x^2*arctan(c/x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arctan(c/x) - (c*arct
an(x/c) - x)*c)*a^2*b - 3/2*((arctan(x/c)^2 - log(c^2 + x^2))*c^2 + 2*(c*a
rctan(x/c) - x)*c*arctan(c/x))*a*b^2 + 1/32*(12*c^2*arctan(c/x)^2*arctan(x
/c) + 8*c^2*arctan2(c, x)^3 + 8*x^2*arctan2(c, x)^3 + 4*(3*arctan(c/x)*arc
tan(x/c)^2/c + arctan(x/c)^3/c)*c^3 + 12*c*x*arctan2(c, x)^2 + 96*c^3*inte
grate(1/32*log(c^2 + x^2)^2/(c^2 + x^2), x) - 3*c*x*log(c^2 + x^2)^2 + 512
*c^2*integrate(1/32*x*arctan(c/x)^3/(c^2 + x^2), x) + 768*c^2*integrate(1/
32*x*arctan(c/x)/(c^2 + x^2), x) + 384*c*integrate(1/32*x^2*arctan(c/x)^2/
(c^2 + x^2), x) + 96*c*integrate(1/32*x^2*log(c^2 + x^2)^2/(c^2 + x^2), x)
 + 384*c*integrate(1/32*x^2*log(c^2 + x^2)/(c^2 + x^2), x) + 512*integrate
(1/32*x^3*arctan(c/x)^3/(c^2 + x^2), x))*b^3
```

3.149.8 Giac [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3 x\,dx$$

input
```
integrate(x*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3*x, x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int x\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3 dx$$

input
```
int(x*(a + b*atan(c/x))^3,x)
```

output
```
int(x*(a + b*atan(c/x))^3, x)
```

3.150 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx$

3.150.1 Optimal result 970
3.150.2 Mathematica [A] (verified) 971
3.150.3 Rubi [A] (verified) 971
3.150.4 Maple [C] (warning: unable to verify) 974
3.150.5 Fricas [F] 975
3.150.6 Sympy [F] 976
3.150.7 Maxima [F] 976
3.150.8 Giac [F] 976
3.150.9 Mupad [F(-1)] 977

3.150.1 Optimal result

Integrand size = 12, antiderivative size = 119

$$\begin{aligned}\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = {} & ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 \\ & -3bc\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2c}{c+ix}\right) \\ & +3ib^2c\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2c}{c+ix}\right) \\ & -\frac{3}{2}b^3c\operatorname{PolyLog}\left(3,1-\frac{2c}{c+ix}\right)\end{aligned}$$

output
```
I*c*(a+b*arccot(x/c))^3+x*(a+b*arccot(x/c))^3-3*b*c*(a+b*arccot(x/c))^2*ln
(2*c/(c+I*x))+3*I*b^2*c*(a+b*arccot(x/c))*polylog(2,1-2*c/(c+I*x))-3/2*b^3
*c*polylog(3,1-2*c/(c+I*x))
```

3.150.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.81

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = a^3x+3a^2bx\arctan\left(\frac{c}{x}\right)+\frac{3}{2}a^2bc\log\left(c^2+x^2\right) - 3ab^2\left(-\left((ic+x)\arctan\left(\frac{c}{x}\right)^2\right) + 2c\arctan\left(\frac{c}{x}\right)\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right) - ic\,\text{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) - \frac{1}{8}b^3\left(-ic\pi^3+8ic\arctan\left(\frac{c}{x}\right)^3-8x\arctan\left(\frac{c}{x}\right)^3 + 24c\arctan\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right) + 24ic\arctan\left(\frac{c}{x}\right)\text{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right) + 12c\,\text{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c/x])^3,x]
```

output
```
a^3*x + 3*a^2*b*x*ArcTan[c/x] + (3*a^2*b*c*Log[c^2 + x^2])/2 - 3*a*b^2*(-(
(I*c + x)*ArcTan[c/x]^2) + 2*c*ArcTan[c/x]*Log[1 - E^((2*I)*ArcTan[c/x])]
- I*c*PolyLog[2, E^((2*I)*ArcTan[c/x])]) - (b^3*((-I)*c*Pi^3 + (8*I)*c*Arc
Tan[c/x]^3 - 8*x*ArcTan[c/x]^3 + 24*c*ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcT
an[c/x])] + (24*I)*c*ArcTan[c/x]*PolyLog[2, E^((-2*I)*ArcTan[c/x])] + 12*c
*PolyLog[3, E^((-2*I)*ArcTan[c/x])]))/8
```

3.150.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5349, 5346, 27, 5456, 27, 5380, 27, 5530, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx$$

$$\Big\downarrow \text{5349}$$

$$\int \left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3dx$$

$$\Big\downarrow \text{5346}$$

$$\frac{3b\int \frac{c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c^2+x^2}dx}{c}+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3$$

$$\Big\downarrow \text{27}$$

$$3bc\int \frac{x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c^2+x^2}dx+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3$$

$$\Big\downarrow \text{5456}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+3bc\left(\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}-\frac{\int \frac{c\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{ic-x}dx}{c}\right)$$

$$\Big\downarrow \text{27}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+3bc\left(\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}-\int \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{ic-x}dx\right)$$

$$\Big\downarrow \text{5380}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+$$
$$3bc\left(-\frac{2b\int \frac{c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c+ix}\right)}{c^2+x^2}dx}{c}+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}-\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\right)$$

$$\Big\downarrow \text{27}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+$$
$$3bc\left(-2bc\int \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c+ix}\right)}{c^2+x^2}dx+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}-\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\right)$$

$$\Big\downarrow \text{5530}$$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+$$
$$3bc\left(-2bc\left(-\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)}{c^2+x^2}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{2c}\right)+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{3b}\right.$$

$$\Big\downarrow \text{7164}$$

3.150. $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$

$$x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+$$
$$3bc\left(-2bc\left(\frac{b\,\text{PolyLog}\left(3,1-\frac{2c}{c+ix}\right)}{4c}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{2c}\right)+\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}-\log\left(\right.\right.$$

input
```
Int[(a + b*ArcTan[c/x])^3,x]
```

output
```
x*(a + b*ArcCot[x/c])^3 + 3*b*c*(((I/3)*(a + b*ArcCot[x/c])^3)/b - (a + b*
ArcCot[x/c])^2*Log[(2*c)/(c + I*x)] - 2*b*c*(((-1/2*I)*(a + b*ArcCot[x/c])
*PolyLog[2, 1 - (2*c)/(c + I*x)])/c + (b*PolyLog[3, 1 - (2*c)/(c + I*x)])/
(4*c)))
```

3.150.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5346
```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Simp[b*c*n*p   Int[x^n*((a + b*ArcCot[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5349
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*A
rcCot[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0]
```

rule 5380
```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Simp[b*c*(
p/e)   Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5456
```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Simp[
1/(c*d)   Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5530
```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Simp[b*p*(I/2)   Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.150.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 102.88 (sec) , antiderivative size = 2028, normalized size of antiderivative = 17.04

method	result	size
parts	Expression too large to display	2028
derivativedivides	Expression too large to display	2031
default	Expression too large to display	2031

input
```
int((a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

output
```
x*a^3-b^3*c*(-1/c*x*arctan(c/x)^3+3*ln(c/x)*arctan(c/x)^2-3/2*arctan(c/x)^
2*ln(1+c^2/x^2)+3*arctan(c/x)^2*ln((1+I*c/x)/(1+c^2/x^2)^(1/2))-3*arctan(c
/x)^2*ln((1+I*c/x)^2/(1+c^2/x^2)-1)-I*arctan(c/x)^3+3/4*(-I*Pi*csgn(I/((1+
I*c/x)^2/(1+c^2/x^2)+1)^2)*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*csgn(I*(1+I*c/x
)^2/(1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)+2*I*Pi*csgn(((1+I*c/x)^2/(1
+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3+2*I*Pi*csgn(I*((1+I*c/x)^2/(1+
c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3+2*I*Pi*csgn(I*(1+I*c/x)/(1+c^2/
x^2)^(1/2))*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))^2-2*I*Pi*csgn(I*((1+I*c/x)^2/(
1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1
)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2-2*I*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)
)*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2-I*Pi*c
sgn(I*(1+I*c/x)/(1+c^2/x^2)^(1/2))^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))+I*Pi*
csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1))^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)^2
)+I*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^3-2*I*Pi*csgn(I*((1+I*c/x)^2/
(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^2-2*I*Pi*csgn(((1+I*
c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2+2*I*Pi*csgn(I*((1+I*c
/x)^2/(1+c^2/x^2)-1))*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x
)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))+2*I*Pi+I*Pi*csgn(I*(1+I*c/
x)^2/(1+c^2/x^2))*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+
1)^2)^2-I*Pi*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))^3+2*I*Pi*csgn(I*((1+I*c/x)...
```

3.150.5 Fricas [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx=\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3 dx$$

input
```
integrate((a+b*arctan(c/x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3, x)
```

3.150.6 Sympy [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
integrate((a+b*atan(c/x))**3,x)
```

output
```
Integral((a + b*atan(c/x))**3, x)
```

3.150.7 Maxima [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3\,dx$$

input
```
integrate((a+b*arctan(c/x))^3,x, algorithm="maxima")
```

output
```
7/8*b^3*c*arctan(c/x)^3*arctan(x/c) + 3*a*b^2*c*arctan(c/x)^2*arctan(x/c)
+ 1/8*b^3*x*arctan2(c, x)^3 - 3/32*b^3*x*arctan2(c, x)*log(c^2 + x^2)^2 +
(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*a*b^2*c^2 + 7/32*(6*arct
an(c/x)^2*arctan(x/c)^2/c + 4*arctan(c/x)*arctan(x/c)^3/c + arctan(x/c)^4/
c)*b^3*c^2 + 3*b^3*c^2*integrate(1/32*arctan(c/x)*log(c^2 + x^2)^2/(c^2 +
x^2), x) + 12*b^3*c*integrate(1/32*x*arctan(c/x)^2/(c^2 + x^2), x) - 3*b^3
*c*integrate(1/32*x*log(c^2 + x^2)^2/(c^2 + x^2), x) + 3/2*(2*x*arctan(c/x
) + c*log(c^2 + x^2))*a^2*b + a^3*x + 28*b^3*integrate(1/32*x^2*arctan(c/x
)^3/(c^2 + x^2), x) + 3*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)^
2/(c^2 + x^2), x) + 96*a*b^2*integrate(1/32*x^2*arctan(c/x)^2/(c^2 + x^2),
 x) + 12*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)/(c^2 + x^2), x)
```

3.150.8 Giac [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3\,dx$$

input
```
integrate((a+b*arctan(c/x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3, x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3 dx$$

input
```
int((a + b*atan(c/x))^3,x)
```

output
```
int((a + b*atan(c/x))^3, x)
```

3.151 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

3.151.1 Optimal result . . . 978
3.151.2 Mathematica [A] (verified) . . . 979
3.151.3 Rubi [A] (verified) . . . 980
3.151.4 Maple [C] (warning: unable to verify) . . . 983
3.151.5 Fricas [F] . . . 984
3.151.6 Sympy [F] . . . 984
3.151.7 Maxima [F] . . . 984
3.151.8 Giac [F] . . . 985
3.151.9 Mupad [F(-1)] . . . 985

3.151.1 Optimal result

Integrand size = 16, antiderivative size = 230

$$\begin{aligned}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx &= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\\&+\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)\\&-\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right)\\&+\frac{3}{2}b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)\\&-\frac{3}{2}b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{ic}{x}}\right)\\&-\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+\frac{ic}{x}}\right)\\&+\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+\frac{ic}{x}}\right)\end{aligned}$$

output
```
2*(a+b*arccot(x/c))^3*arctanh(-1+2/(1+I*c/x))+3/2*I*b*(a+b*arccot(x/c))^2*
polylog(2,1-2/(1+I*c/x))-3/2*I*b*(a+b*arccot(x/c))^2*polylog(2,-1+2/(1+I*c
/x))+3/2*b^2*(a+b*arccot(x/c))*polylog(3,1-2/(1+I*c/x))-3/2*b^2*(a+b*arcco
t(x/c))*polylog(3,-1+2/(1+I*c/x))-3/4*I*b^3*polylog(4,1-2/(1+I*c/x))+3/4*I
*b^3*polylog(4,-1+2/(1+I*c/x))
```

3.151.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx = a^3\log(x)-\frac{3}{2}ia^2b\left(\text{PolyLog}\left(2,-\frac{ic}{x}\right)-\text{PolyLog}\left(2,\frac{ic}{x}\right)\right) + 3ab^2\left(\frac{i\pi^3}{24}-\frac{2}{3}i\arctan\left(\frac{c}{x}\right)^3 - \arctan\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right) + \arctan\left(\frac{c}{x}\right)^2\log\left(1+e^{2i\arctan\left(\frac{c}{x}\right)}\right) - i\arctan\left(\frac{c}{x}\right)\text{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right) - i\arctan\left(\frac{c}{x}\right)\text{PolyLog}\left(2,-e^{2i\arctan\left(\frac{c}{x}\right)}\right) - \frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) + \frac{1}{64}ib^3\left(\pi^4-32\arctan\left(\frac{c}{x}\right)^4 + 64i\arctan\left(\frac{c}{x}\right)^3\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right) - 64i\arctan\left(\frac{c}{x}\right)^3\log\left(1+e^{2i\arctan\left(\frac{c}{x}\right)}\right) - 96\arctan\left(\frac{c}{x}\right)^2\text{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right) - 96\arctan\left(\frac{c}{x}\right)^2\text{PolyLog}\left(2,-e^{2i\arctan\left(\frac{c}{x}\right)}\right) + 96i\arctan\left(\frac{c}{x}\right)\text{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right) - 96i\arctan\left(\frac{c}{x}\right)\text{PolyLog}\left(3,-e^{2i\arctan\left(\frac{c}{x}\right)}\right) + 48\,\text{PolyLog}\left(4,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)+48\,\text{PolyLog}\left(4,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[(a + b*ArcTan[c/x])^3/x,x]
```

output
```
a^3*Log[x] - ((3*I)/2)*a^2*b*(PolyLog[2, ((-I)*c)/x] - PolyLog[2, (I*c)/x]
) + 3*a*b^2*((I/24)*Pi^3 - ((2*I)/3)*ArcTan[c/x]^3 - ArcTan[c/x]^2*Log[1 -
 E^((-2*I)*ArcTan[c/x])] + ArcTan[c/x]^2*Log[1 + E^((2*I)*ArcTan[c/x])] -
I*ArcTan[c/x]*PolyLog[2, E^((-2*I)*ArcTan[c/x])] - I*ArcTan[c/x]*PolyLog[2
, -E^((2*I)*ArcTan[c/x])] - PolyLog[3, E^((-2*I)*ArcTan[c/x])]/2 + PolyLog
[3, -E^((2*I)*ArcTan[c/x])]/2) + (I/64)*b^3*(Pi^4 - 32*ArcTan[c/x]^4 + (64
*I)*ArcTan[c/x]^3*Log[1 - E^((-2*I)*ArcTan[c/x])] - (64*I)*ArcTan[c/x]^3*L
og[1 + E^((2*I)*ArcTan[c/x])] - 96*ArcTan[c/x]^2*PolyLog[2, E^((-2*I)*ArcT
an[c/x])] - 96*ArcTan[c/x]^2*PolyLog[2, -E^((2*I)*ArcTan[c/x])] + (96*I)*A
rcTan[c/x]*PolyLog[3, E^((-2*I)*ArcTan[c/x])] - (96*I)*ArcTan[c/x]*PolyLog
[3, -E^((2*I)*ArcTan[c/x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[c/x])] + 48*P
olyLog[4, -E^((2*I)*ArcTan[c/x])])
```

3.151.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5359, 5357, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$-\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3d\frac{1}{x}$$

$$\Big\downarrow \text{5357}$$

$$6bc\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\text{arctanh}\left(1-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3$$

$$\Big\downarrow \text{5523}$$

$$6bc\left(\frac{1}{2}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\log\left(2-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{1}{2}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)-2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3$$

$$\Big\downarrow \; 5529$$

$$6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2c}-ib\int\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}\right)+\frac{1}{2}\right.$$
$$2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3$$

$$\Big\downarrow \; 5533$$

$$6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2c}-ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}-\frac{1}{2}ib\int\right.\right.\right.$$
$$2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3$$

$$\Big\downarrow \; 7164$$

$$6bc\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2c}-ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}+\frac{b\,\text{PolyI}}{}\right.\right.\right.$$
$$2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3$$

input
```
Int[(a + b*ArcTan[c/x])^3/x,x]
```

output
```
-2*(a + b*ArcTan[c/x])^3*ArcTanh[1 - 2/(1 + (I*c)/x)] + 6*b*c*((((I/2)*(a
+ b*ArcTan[c/x])^2*PolyLog[2, 1 - 2/(1 + (I*c)/x)])/c - I*b*(((I/2)*(a + b
*ArcTan[c/x])*PolyLog[3, 1 - 2/(1 + (I*c)/x)])/c + (b*PolyLog[4, 1 - 2/(1
+ (I*c)/x)])/(4*c)))/2 + (((-1/2*I)*(a + b*ArcTan[c/x])^2*PolyLog[2, -1 +
2/(1 + (I*c)/x)])/c + I*b*(((I/2)*(a + b*ArcTan[c/x])*PolyLog[3, -1 + 2/(1
 + (I*c)/x)])/c + (b*PolyLog[4, -1 + 2/(1 + (I*c)/x)])/(4*c)))/2)
```

3.151. $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

3.151.3.1 Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.151. $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

3.151.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.33 (sec) , antiderivative size = 2225, normalized size of antiderivative = 9.67

method	result	size
parts	Expression too large to display	2225
derivativedivides	Expression too large to display	2226
default	Expression too large to display	2226

input
```
int((a+b*arctan(c/x))^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^3*ln(x)+b^3*(-ln(c/x)*arctan(c/x)^3+arctan(c/x)^3*ln((1+I*c/x)^2/(1+c^2/
x^2)-1)-arctan(c/x)^3*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))+3*I*arctan(c/x)^2*
polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*arctan(c/x)*polylog(3,(1+I*c/x)/(
1+c^2/x^2)^(1/2))-6*I*polylog(4,(1+I*c/x)/(1+c^2/x^2)^(1/2))-arctan(c/x)^3
*ln((1+I*c/x)/(1+c^2/x^2)^(1/2)+1)+3*I*arctan(c/x)^2*polylog(2,-(1+I*c/x)/
(1+c^2/x^2)^(1/2))-6*arctan(c/x)*polylog(3,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-6
*I*polylog(4,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-1/2*I*Pi*(csgn(I*((1+I*c/x)^2/(
1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1
)/((1+I*c/x)^2/(1+c^2/x^2)+1))-csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)
^2/(1+c^2/x^2)+1))^2-csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)
^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2+csgn(I/((1+I*c/x)^2/(1+c^
2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))
*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))+csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((
1+I*c/x)^2/(1+c^2/x^2)+1))^3-csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)
^2/(1+c^2/x^2)+1))^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))-csgn(I*((1+I*c/x)
^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^
2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2+csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+
I*c/x)^2/(1+c^2/x^2)+1))^3+1)*arctan(c/x)^3-3/2*I*arctan(c/x)^2*polylog(2,
-(1+I*c/x)^2/(1+c^2/x^2))+3/2*arctan(c/x)*polylog(3,-(1+I*c/x)^2/(1+c^2/x^
2))+3/4*I*polylog(4,-(1+I*c/x)^2/(1+c^2/x^2)))+3*a^2*b*(-ln(c/x)*arctan...
```

3.151.5 Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x)
+ a^3)/x, x)
```

3.151.6 Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

input
```
integrate((a+b*atan(c/x))**3/x,x)
```

output
```
Integral((a + b*atan(c/x))**3/x, x)
```

3.151.7 Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + 1/32*integrate((28*b^3*arctan2(c, x)^3 + 3*b^3*arctan2(c, x)*
log(c^2 + x^2)^2 + 96*a*b^2*arctan2(c, x)^2 + 96*a^2*b*arctan2(c, x))/x, x
)
```

3.151.8 Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3/x, x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

input
```
int((a + b*atan(c/x))^3/x,x)
```

output
```
int((a + b*atan(c/x))^3/x, x)
```

3.152 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

3.152.1 Optimal result 986
3.152.2 Mathematica [A] (verified) 987
3.152.3 Rubi [A] (verified) 987
3.152.4 Maple [B] (verified) 990
3.152.5 Fricas [F] 990
3.152.6 Sympy [F] 991
3.152.7 Maxima [F] 991
3.152.8 Giac [F] 992
3.152.9 Mupad [F(-1)] 992

3.152.1 Optimal result

Integrand size = 16, antiderivative size = 136

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{c}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x} - \frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c} - \frac{3ib^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{c} - \frac{3b^3\,\text{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)}{2c}$$

output
```
-I*(a+b*arccot(x/c))^3/c-(a+b*arccot(x/c))^3/x-3*b*(a+b*arccot(x/c))^2*ln(
2/(1+I*c/x))/c-3*I*b^2*(a+b*arccot(x/c))*polylog(2,1-2/(1+I*c/x))/c-3/2*b^
3*polylog(3,1-2/(1+I*c/x))/c
```

3.152.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.63

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx =$$
$$-\frac{2a^3c+6a^2bc\arctan\left(\frac{c}{x}\right)+6ab^2c\arctan\left(\frac{c}{x}\right)^2-6iab^2x\arctan\left(\frac{c}{x}\right)^2+2b^3c\arctan\left(\frac{c}{x}\right)^3-2ib^3x\arctan\left(\frac{c}{x}\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c/x])^3/x^2,x]
```

output
```
-1/2*(2*a^3*c + 6*a^2*b*c*ArcTan[c/x] + 6*a*b^2*c*ArcTan[c/x]^2 - (6*I)*a*
b^2*x*ArcTan[c/x]^2 + 2*b^3*c*ArcTan[c/x]^3 - (2*I)*b^3*x*ArcTan[c/x]^3 +
12*a*b^2*x*ArcTan[c/x]*Log[1 + E^((2*I)*ArcTan[c/x])] + 6*b^3*x*ArcTan[c/x
]^2*Log[1 + E^((2*I)*ArcTan[c/x])] - 3*a^2*b*x*Log[1 + c^2/x^2] - (6*I)*b^
2*x*(a + b*ArcTan[c/x])*PolyLog[2, -E^((2*I)*ArcTan[c/x])] + 3*b^3*x*PolyL
og[3, -E^((2*I)*ArcTan[c/x])])/(c*x)
```

3.152.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5363, 5345, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$
$$\Big\downarrow \text{5363}$$
$$-\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3d\frac{1}{x}$$
$$\Big\downarrow \text{5345}$$
$$3bc\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\left(\frac{c^2}{x^2}+1\right)x}d\frac{1}{x}-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}$$
$$\Big\downarrow \text{5455}$$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}+3bc\left(-\frac{\int\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{i-\frac{c}{x}}d\frac{1}{x}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^2}\right)$$

$\Big\downarrow$ 5379

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}+3bc\left(-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{c}-2b\int\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^2}\right)$$

$\Big\downarrow$ 5529

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}+3bc\left(-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{c}-2b\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}\right)}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^2}\right)$$

$\Big\downarrow$ 7164

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}+3bc\left(-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^2}-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{c}-2b\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2c}-\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{\frac{ic}{x}+1}\right)}{4c}\right)}{c}\right)$$

input
```
Int[(a + b*ArcTan[c/x])^3/x^2,x]
```

output
```
-((a + b*ArcTan[c/x])^3/x) + 3*b*c*(((-1/3*I)*(a + b*ArcTan[c/x])^3)/(b*c^
2) - (((a + b*ArcTan[c/x])^2*Log[2/(1 + (I*c)/x)])/c - 2*b*(((-1/2*I)*(a +
 b*ArcTan[c/x])*PolyLog[2, 1 - 2/(1 + (I*c)/x)])/c - (b*PolyLog[3, 1 - 2/(
1 + (I*c)/x)])/(4*c)))/c)
```

3.152. $\int\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

3.152.3.1 Defintions of rubi rules used

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.152.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(129) = 258.

Time = 9.31 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.02

method	result
derivativedivides	$-\frac{\frac{c a^3}{x}+b^3\left(\arctan\left(\frac{c}{x}\right)^3\left(\frac{c}{x}+i\right)-2i\arctan\left(\frac{c}{x}\right)^3+3\arctan\left(\frac{c}{x}\right)^2\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)-3i\arctan\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)\right.}{}$
default	$-\frac{\frac{c a^3}{x}+b^3\left(\arctan\left(\frac{c}{x}\right)^3\left(\frac{c}{x}+i\right)-2i\arctan\left(\frac{c}{x}\right)^3+3\arctan\left(\frac{c}{x}\right)^2\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)-3i\arctan\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)\right.}{}$
parts	$-\frac{a^3}{x}-\frac{b^3\left(\arctan\left(\frac{c}{x}\right)^3\left(\frac{c}{x}+i\right)-2i\arctan\left(\frac{c}{x}\right)^3+3\arctan\left(\frac{c}{x}\right)^2\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)-3i\arctan\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right.\right.}{c}$

input
```
int((a+b*arctan(c/x))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c*(c/x*a^3+b^3*(arctan(c/x)^3*(c/x+I)-2*I*arctan(c/x)^3+3*arctan(c/x)^2
*ln((1+I*c/x)^2/(1+c^2/x^2)+1)-3*I*arctan(c/x)*polylog(2,-(1+I*c/x)^2/(1+c
^2/x^2))+3/2*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2)))+3*a*b^2*(arctan(c/x)^2*(
c/x+I)+2*arctan(c/x)*ln((1+I*c/x)^2/(1+c^2/x^2)+1)-2*I*arctan(c/x)^2-I*pol
ylog(2,-(1+I*c/x)^2/(1+c^2/x^2)))+3*a^2*b*(c/x*arctan(c/x)-1/2*ln(1+c^2/x^
2)))
```

3.152.5 Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x)
+ a^3)/x^2, x)
```

3.152.6 Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*atan(c/x))**3/x**2,x)
```

output
```
Integral((a + b*atan(c/x))**3/x**2, x)
```

3.152.7 Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="maxima")
```

output
```
-3/2*a^2*b*(2*c*arctan(c/x)/x - log(c^2/x^2 + 1))/c - a^3/x - 1/32*(4*b^3*
arctan2(c, x)^3 - 3*b^3*arctan2(c, x)*log(c^2 + x^2)^2 - (28*b^3*arctan(c/
x)^3*arctan(x/c)/c + 896*b^3*c^2*integrate(1/32*arctan(c/x)^3/(c^2*x^2 + x
^4), x) + 96*b^3*c^2*integrate(1/32*arctan(c/x)*log(c^2 + x^2)^2/(c^2*x^2
+ x^4), x) + 3072*a*b^2*c^2*integrate(1/32*arctan(c/x)^2/(c^2*x^2 + x^4),
x) + 96*a*b^2*arctan(c/x)^2*arctan(x/c)/c - 384*b^3*c*integrate(1/32*x*arc
tan(c/x)^2/(c^2*x^2 + x^4), x) + 96*b^3*c*integrate(1/32*x*log(c^2 + x^2)^
2/(c^2*x^2 + x^4), x) + 32*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/
c)*a*b^2 + 7*(6*arctan(c/x)^2*arctan(x/c)^2/c + 4*arctan(c/x)*arctan(x/c)^
3/c + arctan(x/c)^4/c)*b^3 + 96*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2
 + x^2)^2/(c^2*x^2 + x^4), x) - 384*b^3*integrate(1/32*x^2*arctan(c/x)*log
(c^2 + x^2)/(c^2*x^2 + x^4), x))*x)/x
```

3.152.8 Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3/x^2, x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

input
```
int((a + b*atan(c/x))^3/x^2,x)
```

output
```
int((a + b*atan(c/x))^3/x^2, x)
```

3.153 $\int \frac{\left(a+b \arctan\left(\frac{c}{x}\right)\right)^3}{x^3} \, dx$

3.153.1 Optimal result 993
3.153.2 Mathematica [A] (verified) 993
3.153.3 Rubi [A] (verified) 994
3.153.4 Maple [B] (verified) 997
3.153.5 Fricas [F] 998
3.153.6 Sympy [F] 999
3.153.7 Maxima [F] 999
3.153.8 Giac [F] 1000
3.153.9 Mupad [F(-1)] 1000

3.153.1 Optimal result

Integrand size = 16, antiderivative size = 147

$$\begin{aligned}\int \frac{\left(a+b \arctan\left(\frac{c}{x}\right)\right)^3}{x^3} \, dx &= \frac{3ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2} + \frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx} \\ &\quad - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2} \\ &\quad + \frac{3b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c^2} + \frac{3ib^3 \operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{2c^2}\end{aligned}$$

output
```
3/2*I*b*(a+b*arccot(x/c))^2/c^2+3/2*b*(a+b*arccot(x/c))^2/c/x-1/2*(a+b*arc
cot(x/c))^3/c^2-1/2*(a+b*arccot(x/c))^3/x^2+3*b^2*(a+b*arccot(x/c))*ln(2/(
1+I*c/x))/c^2+3/2*I*b^3*polylog(2,1-2/(1+I*c/x))/c^2
```

3.153.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.21

$$\int \frac{\left(a+b \arctan\left(\frac{c}{x}\right)\right)^3}{x^3} \, dx$$

$$= \frac{3b^2(c-ix)(-a(c+ix)+bx)\arctan\left(\frac{c}{x}\right)^2 - b^3(c^2+x^2)\arctan\left(\frac{c}{x}\right)^3 - 3b\arctan\left(\frac{c}{x}\right)\left(a(-2bcx+a(c^2+x\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c/x])^3/x^3,x]
```

output
```
(3*b^2*(c - I*x)*(-(a*(c + I*x)) + b*x)*ArcTan[c/x]^2 - b^3*(c^2 + x^2)*Ar
cTan[c/x]^3 - 3*b*ArcTan[c/x]*(a*(-2*b*c*x + a*(c^2 + x^2)) - 2*b^2*x^2*Lo
g[1 + E^((2*I)*ArcTan[c/x])]] + a*(a*c*(-(a*c) + 3*b*x) + 6*b^2*x^2*Log[1/
Sqrt[1 + c^2/x^2]]) - (3*I)*b^3*x^2*PolyLog[2, -E^((2*I)*ArcTan[c/x])])/(2
*c^2*x^2)
```

3.153.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5363, 5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}dx$$

$$\Big\downarrow \text{5363}$$

$$-\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{5361}$$

$$\frac{3}{2}bc\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\left(\frac{c^2}{x^2}+1\right)x^2}d\frac{1}{x}-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \text{5451}$$

$$\frac{3}{2}bc\left(\frac{\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}}{c^2}-\frac{\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c^2}\right)-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \text{5345}$$

$$\frac{3}{2}bc\left(\frac{\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}-2bc\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{\left(\frac{c^2}{x^2}+1\right)x}d\frac{1}{x}}{c^2}-\frac{\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c^2}\right)-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{3}{2}bc\left(\frac{\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}-2bc\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{\left(\frac{c^2}{x^2}+1\right)x}d\frac{1}{x}}{c^2}-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^3}\right)-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$\downarrow$ 5455

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}+\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^3}+\frac{\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}-2bc\left(-\frac{\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{i-\frac{c}{x}}d\frac{1}{x}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)}{c^2}\right)$$

$\downarrow$ 5379

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}+\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^3}+\frac{\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c}-b\int\frac{\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{\frac{c^2}{x^2}+1}d\frac{1}{x}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)}{c^2}\right)$$

$\downarrow$ 2849

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}+\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^3}+\frac{\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}-2bc\left(-\frac{\frac{ib\int\frac{\log\left(\frac{2}{\frac{ic}{x}+1}\right)}{1-\frac{2}{\frac{ic}{x}+1}}d\frac{1}{\frac{ic}{x}+1}}{c}+\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c}}{c}-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{2bc^2}\right)}{c^2}\right)$$

$\downarrow$ 2752

3.153. $\int\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}dx$

$$-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{2x^2}+$$

$$\frac{3}{2}bc\left(-\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{3bc^3}+\frac{\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}-2bc\left(-\frac{i\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\arctan\left(\frac{c}{x}\right)\right)}{c}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)}{2c}}{c}\right.}{c^2}\right.$$

input
```
Int[(a + b*ArcTan[c/x])^3/x^3,x]
```

output
```
-1/2*(a + b*ArcTan[c/x])^3/x^2 + (3*b*c*(-1/3*(a + b*ArcTan[c/x])^3/(b*c^3
) + ((a + b*ArcTan[c/x])^2/x - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c/x])^2)/(b*
c^2) - (((a + b*ArcTan[c/x])*Log[2/(1 + (I*c)/x)])/c + ((I/2)*b*PolyLog[2,
 1 - 2/(1 + (I*c)/x)])/c)/c))/c^2))/2
```

3.153.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.153. $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$

rule 5363
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
 x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplif
y[(m + 1)/n]]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

3.153.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(133) = 266$.

Time = 14.68 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.18

method	result
derivativedivides	$-\frac{\frac{a^3c^2}{2x^2}+b^3\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^3}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3\arctan\left(\frac{c}{x}\right)^2c}{2x}+\frac{3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\frac{3i\left(\ln\left(\frac{c}{x}-i\right)\ln\left(1+\frac{c^2}{x^2}\right)-\frac{\ln\left(\frac{c}{x}-i\right)^2}{2}-\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{\frac{a^3c^2}{2x^2}+b^3\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^3}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3\arctan\left(\frac{c}{x}\right)^2c}{2x}+\frac{3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\frac{3i\left(\ln\left(\frac{c}{x}-i\right)\ln\left(1+\frac{c^2}{x^2}\right)-\frac{\ln\left(\frac{c}{x}-i\right)^2}{2}-\cdots\right.}{\cdots}\right.}{\cdots}$
parts	$-\frac{a^3}{2x^2}-\frac{b^3\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^3}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3\arctan\left(\frac{c}{x}\right)^2c}{2x}+\frac{3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\frac{3i\left(\ln\left(\frac{c}{x}-i\right)\ln\left(1+\frac{c^2}{x^2}\right)-\frac{\ln\left(\frac{c}{x}-i\right)^2}{2}-\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((a+b*arctan(c/x))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(1/2*a^3*c^2/x^2+b^3*(1/2*c^2/x^2*arctan(c/x)^3+1/2*arctan(c/x)^3-3
/2*arctan(c/x)^2*c/x+3/2*arctan(c/x)*ln(1+c^2/x^2)+3/4*I*(ln(c/x-I)*ln(1+c
^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c/x+I))-ln(c/x-I)*ln(-1/2*I*(c/x+I))
)-3/4*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln(c/x+I)^2-dilog(1/2*I*(c/x-I))-ln(c
/x+I)*ln(1/2*I*(c/x-I))))+3*a*b^2*(1/2*c^2/x^2*arctan(c/x)^2+1/2*arctan(c/
x)^2-c/x*arctan(c/x)+1/2*ln(1+c^2/x^2))+3*a^2*b*(1/2*c^2/x^2*arctan(c/x)-1
/2*c/x+1/2*arctan(c/x)))
```

3.153.5 Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x)
+ a^3)/x^3, x)
```

3.153.6 Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*atan(c/x))**3/x**3,x)
```

output
```
Integral((a + b*atan(c/x))**3/x**3, x)
```

3.153.7 Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="maxima")
```

output
```
3/2*(c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*a^2*b + 3/2*(2*c*(
arctan(x/c)/c^3 + 1/(c^2*x))*arctan(c/x) + (arctan(x/c)^2 - log(c^2 + x^2)
 + 2*log(x))/c^2)*a*b^2 - 3/2*a*b^2*arctan(c/x)^2/x^2 - 1/2*a^3/x^2 + 1/32
*(4*(128*c^3*integrate(1/32*arctan(c/x)^3/(c^3*x^3 + c*x^5), x) - 96*c^2*i
ntegrate(1/32*x*arctan(c/x)^2/(c^3*x^3 + c*x^5), x) - 24*c^2*integrate(1/3
2*x*log(c^2 + x^2)^2/(c^3*x^3 + c*x^5), x) + 128*c*integrate(1/32*x^2*arct
an(c/x)^3/(c^3*x^3 + c*x^5), x) + 192*c*integrate(1/32*x^2*arctan(c/x)/(c^
3*x^3 + c*x^5), x) - 3*arctan(c/x)^2*arctan(x/c)/c^2 - 3*arctan(c/x)*arcta
n(x/c)^2/c^2 - arctan(x/c)^3/c^2 - 24*integrate(1/32*x^3*log(c^2 + x^2)^2/
(c^3*x^3 + c*x^5), x) + 96*integrate(1/32*x^3*log(c^2 + x^2)/(c^3*x^3 + c*
x^5), x))*c^2*x^2 - 8*c^2*arctan2(c, x)^3 - 8*x^2*arctan2(c, x)^3 + 12*c*x
*arctan2(c, x)^2 - 3*c*x*log(c^2 + x^2)^2)*b^3/(c^2*x^2)
```

3.153.8 Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c/x) + a)^3/x^3, x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*atan(c/x))^3/x^3,x)
```

output
```
int((a + b*atan(c/x))^3/x^3, x)
```

3.154 $\int x^2 \arctan(\sqrt{x})\, dx$

3.154.1 Optimal result 1001
3.154.2 Mathematica [A] (verified) 1001
3.154.3 Rubi [A] (verified) 1002
3.154.4 Maple [A] (verified) 1003
3.154.5 Fricas [A] (verification not implemented) 1004
3.154.6 Sympy [A] (verification not implemented) 1004
3.154.7 Maxima [A] (verification not implemented) 1004
3.154.8 Giac [A] (verification not implemented) 1005
3.154.9 Mupad [B] (verification not implemented) 1005

3.154.1 Optimal result

Integrand size = 10, antiderivative size = 51

$$\int x^2 \arctan(\sqrt{x})\, dx = -\frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{\arctan(\sqrt{x})}{3} + \frac{1}{3}x^3 \arctan(\sqrt{x})$$

output
```
1/9*x^(3/2)-1/15*x^(5/2)+1/3*arctan(x^(1/2))+1/3*x^3*arctan(x^(1/2))-1/3*x
^(1/2)
```

3.154.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int x^2 \arctan(\sqrt{x})\, dx = \frac{1}{45}\left(\sqrt{x}(-15 + 5x - 3x^2) + 15(1 + x^3) \arctan(\sqrt{x})\right)$$

input
```
Integrate[x^2*ArcTan[Sqrt[x]],x]
```

output
```
(Sqrt[x]*(-15 + 5*x - 3*x^2) + 15*(1 + x^3)*ArcTan[Sqrt[x]])/45
```

3.154.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5361, 60, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{1}{3}x^3 \arctan\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{5/2}}{x+1}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{6}\left(\int \frac{x^{3/2}}{x+1}dx - \frac{2x^{5/2}}{5}\right) + \frac{1}{3}x^3 \arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{6}\left(-\int \frac{\sqrt{x}}{x+1}dx - \frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3}\right) + \frac{1}{3}x^3 \arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{6}\left(\int \frac{1}{\sqrt{x}(x+1)}dx - \frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3} - 2\sqrt{x}\right) + \frac{1}{3}x^3 \arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{6}\left(2\int \frac{1}{x+1}d\sqrt{x} - \frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3} - 2\sqrt{x}\right) + \frac{1}{3}x^3 \arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{6}\left(2\arctan\left(\sqrt{x}\right) - \frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3} - 2\sqrt{x}\right) + \frac{1}{3}x^3 \arctan\left(\sqrt{x}\right)$$

input
```
Int[x^2*ArcTan[Sqrt[x]],x]
```

output
```
(x^3*ArcTan[Sqrt[x]])/3 + (-2*Sqrt[x] + (2*x^(3/2))/3 - (2*x^(5/2))/5 + 2*
ArcTan[Sqrt[x]])/6
```

3.154.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.154.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

method	result	size
meijerg	$-\frac{\sqrt{x}\,(21x^2-35x+105)}{315}+\frac{(7x^3+7)\arctan(\sqrt{x})}{21}$	30
derivativedivides	$\frac{x^{\frac{3}{2}}}{9}-\frac{x^{\frac{5}{2}}}{15}+\frac{\arctan(\sqrt{x})}{3}+\frac{x^3\arctan(\sqrt{x})}{3}-\frac{\sqrt{x}}{3}$	32
default	$\frac{x^{\frac{3}{2}}}{9}-\frac{x^{\frac{5}{2}}}{15}+\frac{\arctan(\sqrt{x})}{3}+\frac{x^3\arctan(\sqrt{x})}{3}-\frac{\sqrt{x}}{3}$	32
parts	$\frac{x^{\frac{3}{2}}}{9}-\frac{x^{\frac{5}{2}}}{15}+\frac{\arctan(\sqrt{x})}{3}+\frac{x^3\arctan(\sqrt{x})}{3}-\frac{\sqrt{x}}{3}$	32

input
```
int(x^2*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/315*x^(1/2)*(21*x^2-35*x+105)+1/21*(7*x^3+7)*arctan(x^(1/2))
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{3}\left(x^3+1\right)\arctan\left(\sqrt{x}\right) - \frac{1}{45}\left(3\,x^2-5\,x+15\right)\sqrt{x}$$

input
```
integrate(x^2*arctan(x^(1/2)),x, algorithm="fricas")
```

output
```
1/3*(x^3 + 1)*arctan(sqrt(x)) - 1/45*(3*x^2 - 5*x + 15)*sqrt(x)
```

3.154.6 Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = -\frac{x^{\frac{5}{2}}}{15} + \frac{x^{\frac{3}{2}}}{9} - \frac{\sqrt{x}}{3} + \frac{x^3\,\mathrm{atan}\left(\sqrt{x}\right)}{3} + \frac{\mathrm{atan}\left(\sqrt{x}\right)}{3}$$

input
```
integrate(x**2*atan(x**(1/2)),x)
```

output
```
-x**(5/2)/15 + x**(3/2)/9 - sqrt(x)/3 + x**3*atan(sqrt(x))/3 + atan(sqrt(x
))/3
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{3}\,x^3\arctan\left(\sqrt{x}\right) - \frac{1}{15}\,x^{\frac{5}{2}} + \frac{1}{9}\,x^{\frac{3}{2}} - \frac{1}{3}\,\sqrt{x} + \frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x^2*arctan(x^(1/2)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(sqrt(x)) - 1/15*x^(5/2) + 1/9*x^(3/2) - 1/3*sqrt(x) + 1/3*a
rctan(sqrt(x))
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{3}\,x^3 \arctan\left(\sqrt{x}\right) - \frac{1}{15}\,x^{\frac{5}{2}} + \frac{1}{9}\,x^{\frac{3}{2}} - \frac{1}{3}\,\sqrt{x} + \frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x^2*arctan(x^(1/2)),x, algorithm="giac")
```

output
```
1/3*x^3*arctan(sqrt(x)) - 1/15*x^(5/2) + 1/9*x^(3/2) - 1/3*sqrt(x) + 1/3*a
rctan(sqrt(x))
```

3.154.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{\mathrm{atan}\left(\sqrt{x}\right)}{3} + \frac{x^3\,\mathrm{atan}\left(\sqrt{x}\right)}{3} - \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15}$$

input
```
int(x^2*atan(x^(1/2)),x)
```

output
```
atan(x^(1/2))/3 + (x^3*atan(x^(1/2)))/3 - x^(1/2)/3 + x^(3/2)/9 - x^(5/2)/
15
```

3.155 $\int x \arctan\left(\sqrt{x}\right) dx$

3.155.1 Optimal result 1006
3.155.2 Mathematica [A] (verified) 1006
3.155.3 Rubi [A] (verified) 1007
3.155.4 Maple [A] (verified) 1008
3.155.5 Fricas [A] (verification not implemented) 1009
3.155.6 Sympy [A] (verification not implemented) 1009
3.155.7 Maxima [A] (verification not implemented) 1009
3.155.8 Giac [A] (verification not implemented) 1010
3.155.9 Mupad [B] (verification not implemented) 1010

3.155.1 Optimal result

Integrand size = 8, antiderivative size = 42

$$\int x \arctan\left(\sqrt{x}\right) dx = \frac{\sqrt{x}}{2} - \frac{x^{3/2}}{6} - \frac{\arctan\left(\sqrt{x}\right)}{2} + \frac{1}{2}x^2 \arctan\left(\sqrt{x}\right)$$

output
```
-1/6*x^(3/2)-1/2*arctan(x^(1/2))+1/2*x^2*arctan(x^(1/2))+1/2*x^(1/2)
```

3.155.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int x \arctan\left(\sqrt{x}\right) dx = \frac{1}{6}\left(-\left((-3+x)\sqrt{x}\right) + 3\left(-1+x^2\right) \arctan\left(\sqrt{x}\right)\right)$$

input
```
Integrate[x*ArcTan[Sqrt[x]],x]
```

output
```
(-((-3 + x)*Sqrt[x]) + 3*(-1 + x^2)*ArcTan[Sqrt[x]])/6
```

3.155.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5361, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan\left(\sqrt{x}\right)dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)-\frac{1}{4}\int \frac{x^{3/2}}{x+1}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\int \frac{\sqrt{x}}{x+1}dx-\frac{2x^{3/2}}{3}\right)+\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(-\int \frac{1}{\sqrt{x}(x+1)}dx-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)+\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-2\int \frac{1}{x+1}d\sqrt{x}-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)+\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(-2\arctan\left(\sqrt{x}\right)-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)+\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)$$

input
```
Int[x*ArcTan[Sqrt[x]],x]
```

output
```
(2*Sqrt[x] - (2*x^(3/2))/3 - 2*ArcTan[Sqrt[x]])/4 + (x^2*ArcTan[Sqrt[x]])/
2
```

3.155.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.155.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

method	result	size
meijerg	$\frac{\sqrt{x}\,(-5x+15)}{30}-\frac{(-5x^2+5)\arctan(\sqrt{x})}{10}$	25
derivativedivides	$-\frac{x^{\frac{3}{2}}}{6}-\frac{\arctan(\sqrt{x})}{2}+\frac{x^2\arctan(\sqrt{x})}{2}+\frac{\sqrt{x}}{2}$	27
default	$-\frac{x^{\frac{3}{2}}}{6}-\frac{\arctan(\sqrt{x})}{2}+\frac{x^2\arctan(\sqrt{x})}{2}+\frac{\sqrt{x}}{2}$	27
parts	$-\frac{x^{\frac{3}{2}}}{6}-\frac{\arctan(\sqrt{x})}{2}+\frac{x^2\arctan(\sqrt{x})}{2}+\frac{\sqrt{x}}{2}$	27

input
```
int(x*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/30*x^(1/2)*(-5*x+15)-1/10*(-5*x^2+5)*arctan(x^(1/2))
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int x \arctan\left(\sqrt{x}\right) \,dx = \frac{1}{2}\left(x^2-1\right)\arctan\left(\sqrt{x}\right) - \frac{1}{6}(x-3)\sqrt{x}$$

input
```
integrate(x*arctan(x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(x^2 - 1)*arctan(sqrt(x)) - 1/6*(x - 3)*sqrt(x)
```

3.155.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int x \arctan\left(\sqrt{x}\right) \,dx = -\frac{x^{\frac{3}{2}}}{6} + \frac{\sqrt{x}}{2} + \frac{x^2 \operatorname{atan}\left(\sqrt{x}\right)}{2} - \frac{\operatorname{atan}\left(\sqrt{x}\right)}{2}$$

input
```
integrate(x*atan(x**(1/2)),x)
```

output
```
-x**(3/2)/6 + sqrt(x)/2 + x**2*atan(sqrt(x))/2 - atan(sqrt(x))/2
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x \arctan\left(\sqrt{x}\right) \,dx = \frac{1}{2}\,x^2\arctan\left(\sqrt{x}\right) - \frac{1}{6}\,x^{\frac{3}{2}} + \frac{1}{2}\,\sqrt{x} - \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x*arctan(x^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(sqrt(x)) - 1/6*x^(3/2) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x \arctan\left(\sqrt{x}\right) \, dx = \frac{1}{2} \, x^{2} \arctan\left(\sqrt{x}\right) - \frac{1}{6} \, x^{\frac{3}{2}} + \frac{1}{2} \, \sqrt{x} - \frac{1}{2} \, \arctan\left(\sqrt{x}\right)$$

input
```
integrate(x*arctan(x^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2*arctan(sqrt(x)) - 1/6*x^(3/2) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

3.155.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x \arctan\left(\sqrt{x}\right) \, dx = \frac{x^2 \operatorname{atan}\left(\sqrt{x}\right)}{2} - \frac{\operatorname{atan}\left(\sqrt{x}\right)}{2} + \frac{\sqrt{x}}{2} - \frac{x^{3/2}}{6}$$

input
```
int(x*atan(x^(1/2)),x)
```

output
```
(x^2*atan(x^(1/2)))/2 - atan(x^(1/2))/2 + x^(1/2)/2 - x^(3/2)/6
```

3.156 $\int \arctan\left(\sqrt{x}\right) dx$

3.156.1 Optimal result 1011
3.156.2 Mathematica [A] (verified) 1011
3.156.3 Rubi [A] (verified) 1012
3.156.4 Maple [A] (verified) 1013
3.156.5 Fricas [A] (verification not implemented) 1014
3.156.6 Sympy [A] (verification not implemented) 1014
3.156.7 Maxima [A] (verification not implemented) 1014
3.156.8 Giac [A] (verification not implemented) 1015
3.156.9 Mupad [B] (verification not implemented) 1015

3.156.1 Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \arctan\left(\sqrt{x}\right) dx = -\sqrt{x} + \arctan\left(\sqrt{x}\right) + x \arctan\left(\sqrt{x}\right)$$

output
```
arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

3.156.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \arctan\left(\sqrt{x}\right) dx = -\sqrt{x} + (1+x) \arctan\left(\sqrt{x}\right)$$

input
```
Integrate[ArcTan[Sqrt[x]],x]
```

output
```
-Sqrt[x] + (1 + x)*ArcTan[Sqrt[x]]
```

3.156.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5345, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \; 5345$$

$$x\arctan\left(\sqrt{x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{x+1}dx$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{x}(x+1)}dx-2\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(2\int \frac{1}{x+1}d\sqrt{x}-2\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)$$

$$\Big\downarrow \; 216$$

$$x\arctan\left(\sqrt{x}\right)+\frac{1}{2}\left(2\arctan\left(\sqrt{x}\right)-2\sqrt{x}\right)$$

input
```
Int[ArcTan[Sqrt[x]],x]
```

output
```
x*ArcTan[Sqrt[x]] + (-2*Sqrt[x] + 2*ArcTan[Sqrt[x]])/2
```

3.156.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

3.156.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
default	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
parts	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
meijerg	$-\sqrt{x}+\frac{(3x+3)\arctan\left(\sqrt{x}\right)}{3}$	18

input
```
int(arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \arctan\left(\sqrt{x}\right)\, dx = (x+1)\arctan\left(\sqrt{x}\right) - \sqrt{x}$$

input
```
integrate(arctan(x^(1/2)),x, algorithm="fricas")
```

output
```
(x + 1)*arctan(sqrt(x)) - sqrt(x)
```

3.156.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \arctan\left(\sqrt{x}\right)\, dx = -\sqrt{x} + x\,\mathrm{atan}\left(\sqrt{x}\right) + \mathrm{atan}\left(\sqrt{x}\right)$$

input
```
integrate(atan(x**(1/2)),x)
```

output
```
-sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\, dx = x\arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2)),x, algorithm="maxima")
```

output
```
x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = x\arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2)),x, algorithm="giac")
```

output
```
x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

3.156.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = \operatorname{atan}\left(\sqrt{x}\right) + x\operatorname{atan}\left(\sqrt{x}\right) - \sqrt{x}$$

input
```
int(atan(x^(1/2)),x)
```

output
```
atan(x^(1/2)) + x*atan(x^(1/2)) - x^(1/2)
```

3.157 $\int \frac{\arctan(\sqrt{x})}{x} \, dx$

3.157.1 Optimal result 1016
3.157.2 Mathematica [A] (verified) 1016
3.157.3 Rubi [A] (verified) 1017
3.157.4 Maple [A] (verified) 1018
3.157.5 Fricas [F] 1018
3.157.6 Sympy [F] 1019
3.157.7 Maxima [B] (verification not implemented) 1019
3.157.8 Giac [F] 1019
3.157.9 Mupad [B] (verification not implemented) 1020

3.157.1 Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{\arctan(\sqrt{x})}{x} \, dx = i \operatorname{PolyLog}(2, -i\sqrt{x}) - i \operatorname{PolyLog}(2, i\sqrt{x})$$

output
```
I*polylog(2,-I*x^(1/2))-I*polylog(2,I*x^(1/2))
```

3.157.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(\sqrt{x})}{x} \, dx = i \operatorname{PolyLog}(2, -i\sqrt{x}) - i \operatorname{PolyLog}(2, i\sqrt{x})$$

input
```
Integrate[ArcTan[Sqrt[x]]/x,x]
```

output
```
I*PolyLog[2, (-I)*Sqrt[x]] - I*PolyLog[2, I*Sqrt[x]]
```

3.157.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx$$

$$\Big\downarrow \text{5359}$$

$$2\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{5355}$$

$$2\left(\frac{1}{2}i\int \frac{\log\left(1-i\sqrt{x}\right)}{\sqrt{x}}d\sqrt{x}-\frac{1}{2}i\int \frac{\log\left(i\sqrt{x}+1\right)}{\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{2838}$$

$$2\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-i\sqrt{x}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,i\sqrt{x}\right)\right)$$

input
```
Int[ArcTan[Sqrt[x]]/x,x]
```

output
```
2*((I/2)*PolyLog[2, (-I)*Sqrt[x]] - (I/2)*PolyLog[2, I*Sqrt[x]])
```

3.157.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)    Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)    Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

3.157.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result
meijerg	$i\,\text{polylog}\left(2,-i\sqrt{x}\right)-i\,\text{polylog}\left(2,i\sqrt{x}\right)$
derivativedivides	$\ln\left(x\right)\arctan\left(\sqrt{x}\right)+\frac{i\ln(x)\ln(1+i\sqrt{x})}{2}-\frac{i\ln(x)\ln(1-i\sqrt{x})}{2}+i\,\text{dilog}\left(1+i\sqrt{x}\right)-i\,\text{dilog}\left(1-i\right.$
default	$\ln\left(x\right)\arctan\left(\sqrt{x}\right)+\frac{i\ln(x)\ln(1+i\sqrt{x})}{2}-\frac{i\ln(x)\ln(1-i\sqrt{x})}{2}+i\,\text{dilog}\left(1+i\sqrt{x}\right)-i\,\text{dilog}\left(1-i\right.$
parts	$\ln\left(x\right)\arctan\left(\sqrt{x}\right)+\frac{i\ln(x)\ln(1+i\sqrt{x})}{2}-\frac{i\ln(x)\ln(1-i\sqrt{x})}{2}+i\,\text{dilog}\left(1+i\sqrt{x}\right)-i\,\text{dilog}\left(1-i\right.$

input
```
int(arctan(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
I*polylog(2,-I*x^(1/2))-I*polylog(2,I*x^(1/2))
```

3.157.5 Fricas [F]

$$\int\frac{\arctan\left(\sqrt{x}\right)}{x}\,dx=\int\frac{\arctan\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arctan(x^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arctan(sqrt(x))/x, x)
```

3.157.6 Sympy [F]

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{atan}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(atan(x**(1/2))/x,x)
```

output
```
Integral(atan(sqrt(x))/x, x)
```

3.157.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = -\frac{1}{2}\pi\log\left(x+1\right) + \arctan\left(\sqrt{x}\right)\log\left(x\right) - i\operatorname{Li}_2\left(i\sqrt{x}+1\right) + i\operatorname{Li}_2\left(-i\sqrt{x}+1\right)$$

input
```
integrate(arctan(x^(1/2))/x,x, algorithm="maxima")
```

output
```
-1/2*pi*log(x + 1) + arctan(sqrt(x))*log(x) - I*dilog(I*sqrt(x) + 1) + I*d
ilog(-I*sqrt(x) + 1)
```

3.157.8 Giac [F]

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arctan(x^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(arctan(sqrt(x))/x, x)
```

3.157.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = -\mathrm{Li}_2\big(1-\sqrt{x}\,1\mathrm{i}\big)\ 1\mathrm{i} + \mathrm{polylog}\big(2,-\sqrt{x}\,1\mathrm{i}\big)\ 1\mathrm{i}$$

input
```
int(atan(x^(1/2))/x,x)
```

output
```
polylog(2, -x^(1/2)*1i)*1i - dilog(1 - x^(1/2)*1i)*1i
```

3.158 $\int \frac{\arctan(\sqrt{x})}{x^2} dx$

3.158.1 Optimal result 1021
3.158.2 Mathematica [C] (verified) 1021
3.158.3 Rubi [A] (verified) 1022
3.158.4 Maple [A] (verified) 1023
3.158.5 Fricas [A] (verification not implemented) 1024
3.158.6 Sympy [B] (verification not implemented) 1024
3.158.7 Maxima [A] (verification not implemented) 1024
3.158.8 Giac [A] (verification not implemented) 1025
3.158.9 Mupad [B] (verification not implemented) 1025

3.158.1 Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2} dx = -\frac{1}{\sqrt{x}} - \arctan\left(\sqrt{x}\right) - \frac{\arctan\left(\sqrt{x}\right)}{x}$$

output
```
-arctan(x^(1/2))-arctan(x^(1/2))/x-1/x^(1/2)
```

3.158.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2} dx = -\frac{\arctan\left(\sqrt{x}\right)}{x} - \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -x\right)}{\sqrt{x}}$$

input
```
Integrate[ArcTan[Sqrt[x]]/x^2,x]
```

output
```
-(ArcTan[Sqrt[x]]/x) - Hypergeometric2F1[-1/2, 1, 1/2, -x]/Sqrt[x]
```

3.158.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5361, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}dx$$

$$\Big\downarrow \ 5361$$

$$\frac{1}{2}\int \frac{1}{x^{3/2}(x+1)}dx-\frac{\arctan\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \ 61$$

$$\frac{1}{2}\left(-\int \frac{1}{\sqrt{x}(x+1)}dx-\frac{2}{\sqrt{x}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(-2\int \frac{1}{x+1}d\sqrt{x}-\frac{2}{\sqrt{x}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{2}\left(-2\arctan\left(\sqrt{x}\right)-\frac{2}{\sqrt{x}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}$$

input
```
Int[ArcTan[Sqrt[x]]/x^2,x]
```

output
```
(-2/Sqrt[x] - 2*ArcTan[Sqrt[x]])/2 - ArcTan[Sqrt[x]]/x
```

3.158.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.158.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

method	result	size
meijerg	$-\frac{1}{\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)\left(x+1\right)}{x}$	19
derivativedivides	$-\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}$	22
default	$-\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}$	22
parts	$-\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}$	22

input
```
int(arctan(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x^(1/2)-1/x*arctan(x^(1/2))*(x+1)
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{(x+1)\arctan\left(\sqrt{x}\right)+\sqrt{x}}{x}$$

input
```
integrate(arctan(x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-((x + 1)*arctan(sqrt(x)) + sqrt(x))/x
```

3.158.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(22) = 44.

Time = 0.56 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.48

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{x^2}{x^{\frac{5}{2}}+x^{\frac{3}{2}}} - \frac{x}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}$$

input
```
integrate(atan(x**(1/2))/x**2,x)
```

output
```
-x**(5/2)*atan(sqrt(x))/(x**(5/2) + x**(3/2)) - 2*x**(3/2)*atan(sqrt(x))/(
x**(5/2) + x**(3/2)) - sqrt(x)*atan(sqrt(x))/(x**(5/2) + x**(3/2)) - x**2/
(x**(5/2) + x**(3/2)) - x/(x**(5/2) + x**(3/2))
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\arctan\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}} - \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-arctan(sqrt(x))/x - 1/sqrt(x) - arctan(sqrt(x))
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\arctan\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}} - \arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2))/x^2,x, algorithm="giac")
```

output
```
-arctan(sqrt(x))/x - 1/sqrt(x) - arctan(sqrt(x))
```

3.158.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\mathrm{atan}\left(\sqrt{x}\right) - \frac{\mathrm{atan}\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}}$$

input
```
int(atan(x^(1/2))/x^2,x)
```

output
```
- atan(x^(1/2)) - atan(x^(1/2))/x - 1/x^(1/2)
```

3.159 $\int \frac{\arctan(\sqrt{x})}{x^3}\,dx$

3.159.1 Optimal result 1026
3.159.2 Mathematica [C] (verified) 1026
3.159.3 Rubi [A] (verified) 1027
3.159.4 Maple [A] (verified) 1028
3.159.5 Fricas [A] (verification not implemented) 1029
3.159.6 Sympy [B] (verification not implemented) 1029
3.159.7 Maxima [A] (verification not implemented) 1030
3.159.8 Giac [A] (verification not implemented) 1030
3.159.9 Mupad [B] (verification not implemented) 1030

3.159.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{1}{6x^{3/2}}+\frac{1}{2\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}$$

output
```
-1/6/x^(3/2)+1/2*arctan(x^(1/2))-1/2*arctan(x^(1/2))/x^2+1/2/x^(1/2)
```

3.159.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\arctan\left(\sqrt{x}\right)}{2x^2}-\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-x\right)}{6x^{3/2}}$$

input
```
Integrate[ArcTan[Sqrt[x]]/x^3,x]
```

output
```
-1/2*ArcTan[Sqrt[x]]/x^2 - Hypergeometric2F1[-3/2, 1, -1/2, -x]/(6*x^(3/2)
)
```

3.159.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5361, 61, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{4}\int \frac{1}{x^{5/2}(x+1)}dx-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(-\int \frac{1}{x^{3/2}(x+1)}dx-\frac{2}{3x^{3/2}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(\int \frac{1}{\sqrt{x}(x+1)}dx-\frac{2}{3x^{3/2}}+\frac{2}{\sqrt{x}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(2\int \frac{1}{x+1}d\sqrt{x}-\frac{2}{3x^{3/2}}+\frac{2}{\sqrt{x}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(2\arctan\left(\sqrt{x}\right)-\frac{2}{3x^{3/2}}+\frac{2}{\sqrt{x}}\right)-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}$$

input
```
Int[ArcTan[Sqrt[x]]/x^3,x]
```

output
```
-1/2*ArcTan[Sqrt[x]]/x^2 + (-2/(3*x^(3/2)) + 2/Sqrt[x] + 2*ArcTan[Sqrt[x]]
)/4
```

3.159.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.159.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^{2}}+\frac{1}{2\sqrt{x}}$	27
default	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^{2}}+\frac{1}{2\sqrt{x}}$	27
parts	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^{2}}+\frac{1}{2\sqrt{x}}$	27
meijerg	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{1}{2\sqrt{x}}-\frac{4\left(-\frac{3x^{2}}{8}+\frac{3}{8}\right)\arctan\left(\sqrt{x}\right)}{3x^{2}}$	28

input
```
int(arctan(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^(3/2)+1/2*arctan(x^(1/2))-1/2*arctan(x^(1/2))/x^2+1/2/x^(1/2)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\,(x^2-1)\arctan\left(\sqrt{x}\right)+(3\,x-1)\sqrt{x}}{6\,x^2}$$

input
```
integrate(arctan(x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/6*(3*(x^2 - 1)*arctan(sqrt(x)) + (3*x - 1)*sqrt(x))/x^2
```

3.159.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(36) = 72.

Time = 1.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.81

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{3x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{3x^3}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{2x^2}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{x}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}$$

input
```
integrate(atan(x**(1/2))/x**3,x)
```

output
```
3*x**(7/2)*atan(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**(5/2)*atan(sqrt(
x))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**(3/2)*atan(sqrt(x))/(6*x**(7/2) + 6*x
**(5/2)) - 3*sqrt(x)*atan(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**3/(6*x
**(7/2) + 6*x**(5/2)) + 2*x**2/(6*x**(7/2) + 6*x**(5/2)) - x/(6*x**(7/2) +
 6*x**(5/2))
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\,x-1}{6\,x^{\frac{3}{2}}} - \frac{\arctan\left(\sqrt{x}\right)}{2\,x^2} + \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
1/6*(3*x - 1)/x^(3/2) - 1/2*arctan(sqrt(x))/x^2 + 1/2*arctan(sqrt(x))
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\,x-1}{6\,x^{\frac{3}{2}}} - \frac{\arctan\left(\sqrt{x}\right)}{2\,x^2} + \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(arctan(x^(1/2))/x^3,x, algorithm="giac")
```

output
```
1/6*(3*x - 1)/x^(3/2) - 1/2*arctan(sqrt(x))/x^2 + 1/2*arctan(sqrt(x))
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{\mathrm{atan}\left(\sqrt{x}\right)}{2} + \frac{x-\frac{1}{3}}{2\,x^{3/2}} - \frac{\mathrm{atan}\left(\sqrt{x}\right)}{2\,x^2}$$

input
```
int(atan(x^(1/2))/x^3,x)
```

output
```
atan(x^(1/2))/2 + (x - 1/3)/(2*x^(3/2)) - atan(x^(1/2))/(2*x^2)
```

3.160 $\int x^{3/2} \arctan\left(\sqrt{x}\right) dx$

3.160.1 Optimal result 1031
3.160.2 Mathematica [A] (verified) 1031
3.160.3 Rubi [A] (verified) 1032
3.160.4 Maple [A] (verified) 1033
3.160.5 Fricas [A] (verification not implemented) 1033
3.160.6 Sympy [B] (verification not implemented) 1034
3.160.7 Maxima [A] (verification not implemented) 1034
3.160.8 Giac [A] (verification not implemented) 1034
3.160.9 Mupad [B] (verification not implemented) 1035

3.160.1 Optimal result

Integrand size = 12, antiderivative size = 36

$$\int x^{3/2} \arctan\left(\sqrt{x}\right) dx = \frac{x}{5} - \frac{x^2}{10} + \frac{2}{5} x^{5/2} \arctan\left(\sqrt{x}\right) - \frac{1}{5} \log(1+x)$$

output
```
1/5*x-1/10*x^2+2/5*x^(5/2)*arctan(x^(1/2))-1/5*ln(1+x)
```

3.160.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int x^{3/2} \arctan\left(\sqrt{x}\right) dx = \frac{1}{10}\left(-((-2+x)x) + 4x^{5/2} \arctan\left(\sqrt{x}\right) - 2\log(1+x)\right)$$

input
```
Integrate[x^(3/2)*ArcTan[Sqrt[x]],x]
```

output
```
(-((-2 + x)*x) + 4*x^(5/2)*ArcTan[Sqrt[x]] - 2*Log[1 + x])/10
```

3.160.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5361, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} \arctan \left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{5} x^{5/2} \arctan \left(\sqrt{x}\right)-\frac{1}{5} \int \frac{x^2}{x+1}dx$$

$$\Big\downarrow \text{49}$$

$$\frac{2}{5} x^{5/2} \arctan \left(\sqrt{x}\right)-\frac{1}{5} \int \left(x+\frac{1}{x+1}-1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5} x^{5/2} \arctan \left(\sqrt{x}\right)+\frac{1}{5} \left(-\frac{x^2}{2}+x-\log (x+1)\right)$$

input
```
Int[x^(3/2)*ArcTan[Sqrt[x]],x]
```

output
```
(2*x^(5/2)*ArcTan[Sqrt[x]])/5 + (x - x^2/2 - Log[1 + x])/5
```

3.160.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.160.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{x}{5}-\frac{x^{2}}{10}+\frac{2 x^{\frac{5}{2}} \arctan\left(\sqrt{x}\right)}{5}-\frac{\ln\left(x+1\right)}{5}$	25
default	$\frac{x}{5}-\frac{x^{2}}{10}+\frac{2 x^{\frac{5}{2}} \arctan\left(\sqrt{x}\right)}{5}-\frac{\ln\left(x+1\right)}{5}$	25
meijerg	$\frac{x\left(-3x+6\right)}{30}+\frac{2 x^{\frac{5}{2}} \arctan\left(\sqrt{x}\right)}{5}-\frac{\ln\left(x+1\right)}{5}$	25

input
```
int(x^(3/2)*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/5*x-1/10*x^2+2/5*x^(5/2)*arctan(x^(1/2))-1/5*ln(x+1)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}\,x^{2}+\frac{1}{5}\,x-\frac{1}{5}\,\log\left(x+1\right)$$

input
```
integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="fricas")
```

output
```
2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

3.160.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(29) = 58.

Time = 1.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx=\frac{4x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{10x+10}+\frac{4x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{10x+10}-\frac{x^{3}}{10x+10}+\frac{x^{2}}{10x+10}-\frac{2x\log\left(x+1\right)}{10x+10}-\frac{2\log\left(x+1\right)}{10x+10}-\frac{2}{10x+10}$$

input
```
integrate(x**(3/2)*atan(x**(1/2)),x)
```

output
```
4*x**(7/2)*atan(sqrt(x))/(10*x + 10) + 4*x**(5/2)*atan(sqrt(x))/(10*x + 10
) - x**3/(10*x + 10) + x**2/(10*x + 10) - 2*x*log(x + 1)/(10*x + 10) - 2*l
og(x + 1)/(10*x + 10) - 2/(10*x + 10)
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}\,x^{2}+\frac{1}{5}\,x-\frac{1}{5}\,\log\left(x+1\right)$$

input
```
integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="maxima")
```

output
```
2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}\,x^{2}+\frac{1}{5}\,x-\frac{1}{5}\,\log\left(x+1\right)$$

input
```
integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="giac")
```

output
```
2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

3.160.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2} \arctan \left(\sqrt{x}\right) \, dx = \frac{x}{5} - \frac{\ln (x+1)}{5} + \frac{2\, x^{5/2} \operatorname{atan}\left(\sqrt{x}\right)}{5} - \frac{x^2}{10}$$

input
```
int(x^(3/2)*atan(x^(1/2)),x)
```

output
```
x/5 - log(x + 1)/5 + (2*x^(5/2)*atan(x^(1/2)))/5 - x^2/10
```

3.161 $\int \sqrt{x} \arctan\left(\sqrt{x}\right) dx$

3.161.1 Optimal result . . . 1036
3.161.2 Mathematica [A] (verified) . . . 1036
3.161.3 Rubi [A] (verified) . . . 1037
3.161.4 Maple [A] (verified) . . . 1038
3.161.5 Fricas [A] (verification not implemented) . . . 1038
3.161.6 Sympy [A] (verification not implemented) . . . 1039
3.161.7 Maxima [A] (verification not implemented) . . . 1039
3.161.8 Giac [A] (verification not implemented) . . . 1039
3.161.9 Mupad [F(-1)] . . . 1040

3.161.1 Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \sqrt{x} \arctan\left(\sqrt{x}\right) dx = -\frac{x}{3} + \frac{2}{3}x^{3/2} \arctan\left(\sqrt{x}\right) + \frac{1}{3}\log(1+x)$$

output
```
-1/3*x+2/3*x^(3/2)*arctan(x^(1/2))+1/3*ln(1+x)
```

3.161.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \sqrt{x} \arctan\left(\sqrt{x}\right) dx = \frac{1}{3}\left(-x + 2x^{3/2} \arctan\left(\sqrt{x}\right) + \log(1+x)\right)$$

input
```
Integrate[Sqrt[x]*ArcTan[Sqrt[x]],x]
```

output
```
(-x + 2*x^(3/2)*ArcTan[Sqrt[x]] + Log[1 + x])/3
```

3.161.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5361, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x} \arctan\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{3} x^{3/2} \arctan\left(\sqrt{x}\right) - \frac{1}{3} \int \frac{x}{x+1} dx$$

$$\Big\downarrow \text{49}$$

$$\frac{2}{3} x^{3/2} \arctan\left(\sqrt{x}\right) - \frac{1}{3} \int \left(1 + \frac{1}{-x-1}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3} x^{3/2} \arctan\left(\sqrt{x}\right) + \frac{1}{3} (\log(x+1) - x)$$

input
```
Int[Sqrt[x]*ArcTan[Sqrt[x]],x]
```

output
```
(2*x^(3/2)*ArcTan[Sqrt[x]])/3 + (-x + Log[1 + x])/3
```

3.161.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.161.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{x}{3}+\frac{2x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)}{3}+\frac{\ln\left(x+1\right)}{3}$	20
default	$-\frac{x}{3}+\frac{2x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)}{3}+\frac{\ln\left(x+1\right)}{3}$	20
meijerg	$-\frac{x}{3}+\frac{2x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)}{3}+\frac{\ln\left(x+1\right)}{3}$	20

input
```
int(x^(1/2)*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x+2/3*x^(3/2)*arctan(x^(1/2))+1/3*ln(x+1)
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\arctan\left(\sqrt{x}\right)\,dx=\frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x+1\right)$$

input
```
integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="fricas")
```

output
```
2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

3.161.6 Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \sqrt{x} \arctan\left(\sqrt{x}\right)\, dx = \frac{2x^{\frac{3}{2}} \operatorname{atan}\left(\sqrt{x}\right)}{3} - \frac{x}{3} + \frac{\log\left(x+1\right)}{3}$$

input
```
integrate(x**(1/2)*atan(x**(1/2)),x)
```

output
```
2*x**(3/2)*atan(sqrt(x))/3 - x/3 + log(x + 1)/3
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x} \arctan\left(\sqrt{x}\right)\, dx = \frac{2}{3} x^{\frac{3}{2}} \arctan\left(\sqrt{x}\right) - \frac{1}{3} x + \frac{1}{3} \log\left(x+1\right)$$

input
```
integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="maxima")
```

output
```
2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x} \arctan\left(\sqrt{x}\right)\, dx = \frac{2}{3} x^{\frac{3}{2}} \arctan\left(\sqrt{x}\right) - \frac{1}{3} x + \frac{1}{3} \log\left(x+1\right)$$

input
```
integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="giac")
```

output
```
2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\arctan\left(\sqrt{x}\right)\,dx=\int \sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right)\,dx$$

input
```
int(x^(1/2)*atan(x^(1/2)),x)
```

output
```
int(x^(1/2)*atan(x^(1/2)), x)
```

3.162 $\int \frac{\arctan(\sqrt{x})}{\sqrt{x}} dx$

3.162.1 Optimal result 1041
3.162.2 Mathematica [A] (verified) 1041
3.162.3 Rubi [A] (verified) 1042
3.162.4 Maple [A] (verified) 1043
3.162.5 Fricas [A] (verification not implemented) 1043
3.162.6 Sympy [A] (verification not implemented) 1043
3.162.7 Maxima [A] (verification not implemented) 1044
3.162.8 Giac [A] (verification not implemented) 1044
3.162.9 Mupad [B] (verification not implemented) 1044

3.162.1 Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{\arctan(\sqrt{x})}{\sqrt{x}} dx = 2\sqrt{x}\arctan(\sqrt{x}) - \log(1+x)$$

output
```
-ln(1+x)+2*x^(1/2)*arctan(x^(1/2))
```

3.162.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(\sqrt{x})}{\sqrt{x}} dx = 2\sqrt{x}\arctan(\sqrt{x}) - \log(1+x)$$

input
```
Integrate[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

3.162.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5361, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 5361}$$

$$2\sqrt{x}\arctan\left(\sqrt{x}\right)-\int \frac{1}{x+1}dx$$

$$\Big\downarrow \text{ 16}$$

$$2\sqrt{x}\arctan\left(\sqrt{x}\right)-\log(x+1)$$

input
```
Int[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

3.162.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.162.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\ln(x+1)+2\sqrt{x}\,\arctan(\sqrt{x})$	17
default	$-\ln(x+1)+2\sqrt{x}\,\arctan(\sqrt{x})$	17
meijerg	$-\ln(x+1)+2\sqrt{x}\,\arctan(\sqrt{x})$	17

input
```
int(arctan(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(x+1)+2*x^(1/2)*arctan(x^(1/2))
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan(\sqrt{x})}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan(\sqrt{x}) - \log(x+1)$$

input
```
integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

3.162.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\arctan(\sqrt{x})}{\sqrt{x}}\,dx = 2\sqrt{x}\,\mathrm{atan}(\sqrt{x}) - \log(x+1)$$

input
```
integrate(atan(x**(1/2))/x**(1/2),x)
```

output
```
2*sqrt(x)*atan(sqrt(x)) - log(x + 1)
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

input
```
integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

input
```
integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right) - \ln\left(x+1\right)$$

input
```
int(atan(x^(1/2))/x^(1/2),x)
```

output
```
2*x^(1/2)*atan(x^(1/2)) - log(x + 1)
```

3.163 $\int \frac{\arctan(\sqrt{x})}{x^{3/2}}\,dx$

3.163.1 Optimal result 1045
3.163.2 Mathematica [A] (verified) 1045
3.163.3 Rubi [A] (verified) 1046
3.163.4 Maple [A] (verified) 1047
3.163.5 Fricas [A] (verification not implemented) 1047
3.163.6 Sympy [A] (verification not implemented) 1048
3.163.7 Maxima [A] (verification not implemented) 1048
3.163.8 Giac [A] (verification not implemented) 1048
3.163.9 Mupad [B] (verification not implemented) 1049

3.163.1 Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}} + \log(x) - \log(1+x)$$

output
```
ln(x)-ln(1+x)-2*arctan(x^(1/2))/x^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}} + \log(x) - \log(1+x)$$

input
```
Integrate[ArcTan[Sqrt[x]]/x^(3/2),x]
```

output
```
(-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

3.163.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5361, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{5361}$$

$$\int \frac{1}{x(x+1)}dx-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{47}$$

$$\int \frac{1}{x}dx-\int \frac{1}{x+1}dx-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{14}$$

$$-\int \frac{1}{x+1}dx-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}+\log(x)$$

$$\Big\downarrow \text{16}$$

$$-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}+\log(x)-\log(x+1)$$

input
```
Int[ArcTan[Sqrt[x]]/x^(3/2),x]
```

output
```
(-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

3.163.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.163.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\ln(x) - \ln(x+1) - \frac{2\arctan(\sqrt{x})}{\sqrt{x}}$	19
default	$\ln(x) - \ln(x+1) - \frac{2\arctan(\sqrt{x})}{\sqrt{x}}$	19
meijerg	$\ln(x) - \ln(x+1) - \frac{2\arctan(\sqrt{x})}{\sqrt{x}}$	19

input
```
int(arctan(x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-ln(x+1)-2*arctan(x^(1/2))/x^(1/2)
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\arctan(\sqrt{x})}{x^{3/2}}\,dx = -\frac{x\log(x+1) - x\log(x) + 2\sqrt{x}\arctan(\sqrt{x})}{x}$$

input
```
integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="fricas")
```

output
```
-(x*log(x + 1) - x*log(x) + 2*sqrt(x)*arctan(sqrt(x)))/x
```

3.163.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = \log(x) - \log(x+1) - \frac{2\,\mathrm{atan}\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
integrate(atan(x**(1/2))/x**(3/2),x)
```

output
```
log(x) - log(x + 1) - 2*atan(sqrt(x))/sqrt(x)
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\,\arctan\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x+1) + \log(x)$$

input
```
integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="maxima")
```

output
```
-2*arctan(sqrt(x))/sqrt(x) - log(x + 1) + log(x)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\,\arctan\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x+1) + \log(x)$$

input
```
integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(x))/sqrt(x) - log(x + 1) + log(x)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = 2\,\ln\left(\sqrt{x}\right) - \ln\left(x+1\right) - \frac{2\,\mathrm{atan}\left(\sqrt{x}\right)}{\sqrt{x}}$$

input
```
int(atan(x^(1/2))/x^(3/2),x)
```

output
```
2*log(x^(1/2)) - log(x + 1) - (2*atan(x^(1/2)))/x^(1/2)
```

3.164 $\int \frac{\arctan(\sqrt{x})}{x^{5/2}}\,dx$

3.164.1 Optimal result 1050
3.164.2 Mathematica [A] (verified) 1050
3.164.3 Rubi [A] (verified) 1051
3.164.4 Maple [A] (verified) 1052
3.164.5 Fricas [A] (verification not implemented) 1052
3.164.6 Sympy [B] (verification not implemented) 1053
3.164.7 Maxima [A] (verification not implemented) 1053
3.164.8 Giac [A] (verification not implemented) 1053
3.164.9 Mupad [B] (verification not implemented) 1054

3.164.1 Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{1}{3x} - \frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}} - \frac{\log(x)}{3} + \frac{1}{3}\log(1+x)$$

output
```
-1/3/x-2/3*arctan(x^(1/2))/x^(3/2)-1/3*ln(x)+1/3*ln(1+x)
```

3.164.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{1}{3}\left(-\frac{1}{x} - \frac{2\arctan\left(\sqrt{x}\right)}{x^{3/2}} - \log(x) + \log(1+x)\right)$$

input
```
Integrate[ArcTan[Sqrt[x]]/x^(5/2),x]
```

output
```
(-x^(-1) - (2*ArcTan[Sqrt[x]])/x^(3/2) - Log[x] + Log[1 + x])/3
```

3.164.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5361, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{3}\int \frac{1}{x^2(x+1)}dx-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{3}\int \left(\frac{1}{x+1}-\frac{1}{x}+\frac{1}{x^2}\right)dx-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{1}{x}-\log(x)+\log(x+1)\right)-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}}$$

input
```
Int[ArcTan[Sqrt[x]]/x^(5/2),x]
```

output
```
(-2*ArcTan[Sqrt[x]])/(3*x^(3/2)) + (-x^(-1) - Log[x] + Log[1 + x])/3
```

3.164.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

3.164.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{1}{3x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}-\frac{\ln\left(x\right)}{3}+\frac{\ln\left(x+1\right)}{3}$	26
default	$-\frac{1}{3x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}-\frac{\ln\left(x\right)}{3}+\frac{\ln\left(x+1\right)}{3}$	26
meijerg	$-\frac{1}{x}+\frac{2}{9}-\frac{\ln\left(x\right)}{3}+\frac{-10x+30}{45x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}+\frac{\ln\left(x+1\right)}{3}$	37

input
```
int(arctan(x^(1/2))/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x-2/3*arctan(x^(1/2))/x^(3/2)-1/3*ln(x)+1/3*ln(x+1)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx=\frac{x^2\log\left(x+1\right)-x^2\log\left(x\right)-2\sqrt{x}\arctan\left(\sqrt{x}\right)-x}{3\,x^2}$$

input
```
integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="fricas")
```

output
```
1/3*(x^2*log(x + 1) - x^2*log(x) - 2*sqrt(x)*arctan(sqrt(x)) - x)/x^2
```

3.164.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(31) = 62.

Time = 1.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.86

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{3x^3+3x^2} - \frac{2\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{3x^3+3x^2} - \frac{x^3\log\left(x\right)}{3x^3+3x^2} + \frac{x^3\log\left(x+1\right)}{3x^3+3x^2} - \frac{x^2\log\left(x\right)}{3x^3+3x^2} + \frac{x^2\log\left(x+1\right)}{3x^3+3x^2} - \frac{x^2}{3x^3+3x^2} - \frac{x}{3x^3+3x^2}$$

input
```
integrate(atan(x**(1/2))/x**(5/2),x)
```

output
```
-2*x**(3/2)*atan(sqrt(x))/(3*x**3 + 3*x**2) - 2*sqrt(x)*atan(sqrt(x))/(3*x
**3 + 3*x**2) - x**3*log(x)/(3*x**3 + 3*x**2) + x**3*log(x + 1)/(3*x**3 +
3*x**2) - x**2*log(x)/(3*x**3 + 3*x**2) + x**2*log(x + 1)/(3*x**3 + 3*x**2
) - x**2/(3*x**3 + 3*x**2) - x/(3*x**3 + 3*x**2)
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2\arctan\left(\sqrt{x}\right)}{3\,x^{\frac{3}{2}}} - \frac{1}{3\,x} + \frac{1}{3}\log\left(x+1\right) - \frac{1}{3}\log\left(x\right)$$

input
```
integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="maxima")
```

output
```
-2/3*arctan(sqrt(x))/x^(3/2) - 1/3/x + 1/3*log(x + 1) - 1/3*log(x)
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{x-1}{3\,x} - \frac{2\arctan\left(\sqrt{x}\right)}{3\,x^{\frac{3}{2}}} + \frac{1}{3}\log\left(x+1\right) - \frac{1}{3}\log\left(x\right)$$

input
```
integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="giac")
```

output
```
1/3*(x - 1)/x - 2/3*arctan(sqrt(x))/x^(3/2) + 1/3*log(x + 1) - 1/3*log(x)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{\ln\left(x+1\right)}{3} - \frac{2\,\ln\left(\sqrt{x}\right)}{3} - \frac{2\,\mathrm{atan}\left(\sqrt{x}\right)}{3\,x^{3/2}} - \frac{1}{3\,x}$$

input

```
int(atan(x^(1/2))/x^(5/2),x)
```

output

```
log(x + 1)/3 - (2*log(x^(1/2)))/3 - (2*atan(x^(1/2)))/(3*x^(3/2)) - 1/(3*x
)
```

3.165 $\int \frac{\arctan(ax^5)}{x}\,dx$

3.165.1 Optimal result 1055
3.165.2 Mathematica [A] (verified) 1055
3.165.3 Rubi [A] (verified) 1056
3.165.4 Maple [C] (verified) 1057
3.165.5 Fricas [F] 1057
3.165.6 Sympy [F] 1058
3.165.7 Maxima [B] (verification not implemented) 1058
3.165.8 Giac [F] 1058
3.165.9 Mupad [B] (verification not implemented) 1059

3.165.1 Optimal result

Integrand size = 10, antiderivative size = 33

$$\int \frac{\arctan(ax^5)}{x}\,dx = \frac{1}{10} i \operatorname{PolyLog}\left(2,-iax^5\right) - \frac{1}{10} i \operatorname{PolyLog}\left(2,iax^5\right)$$

output
```
1/10*I*polylog(2,-I*a*x^5)-1/10*I*polylog(2,I*a*x^5)
```

3.165.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax^5)}{x}\,dx = \frac{1}{10} i \operatorname{PolyLog}\left(2,-iax^5\right) - \frac{1}{10} i \operatorname{PolyLog}\left(2,iax^5\right)$$

input
```
Integrate[ArcTan[a*x^5]/x,x]
```

output
```
(I/10)*PolyLog[2, (-I)*a*x^5] - (I/10)*PolyLog[2, I*a*x^5]
```

3.165.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax^5)}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{1}{5}\int \frac{\arctan(ax^5)}{x^5}dx^5$$

$$\Big\downarrow \text{5355}$$

$$\frac{1}{5}\left(\frac{1}{2}i\int \frac{\log(1-iax^5)}{x^5}dx^5-\frac{1}{2}i\int \frac{\log(iax^5+1)}{x^5}dx^5\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{5}\left(\frac{1}{2}i\,\text{PolyLog}(2,-iax^5)-\frac{1}{2}i\,\text{PolyLog}(2,iax^5)\right)$$

input
```
Int[ArcTan[a*x^5]/x,x]
```

output
```
((I/2)*PolyLog[2, (-I)*a*x^5] - (I/2)*PolyLog[2, I*a*x^5])/5
```

3.165.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

3.165.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.88 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

method	result
default	$\ln(x)\arctan(a\,x^5)-\frac{\sum_{\_R1=\mathrm{RootOf}\left(a^2\_Z^{10}+1\right)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^5}}{2a}$
parts	$\ln(x)\arctan(a\,x^5)-\frac{\sum_{\_R1=\mathrm{RootOf}\left(a^2\_Z^{10}+1\right)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^5}}{2a}$
meijerg	$-\frac{ia\,x^5\,\mathrm{polylog}\left(2,i\sqrt{a^2x^{10}}\right)}{10\sqrt{a^2x^{10}}}+\frac{ia\,x^5\,\mathrm{polylog}\left(2,-i\sqrt{a^2x^{10}}\right)}{10\sqrt{a^2x^{10}}}$
risch	$\frac{i\ln(x)\ln(-ia\,x^5+1)}{2}-\frac{i\left(\sum_{\_R1=\mathrm{RootOf}\left(a\_Z^5+\mathrm{RootOf}\left(\_Z^2+1,\mathrm{index}=1\right)\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}-\frac{i\ln(x)}{}$

input
```
int(arctan(a*x^5)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arctan(a*x^5)-1/2/a*sum(1/_R1^5*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)
/_R1)),_R1=RootOf(_Z^10*a^2+1))
```

3.165.5 Fricas [F]

$$\int \frac{\arctan(ax^5)}{x}\,dx=\int \frac{\arctan(ax^5)}{x}\,dx$$

input
```
integrate(arctan(a*x^5)/x,x, algorithm="fricas")
```

output
```
integral(arctan(a*x^5)/x, x)
```

3.165.6 Sympy [F]

$$\int \frac{\arctan(ax^5)}{x}\,dx = \int \frac{\operatorname{atan}(ax^5)}{x}\,dx$$

input
```
integrate(atan(a*x**5)/x,x)
```

output
```
Integral(atan(a*x**5)/x, x)
```

3.165.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(19) = 38$.

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{\arctan(ax^5)}{x}\,dx = -\frac{1}{20}\,\pi\log\left(a^2x^{10}+1\right) + \frac{1}{5}\,\arctan\left(ax^5\right)\log\left(ax^5\right) - \frac{1}{10}i\,\mathrm{Li}_2\left(i\,ax^5+1\right) + \frac{1}{10}i\,\mathrm{Li}_2\left(-i\,ax^5+1\right)$$

input
```
integrate(arctan(a*x^5)/x,x, algorithm="maxima")
```

output
```
-1/20*pi*log(a^2*x^10 + 1) + 1/5*arctan(a*x^5)*log(a*x^5) - 1/10*I*dilog(I
*a*x^5 + 1) + 1/10*I*dilog(-I*a*x^5 + 1)
```

3.165.8 Giac [F]

$$\int \frac{\arctan(ax^5)}{x}\,dx = \int \frac{\arctan(ax^5)}{x}\,dx$$

input
```
integrate(arctan(a*x^5)/x,x, algorithm="giac")
```

output
```
integrate(arctan(a*x^5)/x, x)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{\arctan (ax^5)}{x} \, dx = \frac{\operatorname{polylog}(2, -a\, x^5\, 1\mathrm{i})\ 1\mathrm{i}}{10} - \frac{\operatorname{polylog}(2, a\, x^5\, 1\mathrm{i})\ 1\mathrm{i}}{10}$$

input
```
int(atan(a*x^5)/x,x)
```

output
```
(polylog(2, -a*x^5*1i)*1i)/10 - (polylog(2, a*x^5*1i)*1i)/10
```

3.166 $\int \frac{\arctan(ax^n)}{x}\,dx$

3.166.1 Optimal result . . . 1060
3.166.2 Mathematica [A] (verified) . . . 1060
3.166.3 Rubi [A] (verified) . . . 1061
3.166.4 Maple [B] (verified) . . . 1062
3.166.5 Fricas [B] (verification not implemented) . . . 1062
3.166.6 Sympy [F] . . . 1063
3.166.7 Maxima [F] . . . 1063
3.166.8 Giac [F] . . . 1063
3.166.9 Mupad [F(-1)] . . . 1064

3.166.1 Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{\arctan(ax^n)}{x}\,dx = \frac{i\,\text{PolyLog}(2,-iax^n)}{2n} - \frac{i\,\text{PolyLog}(2,iax^n)}{2n}$$

output
```
1/2*I*polylog(2,-I*a*x^n)/n-1/2*I*polylog(2,I*a*x^n)/n
```

3.166.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\arctan(ax^n)}{x}\,dx = \frac{i(\text{PolyLog}(2,-iax^n) - \text{PolyLog}(2,iax^n))}{2n}$$

input
```
Integrate[ArcTan[a*x^n]/x,x]
```

output
```
((I/2)*(PolyLog[2, (-I)*a*x^n] - PolyLog[2, I*a*x^n]))/n
```

3.166.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax^n)}{x} \, dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{\int x^{-n}\arctan(ax^n)\,dx^n}{n}$$

$$\Big\downarrow \text{5355}$$

$$\frac{\frac{1}{2}i\int x^{-n}\log(1-iax^n)\,dx^n-\frac{1}{2}i\int x^{-n}\log(iax^n+1)\,dx^n}{n}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{1}{2}i\,\text{PolyLog}(2,-iax^n)-\frac{1}{2}i\,\text{PolyLog}(2,iax^n)}{n}$$

input
```
Int[ArcTan[a*x^n]/x,x]
```

output
```
((I/2)*PolyLog[2, (-I)*a*x^n] - (I/2)*PolyLog[2, I*a*x^n])/n
```

3.166.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

3.166.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.

Time = 1.90 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

method	result
meijerg	$\frac{-\frac{2ia\,x^{n}\,\mathrm{polylog}\left(2,i\sqrt{a^{2}x^{2n}}\right)}{\sqrt{a^{2}x^{2n}}}+\frac{2ia\,x^{n}\,\mathrm{polylog}\left(2,-i\sqrt{a^{2}x^{2n}}\right)}{\sqrt{a^{2}x^{2n}}}}{4n}$
derivativedivides	$\frac{\ln(a\,x^{n})\arctan(a\,x^{n})+\frac{i\ln(a\,x^{n})\ln(1+ia\,x^{n})}{2}-\frac{i\ln(a\,x^{n})\ln(1-ia\,x^{n})}{2}+\frac{i\,\mathrm{dilog}(1+ia\,x^{n})}{2}-\frac{i\,\mathrm{dilog}(1-ia\,x^{n})}{2}}{n}$
default	$\frac{\ln(a\,x^{n})\arctan(a\,x^{n})+\frac{i\ln(a\,x^{n})\ln(1+ia\,x^{n})}{2}-\frac{i\ln(a\,x^{n})\ln(1-ia\,x^{n})}{2}+\frac{i\,\mathrm{dilog}(1+ia\,x^{n})}{2}-\frac{i\,\mathrm{dilog}(1-ia\,x^{n})}{2}}{n}$
risch	$-\frac{i\ln(x)\ln(1+ia\,x^{n})}{2}-\frac{i\,\mathrm{dilog}(1-ia\,x^{n})}{2n}+\frac{i\ln(-i(-a\,x^{n}+i))\ln(x)}{2}-\frac{i\ln(-i(-a\,x^{n}+i))\ln(-ia\,x^{n})}{2n}-\frac{i\,\mathrm{dilog}(-\cdots}{2n}$

input
```
int(arctan(a*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4/n*(-2*I*a*x^n/(a^2*x^(2*n))^(1/2)*polylog(2,I*(a^2*x^(2*n))^(1/2))+2*I
*a*x^n/(a^2*x^(2*n))^(1/2)*polylog(2,-I*(a^2*x^(2*n))^(1/2)))
```

3.166.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{\arctan(ax^n)}{x}\,dx = \frac{2\,n\arctan(ax^n)\log(x)+i\,n\log(i\,ax^n+1)\log(x)-i\,n\log(-i\,ax^n+1)\log(x)-i\,\mathrm{Li}_2(i\,ax^n)+i\,\mathrm{Li}_2(-i\,a\cdots}{2\,n}$$

input
```
integrate(arctan(a*x^n)/x,x, algorithm="fricas")
```

output
```
1/2*(2*n*arctan(a*x^n)*log(x) + I*n*log(I*a*x^n + 1)*log(x) - I*n*log(-I*a
*x^n + 1)*log(x) - I*dilog(I*a*x^n) + I*dilog(-I*a*x^n))/n
```

3.166.6 Sympy [F]

$$\int \frac{\arctan (ax^n)}{x}\, dx = \int \frac{\operatorname{atan}(ax^n)}{x}\, dx$$

input
```
integrate(atan(a*x**n)/x,x)
```

output
```
Integral(atan(a*x**n)/x, x)
```

3.166.7 Maxima [F]

$$\int \frac{\arctan (ax^n)}{x}\, dx = \int \frac{\arctan (ax^n)}{x}\, dx$$

input
```
integrate(arctan(a*x^n)/x,x, algorithm="maxima")
```

output
```
-a*n*integrate(x^n*log(x)/(a^2*x*x^(2*n) + x), x) + arctan(a*x^n)*log(x)
```

3.166.8 Giac [F]

$$\int \frac{\arctan (ax^n)}{x}\, dx = \int \frac{\arctan (ax^n)}{x}\, dx$$

input
```
integrate(arctan(a*x^n)/x,x, algorithm="giac")
```

output
```
integrate(arctan(a*x^n)/x, x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax^n)}{x}\,dx = \int \frac{\operatorname{atan}(a\,x^n)}{x}\,dx$$

input
```
int(atan(a*x^n)/x,x)
```

output
```
int(atan(a*x^n)/x, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 1065

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
                finalresult={"A"," "}
            ,(*ELSE*)
                finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
            ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
            return 1
        elif isinstance(expn,list):
            return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
        elif  is_sqrt(expn):
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        elif isinstance(expn,Pow):    #type(expn,'`^`')
            if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
                return expnType(expn.args[0])    #ExpnType(op(1,expn))
            elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
                if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                    return 1
                else:
                    return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
            else:
                return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
        elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
            m1 = expnType(expn.args[0])
            m2 = expnType(list(expn.args[1:]))
            return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
        elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
            return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
        elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
            m1 = max(map(expnType, list(expn.args)))
            return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
        elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
            m1 = max(map(expnType, list(expn.args)))
            return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        elif is_appell_function(expn.func):
            m1 = max(map(expnType, list(expn.args)))
            return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        elif isinstance(expn,RootSum):
            m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
            return max(7,m1)
        elif str(expn).find("Integral") != -1:
            m1 = max(map(expnType, list(expn.args)))
            return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
        else:
            return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```